Revisiting the transitional dynamics of business-cycle phases with mixed frequency data

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Abstract

This paper introduces a Markov-Switching model where transition probabilities depend on higher frequency indicators and their lags, through polynomial weighting schemes. The MSV-MIDAS model is estimated via maximum likelihood methods. The estimation relies on a slightly modified version of Hamilton's recursive filter. We use Monte Carlo simulations to assess the robustness of the estimation procedure and related test-statistics. The results show that ML provides accurate estimates, but they suggest some caution in the tests on the parameters involved in the transition probabilities. We apply this new model to the detection and forecast of business cycle turning points. We properly detect recessions in United States and United Kingdom by exploiting the link between GDP growth and higher frequency variables from financial and energy markets. Spread term is a particularly useful indicator to predict recessions in the United States, while stock returns have the strongest explanatory power around British turning points.

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1 Introduction

The failure to detect downturns in economic activity is a major source of error in macroeconomic forecasting. At the onset of the great recession, practitioners in the United States surveyed by the Survey of Professional Forecasters in November 2007 gave a probability around 20 percent of a negative growth in each quarter of 2008 and believed that US activity would grow by 2.5 percent in 2008.¹

This paper introduces a new specification which can be useful for monitoring and predicting the business cycle. We consider a Markov-Switching model where transition probabilities depend on higher frequency indicators (MSV-MIDAS model). As done in Diebold, Lee and Weinbach [1994] and Filardo [1994], the parameters of the model depend on an unobserved state variable following a first-order Markov chain with time-varying transition probabilities.² The innovation of the paper relies in the specification of the transition probabilities that depend on a set of exogenous indicators sampled at a higher frequency. To deal with the discrepancy in the frequencies, we apply the MIDAS (mixed-data sampling) approach developed by Ghysels, Santa-Clara and Valkanov [2004] and Ghysels, Sinko and Valkanov [2007] (see also Foroni and Marcellino [2013] for a recent overview of econometric methods with mixed-frequency data). A parsimonious parameterization of the lagged coefficients of the high frequency variable is obtained through the use of functional polynomial weights.

It is well known that a wide range of indicators produces useful signals for the identification of the current and future state of the economy. In particular, there is a large literature showing that financial indicators can be used to predict business cycle turning points. Among these variables, the yield curve takes a prominent place (see Estrella and Mishkin [1998], Kauppi and Saikkonen [2008] and Rudebusch and Williams [2009], among many others), but other indicators such as stