Extension of Partitional Clustering Methods for Handling Mixed Data

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Abstract

Clustering is an active research topic in data mining and different methods have been proposed in the literature. Most of these methods are based on the use of a distance measure defined either on numerical attributes or on categorical attributes. However, in fields such as road traffic and medicine, datasets are composed of numerical and categorical attributes. Recently, there have been several proposals to develop clustering methods that support mixed attributes. There are three basic categories of clustering methods: partitional methods, hierarchical methods and density-based methods. This paper proposes an extension of partitional clustering methods devoted to mixed attributes. The proposed extension looks to create several partitions by using numerical attributes-based clustering methods and then chooses the one that maximizes a measure—called “homogeneity degree”—of these partitions according to categorical attributes.

1. Introduction

In applications such as telecommunications, road traffic, medicine and banking there is often a need to extract data measuring the evolution of some parameters that are useful for decision making. These parameters can be numerical (e.g. phone consumption, flow of cars, consumption of to-

bacco, balances of the customer) or categorical (e.g. day of the week, antecedents of a disease in a family, unemployed). Decision makers (e.g. traffic experts, doctors, or bank managers) have to understand and analyse these data to make decisions. For example, an expert of traffic should understand why there is a congestion in a period of day to avoid this situation, a doctor needs to know the reasons for which a group of people is ill to better deal with the problem. They should find the factors that influence a given behavior.

Clustering is a widely used technique in data mining applications to achieve this goal. Generally, decision makers have a limited information about datasets that they hold. Clustering allows to enrich these datasets and to identify similar or different behaviors between them. Concretely, it allows to partition a set of objects into classes based on defined criteria so that objects in the same class are more similar than objects within different classes [5, 9].

In the second half of 20th century, data mining community has proposed clustering methods that handle objects described only by numerical attributes such as K-means [12], or only by categorical ones such as K-mode [7]. However, as mentioned above, datasets in real-world applications are composed of both types of attributes. Since the late of 1990th, there have been several proposals to develop clustering methods that support mixed attributes (numerical and categorical ones). Most efforts have been oriented towards the extension of conventional clustering algorithms giving a new distance measure that handles mixed attributes. It

is important to note that both numerical and categorical attributes need to be considered during classification. However, we think that numerical attributes are more discriminative than categorical ones since they explain better the differences that may exist between objects. Categorical attributes can be used latter to explain some behaviors. The use of weighted-sum like aggregation rules by combining both numerical and categorical attributes may be not suitable since the definition of weights is not an obvious task.

In this paper, we present a clustering approach that deals with mixed attributes while giving more importance to numerical ones. There are three basic categories of clustering methods: partitional methods, hierarchical methods and density-based methods. The paper proposes an extension to partitional clustering methods devoted to mixed attributes. The extension looks to create several partitions by using numerical attributes-based clustering methods and then chooses the one that maximizes a measure, that we called “homogeneity degree”, of these partitions according to categorical attributes.

This paper is organized as follows. Section 2 discusses some related work. Section 3 details the proposed approach. Section 4 presents an extended mixed-attributes partitional clustering algorithm. Section 5 presents some numerical results and discusses some important points. Section 6 concludes the paper.

2. Related work

2.1. Clustering and clustering approaches

Clustering allows to partition a set of objects into classes based on defined criteria so that objects in the same class are more similar than objects within different classes [5, 9]. More formally, given a set of n objects $X = \{O_1, O_2, \dots, O_n\}$, clustering aims to find a function

$$f : X \rightarrow \{C_1, C_2, \dots, C_k\}$$

that assigns each object $O_i \in X$ to a class C_j ($j = 1, \dots, k$) (k represents the number of classes).

Three main steps are important to cluster objects [9]: (i) *feature selection* where characteristics of objects (the attributes that describe objects) are identified; (ii) *definition of a proximity measure* which is often a distance function that will be used to measure similarity between objects. The choice of a given proximity measure depends largely on the application’s domain. An example of distance function is the Euclidean distance; and (iii) *creation of classes* where a clustering method is first identified and then used to create classes.

The basic categories of clustering methods are [5]:

- *Partitional methods*: these methods aim to decompose objects into k classes. With this approach the number of classes k is generally a predefined parameter. K-means [12], CLARANS [13] and K-mode [7] are examples of partitional clustering methods.
- *Hierarchical methods*: given n objects, this category of methods aims to create different levels of partitions (from 1 to $n - 1$ levels). The levels are built either by merging at each step the two closest classes until obtaining one class or by splitting, at each step, clusters that contain dispersed objects until obtaining n classes or satisfying a given criterion. Examples are: agglomerative methods, divisive methods [10], BIRCH [15] and CURE [4] methods.
- *Density-based methods*: these methods group objects into classes so that the surface or volume of these classes are dense “enough”. Density is computed using pre-defined thresholds. DBSCAN [3] is one of the most used density-based methods. Squeezer [6] is a density-based method devoted to categorical attributes.

There are other categories of clustering methods such as Grid-based methods but they are used only in specific domains like spatial data mining.

2.2. Mixed attributes-based clustering

There are three intuitive ideas to support mixed attributes clustering:

- The conversion of each categorical attribute into a numerical attribute. The clustering methods based on numerical attributes can then be used. However, it is difficult sometimes to code a categorical attribute without understanding the meaning of this attribute [1]. For example, the conversion of an attribute *color* is more difficult than the conversion of an attribute *month*.
- The discretization of numerical attributes so that each numerical attribute is converted into a categorical one by using a discretization method. Then the objects can be clustered by using one of the clustering methods based on categorical attributes. DSqueezer (*Discretizing before Using Squeezer*) [6] method follows this idea. The principal limit of this type of approach is the loss of information generated by discretizing.
- Conversion of the categorical attributes into binary ones: each categorical attribute A_i with d distinct values is converted into d binary attributes where each attribute takes value 1 if the object is described by this attribute, 0 if not. In [14], Ralambondrainy follows this principle to create classes of objects with mixed

attributes. The disadvantage of this approach is that the conversion increases the number of attributes describing the objects. Moreover, with this approach attributes are less meaningful after conversion [2].

More recently, more emphasized methods allowing to cluster objects described by mixed attributes have been proposed in the literature. These methods represent extensions of conventional methods (numerical attributes-based clustering methods or categorical attributes-based clustering methods). Extensions consist in using a measure distance that supports mixed attributes. The three clustering steps mentioned earlier still apply with these extended methods.

K-prototype [8] is one of the first extended methods proposed in the literature. It is a combination of K-means [12] and K-mode [7] methods. The distance measure used with K-prototype is the sum of Euclidean distance, which is used to compute the distance between numerical attributes, and the simple matching measure used to deal with categorical attributes where the distance between two objects is computed as the number of different attributes values between these objects. To build classes, K-prototype follows the same steps as K-means and K-mode.

The authors of [2] propose a clustering method based on the algorithm of agglomerative hierarchical clustering methods and on a distance hierarchy that expresses similarity between attributes' values. Every attribute is represented by a hierarchy. For categorical attributes this hierarchy is a tree where each node represents a possible value of the attribute and every link is valued by a weight representing the distance between these values. Concerning numerical attributes, the hierarchy is represented by only two nodes corresponding to the minimal and maximal values of the attribute. The distance between two objects is the total of distances between values of objects' attributes based on the hierarchy distance. Given this distance measure, the [2]'s method begins by representing each class by one object and then an additional measure is used iteratively to merge the closest classes.

UsmSqueezer (Squeezer for Unified Similarity Measures) [6] is an extension of the categorical attribute-based Squeezer method. The authors propose a similarity measure between an object O and a class C , which is the sum of similarity measure between categorical attributes and similarity measure between numerical attributes. The first one computed on basis of the proportion of objects of C having the same values as the studied object O . The second one is computed on the basis of the difference between values of each numerical attribute A of O and the mean value of the studied attribute for all objects of the class. The clustering algorithm of UsmSqueezer remains the same as Squeezer.

There are several other methods supporting mixed attributes [1, 11]. Most of them rely on the definition of a

new distance measure that combines numerical and categorical attributes. However and as mentioned previously in Section 1, we think that numerical attributes are more discriminative than categorical. The approach that we propose in the next section is an extension to partitional clustering methods. It looks to create several partitions by using numerical attribute-based clustering methods and then chooses the one that maximizes the "homogeneity degree" of these partitions according to categorical attributes.

3. Proposed homogeneity degree-based clustering approach

The basic idea of the proposed approach is to apply a numerical attributes-based clustering method several times on the same datasets while changing some initial parameters such as the number of classes or the similarity thresholds. Then, for every obtained partition, a measure called *homogeneity degree* is computed. The homogeneity degree measures the homogeneity of a partition according to categorical attributes. Finally, the partition that maximizes the homogeneity degree is selected. Algorithm 1 in Section 4 illustrates this idea. The objective of this section is to show how homogeneity degree is computed

The homogeneity degree of a partition depends on the homogeneity degrees of its classes and the homogeneity degrees of attributes domains. Some useful definitions are introduced first.

3.1. Basic definitions

Definition 3.1 (Domain of categorical attribute). *The domain of a categorical attribute A_i is denoted $DOM(A_i) = \{a_{i1}, a_{i2}, \dots, a_{it}\}$ where a_{ij} is a possible value of A_i and t is the number of possible values.*

Definition 3.2 (Hierarchy of categorical attributes). *A hierarchy organizes attributes according to a "finer than" relationship, along with their level of details. Let A_1 and A_2 be two categorical attributes. Then, $A_1 \subseteq_{HA} A_2$ signifies that A_1 is less general (or more finer) than A_2 . If there is not any hierarchical relation between A_1 and A_2 , A_1 and A_2 are said to be non hierarchical. This is denoted by $A_1 \not\subseteq_{HA} A_2$.*

Example 1. *Let month and trimester be two categorical attributes. Let $DOM(month) = \{1, 2, \dots, 12\}$ and $DOM(trimester) = \{1, 2, 3, 4\}$. Then, $month \subseteq_{HA} trimester$.*

Definition 3.3 (Labels of an object). *A label is a set of values of categorical attributes with no hierarchical relationship among them. Let $A_1, A_2, \dots, A_n$ be n categorical attributes. The set of labels L associated with attributes*

$A_1, A_2, \dots, A_n$ is defined as the Cartesian product of the domains of $A_1, A_2, \dots, A_n$: $DOM(A_1) \times DOM(A_2) \times \dots \times DOM(A_n)$. Accordingly, a label $l_i \in L$ is an instance of $DOM(A_1) \times DOM(A_2) \times \dots \times DOM(A_n)$. The notation $Label(O) = l_i$ means that object O is described by label l_i .

Example 2. Let A_1, A_2 and A_3 be three categorical attributes associated respectively with the day number of the week (from 1 to 7), the type of day (Y if it is a holiday and N if not) and the month number (from 1 to 12). Figure 1 provides the set L of labels. The label l_{12} (1 Y 12) for instance corresponds to a Monday of December, which is also a holiday.

$l_1 : 1 Y 1$
 $l_2 : 1 Y 2$
 $\dots$
 $l_{12} : 1 Y 12$
 $l_{13} : 1 N 1$
 $l_{14} : 1 N 2$
 $\dots$
 $l_{24} : 1 N 12$
 $l_{25} : 2 Y 1$
 $l_{26} : 2 Y 2$
 $\dots$
 $\dots$
 $\dots$
 $l_{167} : 7 N 11$
 $l_{168} : 7 N 12$

Figure 1. Example of labels

3.2. Class homogeneity according to categorical attributes

Naturally, a class is homogeneous according to categorical attributes if its objects have the same label(s). However, a crisp definition of the notion of homogeneity may be very restrictive in practice and makes homogeneity largely depending on the application's domain and on the datasets. The homogeneity degree concept as introduced in the rest of this section provides a fuzzy measure of homogeneity. Indeed, homogeneity degree takes its values in the range $[0,1]$, where 1 indicates that the class is fully homogeneous and 0 indicates that the class is not homogeneous. A value between 0 and 1 represents the level to which a class is homogeneous.

The definition of homogeneity degree requires the use of two thresholds $\alpha \in [0,1]$ and $\beta \in [0,1]$ assuring the co-

herence of results. The practical utility of these parameters will be better explained in 5.3.1.

Next, some useful notations are introduced.

- $X = \{O_1, O_2, \dots, O_n\}$: a set of n objects.
- $L = \{l_1, l_2, \dots, l_m\}$: a set of m labels ($m \leq n$).
- $P_k(C_1, C_2, \dots, C_k)$: a partition of k classes.
- $SC_{ij} = \{O_r \in C_i : Label(O_r) = l_j\}$: the set of objects of class C_i having the label l_j .
- $S_j = \{O_r \in X : Label(O_r) = l_j\}$: the set of objects of X having the label l_j .
- $\|\cdot\|$: set cardinality symbol.

3.2.1 Membership degree of a label in respect to a domain

The membership degree $MD_\alpha(l_j, C_i)$ of a label l_j in a class C_i in respect to a domain expresses the proportion of objects of C_i described by label l_j in respect to the total number of objects of X having the label l_j . $MD_\alpha(l_j, C_i)$ is defined as follows:

$$MD_\alpha(l_j, C_i) = \begin{cases} \frac{\|SC_{ij}\|}{\|S_j\|}, & \text{if } \frac{\|SC_{ij}\|}{\|S_j\|} \geq \alpha \text{ and } \|S_j\| \neq 0; \\ 0, & \text{Otherwise} \end{cases} \quad (1)$$

As it is shown in Eq. 1, $MD_\alpha(l_j, C_i)$ will be equal to 0 any time the proportion of objects described by label l_j is strictly less than α . This ensures that C_i contains at least $\alpha\%$ of objects of X .

3.2.2 Homogeneity degree in respect to a domain

The homogeneity degree of a class C_i in respect to a domain, denoted by $HD_\alpha(C_i)$, is defined through Eq. 2:

$$HD_\alpha(C_i) = \begin{cases} \frac{1}{B} \cdot \sum_{j=1}^m MD_\alpha(l_j, C_i), & \text{if } B \neq 0; \\ 0, & \text{Otherwise.} \end{cases} \quad (2)$$

where $B = \|\{l_k : l_k \in L \wedge MD_\alpha(l_k, C_i) > 0\}\|$, that is, the number of labels l_k such that $MD_\alpha(l_k, C_i)$ is strictly positive.

$HD_\alpha(C_i)$ represents the average of labels' membership degrees. Other formula such as the product of $MD_\alpha(l_k, C_i)$ ($k = 1, \dots, m$) or the maximum or minimum value of $MD_\alpha(l_k, C_i)$ ($k = 1, \dots, m$) may also apply here.

3.2.3 Membership degree of a label in respect to a class

The membership degree $MC_\alpha(l_j, C_i)$ of a label l_j in respect to a class C_i reflects the proportion of objects of C_i described by l_j . In other words, $MC_\alpha(l_j, C_i)$ permits to measure the importance of label l_j to class C_j . Formally, $MC_\alpha(l_j, C_i)$ is computed as follows:

$$MC_\alpha(l_j, C_i) = \begin{cases} \frac{\|SC_{ij}\|}{\|C_i\|}, & \text{if } MD_\alpha(l_j, C_i) \geq \alpha; \\ 0, & \text{Otherwise} \end{cases} \quad (3)$$

As it is shown in Eq. 3, only labels for which $MD_\alpha \geq \alpha$ are included in the definition of $MC_\alpha(l_j, C_i)$. This ensures that only the most important labels are considered in the computing of $MC_\alpha(l_j, C_i)$.

3.2.4 Partial homogeneity degree of a class

The partial homogeneity degree $HC_\alpha(C_i)$ of class C_i is given by Eq. 4:

$$HC_{\alpha,\beta}(C_i) = \begin{cases} \sum_{j=1}^m MC_\alpha(l_j, C_i), & \text{if } \sum_{j=1}^m MC_\alpha(l_j, C_i) \geq \beta; \\ 0, & \text{Otherwise} \end{cases} \quad (4)$$

The threshold β used in the definition of $HC_\alpha(C_i)$ ensures that only important labels are considered for computing the partial homogeneity degree of the class.

3.2.5 Overall homogeneity degree of a class

The overall homogeneity degree of a class C_i takes into account the homogeneity degree of C_i in respect to the domain and the partial homogeneity of C_i . It is denoted by $D_{\alpha,\beta}(C_i)$ and computed through Eq. 5 hereafter:

$$D_{\alpha,\beta}(C_i) = HD_\alpha(C_i) \cdot HC_{\alpha,\beta}(C_i) \quad (5)$$

Intuitively, a class C_i is more homogeneous (resp. less homogeneous) than a class C_j if and only if $D_{\alpha,\beta}(C_i) > D_{\alpha,\beta}(C_j)$ (resp. $D_{\alpha,\beta}(C_i) < D_{\alpha,\beta}(C_j)$).

3.3. Partition homogeneity degree according to categorical attributes

Let $P_k(C_1, C_2, \dots, C_k)$ be a partition of k classes. The homogeneity degree $DP_{\alpha,\beta}(P_k)$ of P_k is defined by the Eq. 6 below:

$$DP_{\alpha,\beta}(P_k) = \frac{1}{k} \cdot \sum_{i=1}^k D_{\alpha,\beta}(C_i) \quad (6)$$

It is important to note that other formula may also apply such as:

$$DP_{\alpha,\beta}(P) = \prod_{i=1}^{i=k} D_{\alpha,\beta}(C_i),$$

$$DP_{\alpha,\beta}(P) = \min(D_{\alpha,\beta}(C_1), \dots, D_{\alpha,\beta}(C_k)),$$

or

$$DP_{\alpha,\beta}(P) = \max(D_{\alpha,\beta}(C_1), \dots, D_{\alpha,\beta}(C_k)).$$

In the rest of this paper we suppose that Eq. 6 is used.

The value of $DP_{\alpha,\beta}(P_k)$ varies between 0 and 1. If $DP_{\alpha,\beta}(P_k)=1$, the partition is said to be fully homogeneous and if $DP_{\alpha,\beta}(P_k)=0$, then the partition is non homogeneous. Otherwise, the partition P_k is homogenous to a level equal to $DP_{\alpha,\beta}(P_k)$.

3.4. Illustrative example

Consider a set X of objects described by two attributes:

- Attribute A_1 indicates if a day is a working day (between Monday and Friday) or not: $DOM(A_1) = \{Y, N\}$.
- Attribute A_2 indicates if a day is a holiday or not: $DOM(A_2) = \{Y, N\}$.

The $L = \{l_1 = YY, l_2 = YN, l_3 = NY, l_4 = NN\}$ be the set of labels. Suppose that $\|S_1\| = 5$, $\|S_2\| = 10$, $\|S_3\| = 6$, and $\|S_4\| = 4$. Let C_1 and C_2 be two classes defined on X . The content of these two classes is shown in Figure 2. Class C_1 contains 13 objects (two are described by label l_1 , nine by l_2 and two by l_4). Class C_2 contains 12 objects (three are described by label l_1 , one by l_2 , six by l_3 and two by l_4).

Consider now class C_1 and suppose that $\alpha = 0.5$ and $\beta = 0.1$. Using Eq. 1 and Eq. 3, we get:

- $MD_{0.5}(l_1, C_1) = 0$; $MC_{0.5}(l_1, C_1) = 0$,
- $MD_{0.5}(l_2, C_1) = 0.9$; $MC_{0.5}(l_2, C_1) = 0.64$,
- $MD_{0.5}(l_3, C_1) = 0$; $MC_{0.5}(l_3, C_1) = 0$,
- $MD_{0.5}(l_4, C_1) = 0.5$; $MC_{0.5}(l_4, C_1) = 0.22$.

Figure 2. Classes C_1 and C_2

For instance: $MD_{0.5}(l_1, C_1) = 0$ (since $\frac{\|SC_{1,1}\|}{\|S_1\|} = \frac{2}{5} = 0.4 < \alpha$) and $MD_{0.5}(l_4, C_1) = \frac{\|SC_{4,1}\|}{\|S_4\|} = \frac{2}{4} = 0.5$.

Then, by using Eq. 2 and Eq. 4, we get:

- $HD_{0.5}(C_1) = \frac{1}{2}(MD_{0.5}(l_2, C_1) + MD_{0.5}(l_4, C_1)) = 0.7$,
- $HC_{0.5,0.1}(C_1) = MC_{0.5}(l_1, C_1) + MC_{0.5}(l_2, C_1) + MC_{0.5}(l_3, C_1) + MC_{0.5}(l_4, C_1) = 0.86$.

Next, the homogeneity degrees of classes C_1 and C_2 can be computed by Eq. 5. This leads to:

$$\begin{aligned} D_{0.5,0.1}(C_1) &= HD_{0.5}(C_1) \cdot HC_{0.5,0.1}(C_1) \\ &= 0.7 \cdot 0.86 \\ &= 0.6 \end{aligned}$$

$$\begin{aligned} D_{0.5,0.1}(C_2) &= HD_{0.5}(C_2) \cdot HC_{0.5,0.1}(C_2) \\ &= 0.64 \end{aligned}$$

Finally, the homogeneity degree of partition $P_2(C_1, C_2)$ is computed by Eq. 6:

$$\begin{aligned} DP_{0.5,0.1}(P_2) &= \frac{1}{2}(D_{0.5,0.1}(C_1) + D_{0.5,0.1}(C_2)) \\ &= \frac{1}{2}(0.6 + 0.64) \\ &= 0.62 \end{aligned}$$

Tables 1, 2 and 3 hereafter give the values of $D_{\alpha,\beta}(C_1)$, $D_{\alpha,\beta}(C_2)$ and $DP_{\alpha,\beta}$ for some values of α and β . It is easy to see that for $\alpha = 0.1$ and $\beta = 0.5$, class C_1 is more homogeneous than class C_2 , while for $\alpha = 0.5$ and $\beta = 0.1$, C_2 is more homogeneous than C_1 .

α	β	$HD_{\alpha}(C_1)$	$HC_{\alpha,\beta}(C_1)$	$D_{\alpha,\beta}(C_1)$
0.1	0.1	0.6	1	0.6
	0.5		1	0.6
	0.9		1	0.6
0.5	0.1	0.7	0.86	0.6
	0.5		0.86	0.6
	0.9		0	0
0.9	0.1	0.9	0.64	0.58
	0.5		0.64	0.58
	0.9		0	0
1	0.1	0	0	0
	0.5		0	0
	0.9		0	0

Table 1. Values of $HD_{\alpha}(C_1)$, $HC_{\alpha}(C_1)$ and $D_{\alpha,\beta}(C_1)$

α	β	$HD_{\alpha}(C_2)$	$HC_{\alpha,\beta}(C_2)$	$D_{\alpha,\beta}(C_2)$
0.1	0.1	0.55	1	0.55
	0.5		1	0.55
	0.9		1	0.55
0.5	0.1	0.7	0.91	0.64
	0.5		0.91	0.64
	0.9		0.91	0.64
0.9	0.1	1	0.36	0.36
	0.5		0	0
	0.9		0	0
0.9	0.1	1	0.36	0.36
	0.5		0	0
	0.9		0	0

Table 2. Values of $HD_{\alpha}(C_2)$, $HC_{\alpha}(C_2)$ and $D_{\alpha,\beta}(C_2)$

α	β	$D_{\alpha,\beta}(C_1)$	$D_{\alpha,\beta}(C_2)$	$DP_{\alpha,\beta}(P_2)$
0.1	0.1	0.6	0.55	0.58
	0.5	0.6	0.55	0.58
	0.9	0.6	0.55	0.58
0.5	0.1	0.6	0.64	0.62
	0.5	0.6	0.64	0.62
	0.9	0	0.64	0
0.9	0.1	0.58	0.36	0.47
	0.5	0.58	0	0
	0.9	0	0	0
1	0.1	0	0.36	0
	0.5	0	0	0
	0.9	0	0	0

Table 3. Values of $DP_{\alpha,\beta}(P_2)$ for different values of α and β

3.5. Propositions

This section presents a series of propositions concerning the relationship between HD_{α} , $HC_{\alpha,\beta}$, MD_{α} and MC_{α} .

Proposition 1 (Relation between HD_{α} and α). *Giving a class C_i ,*

- If $\max_j \frac{\|SC_{ij}\|}{\|S_j\|} < \alpha$, then $HD_{\alpha}(C_i) = 0$ (part 1)
- If $\alpha = 1$ and $\nexists l_j$ such that $\frac{\|SC_{ij}\|}{\|S_j\|} = 1$, then, $HD_{\alpha}(C_i) = 0$ (sub-part 2.1). Otherwise $HD_{\alpha}(C_i) = 1$ (sub-part 2.2)

- If there is at least one label l_j such that $\frac{\|SC_{ij}\|}{\|S_j\|} = 1$, then $\lim_{\alpha \rightarrow 1}(HD_\alpha(C_i)) = 1$ (part 3)

Proof (part 1). Given a class C_i , if for every label l_j ($j = 1, \dots, m$), $\frac{\|SC_{ij}\|}{\|S_j\|} < \alpha$, then for every label l_j ($j = 1, \dots, m$), $MD_\alpha(l_j, C_i) = 0$. Then, according to Eq. 2, $HD_\alpha(C_i) = 0$.

Proof (part 2). The proof of the sub-part 2.1 is trivial because this sub-part is the consequence of part 1. For sub-part 2.2, given a class C_i , $HD_\alpha(C_i)$ is equal to the mean of the $MD_\alpha(l_j, C_i)$, $\forall l_j$, that are different from 0. When $\alpha = 1$, we take into account only the values of $\frac{\|SC_{ij}\|}{\|S_j\|}$, $\forall l_j$, that are greater than 1 to compute $HD_\alpha(C_i)$. Then, if there is at least one label l_j such that $\frac{\|SC_{ij}\|}{\|S_j\|} = 1$, then $HD_\alpha(C_i) = 1$.

Proof (part 3). As mentioned above, $HD_\alpha(C_i)$ is equal to the mean of the $MD_\alpha(l_j, C_i)$ that are different from 0. If α is close to 1, so for the computation of $HD_\alpha(C_i)$, we take into account only the values of $\frac{\|SC_{ij}\|}{\|S_j\|}$, $\forall l_j$, that are greater than α . Then, the mean will be greater than α .

Proposition 2. Let PL_j be a set of r classes containing objects having the label l_j . The following holds:

- $\sum_{i=1}^r MD_\alpha(l_j, C_i) = 1$ if $\forall C_i, MD_\alpha(l_j, C_i) \neq 0$ (part 1)
- If for a given class $C_i \in PL_j$, $MD_\alpha(l_j, C_i) > 0.5$, then $\forall C_m \neq C_i$ and $\in PL_j$, $MD_\alpha(l_j, C_m) < 0.5$ (part 2)

Proof (part 1). According to section 3.2.1, $MD_\alpha(l_j, C_i)$ is equal to the proportion of objects of a class C_i described by a label l_j in respect to the total number of the objects of X having the label l_j when this proportion is greater than α . Next, if $\forall C_i$, $MD_\alpha(l_j, C_i) \neq 0$, Then:

$$\begin{aligned} \sum_{i=1}^r MD_\alpha(l_j, C_i) &= \frac{\|SC_{1j}\|}{\|S_j\|} + \frac{\|SC_{2j}\|}{\|S_j\|} + \dots + \frac{\|SC_{rj}\|}{\|S_j\|} \\ &= \frac{\|SC_{1j}\| + \|SC_{2j}\| + \dots + \|SC_{rj}\|}{\|S_j\|} \\ &= 1 \end{aligned}$$

Otherwise, we will have some proportions that will not be taken into account.

Proof (part 2). Trivial (consequence of part 1)

Proposition 3 (Relation between $HC_{\alpha,\beta}(l_j, C_i)$ and $MD_\alpha(l_j, C_i)$). Let C_i be a class with labels list $Labels(C_i)$. If $\forall l_j \in Labels(C_i)$ $MD_\alpha(l_j, C_i) \geq \alpha$, then $HC_{\alpha,\beta}(C_i) = 1$.

Proof. $MD_\alpha(l_j, C_i) \geq \alpha$ means that $MC_\alpha(l_j, C_i) = \frac{\|SC_{ij}\|}{\|C_i\|}$. Consequently, if $\forall l_j \in Labels(C_i)$, then $MD_\alpha(l_j, C_i) \geq \alpha$. Then,

$$\begin{aligned} HC_{\alpha,\beta}(C_i) &= \frac{\|SC_{i1}\|}{\|C_i\|} + \frac{\|SC_{i2}\|}{\|C_i\|} + \dots + \frac{\|SC_{is}\|}{\|C_i\|} \\ &= \frac{\|SC_{i1}\| + \|SC_{i2}\| + \dots + \|SC_{is}\|}{\|C_i\|} \\ &= 1 \end{aligned}$$

Proposition 4 (Relation between $HC_{\alpha,\beta}$ and HD_α). If $HD_\alpha(C_i) = 0$, then $HC_{\alpha,\beta}(C_i) = 0$.

Proof. $HD_\alpha(C_i) = 0$ means that $\forall l_j, MD_\alpha(l_j, C_i) > 0$. Thus, according to Eq. 3, $MC_\alpha(l_j, C_i) = 0, \forall l_j$. Consequently, $HC_{\alpha,\beta}(C_i) = 0$.

4. Mixed-attributes clustering algorithm

Partitional clustering methods permit to obtain k classes of objects. The number of classes k is generally fixed before running the algorithm. The idea of the proposed mixed-attributes clustering algorithm is to run several times one of the partitional clustering methods by varying the number of classes k from 2 to a maximal number of classes k_{max} corresponding to the number of labels. At each iteration we compute the overall homogeneity degree $DP_{\alpha,\beta}$ for different values of α and β . The values of α and β that maximize $DP_{\alpha,\beta}$ are stored. Then, the partition (and so the number of classes) that maximizes $DP_{\alpha,\beta}$ is picked out.

Algorithm 1 that follows implements this idea. The procedure PARTITIONING used in Algorithm 1 may be any of the conventional partitional clustering methods that permits to obtain a partition of k classes based on numerical attributes.

Procedure Extended_Partitional_Clustering(
X : Set of objects, L : Set of labels)
 $k_{max} \leftarrow \|L\|$
For k **from** 2 **to** k_{max} **do**
     $P_k \leftarrow \text{PARTITIONING}(k, X)$
    For α **from** 0 **to** 1 **by** $step_\alpha$ **do**
        For β **from** 0 **to** 1 **by** $step_\beta$ **do**
            compute $DP_{\alpha,\beta}(P_k)$
        end For
    end For
    save $(\alpha, \beta, DP_{\alpha,\beta}(P_k))$ for which $DP_{\alpha,\beta}(P_k)$ is maximal
end For
return the partition for which $DP_{\alpha,\beta}(P_k)$ is maximal
End

Algorithm 1: Extended partitional clustering algorithm

Thresholds α and β vary from 0 to 1 by step $step_\alpha$ and $step_\beta$, respectively. The parameters $step_\alpha$ and $step_\beta$ are chosen by the user before running the algorithm. Both $step_\alpha$ and $step_\beta$ must be in the range $]0, 1[$. Possible values are: 0.01, 0.1, and 0.2.

The complexity of the algorithm is $O(k_{max} \cdot (CP + \frac{1}{step_\alpha} \frac{1}{step_\beta}))$ where CP is the complexity of PARTITIONING algorithm.

5. Experiment and discussion

This section presents an application of the proposed algorithm. The well-known K-means [12] partitional clustering algorithm is used.

5.1. Datasets

The datasets is composed of 230 objects aleatory generated. Each of these objects is described by 100 numerical attributes. Attributes values used are summed up as follows:

- $E_1 = \{O_1, O_2, \dots, O_{50}\}$: 50 objects where attributes values vary between 0 and 20.
- $E_2 = \{O_{51}, O_{52}, \dots, O_{90}\}$: 40 objects where attributes values vary between 150 and 170.
- $E_3 = \{O_{91}, O_{92}, \dots, O_{130}\}$: 40 objects where attributes values vary between 300 and 320.
- $E_4 = \{O_{131}, O_{132}, \dots, O_{200}\}$: 70 objects attributes values vary between 450 and 470.
- $E_5 = \{O_{201}, O_{202}, \dots, O_{230}\}$: 30 objects attributes values vary between 600 and 620.

For simplicity, the variation range for attributes values is the same for all attributes. This restriction has no consequence on the results.

A set of 10 labels $L = \{l_1, l_2, \dots, l_{10}\}$ is used. These labels are supposed to be created from categorical attributes. The distribution of labels in the different sets is given in Table 4.

Label	Number of objects	E_1	E_2	E_3	E_4	E_5
l_1	30	18	12			
l_2	5			5		
l_3	25	20			5	
l_4	15			14	1	
l_5	4	4				
l_6	21				21	
l_7	19	2		17		
l_8	51	6		4	41	
l_9	30		28		2	
l_{10}	30					30
Total	230	50	40	40	70	30

Table 4. Distribution of labels

5.2. Application of the algorithm

A summary of the results is given in Table 5. As it is shown in Table 5, the maximal value of overall homogeneity degree corresponds to partition $k = 3$ with $\alpha = 0.8$ and $\beta = 0.9$ ($DP_{0.8,0.9}(P_3) = 0.91$)

The justification of this result follows. First, note that for partition P_3 , objects of the sets E_3 and E_4 are in the same class, which is C_2 . Accordingly, all objects described by label l_4 are in class C_2 . This leads to $MD_\alpha(l_4, C_2) = 1$. Moreover, class C_2 contains the majority of objects described by label l_8 . For P_4 , in turn, the values of $MD_\alpha(l_4, C_2)$ and $MD_\alpha(l_4, C_3)$ are less than 1; and the values of $MD_\alpha(l_8, C_2)$ and $MD_\alpha(l_8, C_3)$ will decrease. Consequently, $DP_{\alpha,\beta}(P_4) < DP_{\alpha,\beta}(P_3)$. The same remark holds by comparing partition P_3 to partitions P_2 , and P_5 .

k	α_{max}	β_{max}	Partition	$DP_{\alpha,\beta}$
2	0.8	0.9	$C_1 = \{50 \text{ of } E_1, 40 \text{ of } E_2\}$ $C_2 = \{40 \text{ of } E_3, 70 \text{ of } E_4, 30 \text{ of } E_5\}$	0.88
3	0.8	0.9	$C_1 = \{50 \text{ of } E_1, 40 \text{ of } E_2\}$ $C_2 = \{40 \text{ of } E_3, 70 \text{ of } E_4\}$ $C_3 = \{30 \text{ of } E_5\}$	0.91
4	0.8	0.8	$C_1 = \{50 \text{ of } E_1, 40 \text{ of } E_2\}$ $C_2 = \{40 \text{ of } E_3\}$ $C_3 = \{70 \text{ of } E_4\}$ $C_4 = \{30 \text{ of } E_5\}$	0.87
5	0.4	0.8	$C_1 = \{50 \text{ of } E_1\}$ $C_2 = \{40 \text{ of } E_2\}$ $C_3 = \{40 \text{ of } E_3\}$ $C_4 = \{70 \text{ of } E_4\}$ $C_5 = \{30 \text{ of } E_5\}$	0.79

Table 5. Results of the application of Algorithm 1

5.3. Discussion

This section first explains the role of the parameters α and β and their utility in explaining the behavior of datasets. A practical example illustrating this fact is then provided.

5.3.1 Role of α and β

First, we note that to obtain coherent results, the values of α and β should be greater or equal to 0.5. This corresponds to the majority rule. Indeed, according to Proposition 2, if $\alpha \geq 0.5$ and if $MD_\alpha(l_j, C_i) > 0$, then class C_i will contain more than 50% of objects having the label l_i . Moreover the fact that β have to be greater than 0.5 assures that the number of labels l_j of which $MD_\alpha(l_j, C_i) > 0$ represents a proportion greater than 50% in the class. For instance, for partition P_3 in Table 5, we have:

- For C_1 :

- $MD_{0.8}(l_1, C_1) = 1$,
- $MD_{0.8}(l_3, C_1) = 0.8$,
- $MD_{0.8}(l_5, C_1) = 1$,
- $MD_{0.8}(l_9, C_1) = 0.93$ and
- $HC_{0.8,0.9}(C_1) \geq 0.9$.

Then, class C_1 is described by labels l_1, l_3, l_5 and l_9 .

– For C_2 :

- $MD_{0.8}(l_2, C_2) = 1$,
- $MD_{0.8}(l_4, C_2) = 1$,
- $MD_{0.8}(l_6, C_2) = 1$,
- $MD_{0.8}(l_7, C_2) = 0.89$,
- $MD_{0.8}(l_8, C_2) = 0.8$ and
- $HC_{0.8,0.9}(C_2) \geq 0.9$.

Then, C_2 is described by labels l_2, l_4, l_6, l_7 and l_8 .

– For C_3 :

- $MD_{0.8}(l_{10}, C_3) = 1$, and
- $HC_{0.8,0.9}(C_3) \geq 0.9$.

Then, C_3 is described by label l_{10} .

This type of result can not be obtained with methods described in Section 2 since the use of a distance measure that handles mixed attributes does not guarantee that the obtained classes are described by distinct labels.

Results such as ones presented above allow, in a second stage, to make decisions such as:

- Forecasting: the fact that each class is described with distinct labels permits to conclude that each object having a label l_j must be included in a class for which label l_j is representative. This permits to predict the behavior of any object.
- Detection of outlier: a new object O_r described by a label l_j is included in the nearest class C_i in terms of numerical attributes. If the label l_j is not representative of class C_i , then object O_r is an outlier.

5.3.2 Road traffic application

Consider that a road traffic dataset representing the number of vehicles passing in a point x of a road network each hour during one day. Each object is then described by 24 numerical attributes $\{A_1, A_2, \dots, A_{24}\}$: A_1 is the number of vehicles on 00h00, A_2 is the number of vehicles on 01h00, and so on. The objects are also described by 2 categorical attributes A_{25} and A_{26} that represent respectively

working day and *holiday* with $DOM(A_{25} = \{Y, N\})$ and $DOM(A_{26}) = \{Y, N\}$.

By applying one of numerical attributes-based clustering methods, the proposed approach assures that the objects in the same class are more similar than objects within different classes according to only numerical attributes. After that, the identification of the partition that maximizes the overall homogeneity degree permits to detect factors that influence the behaviors of datasets. For example, obtaining a class C_1 described by the label NN and NY (α and β should be greater or equal to 0.5) indicates that the majority of the objects described by label NN or label NY are in the same class. We can next conclude that the traffic during weekend is similar to the traffic during the holiday and that any new object whose values are recorded in a holiday or a weekend will have the same behavior as the representative labels of class C_1 . Moreover, we can detect the outliers as mentioned above and we can explain some facts like accidents or traffic lights problems.

6. Conclusion

In this paper, we first introduced the concept of homogeneity degree. This concept permits to measure the level to which a class is homogenous in respect to categorical attributes. Then, we proposed an extended mixed-attributes clustering algorithm based on the notion of homogeneity degree. Currently, the extended algorithm requires the use of a conventional partitioning clustering method. However, we intend to explore the possibility to extend the algorithm to apply with other type of clustering methods such as hierarchical or density-based ones.

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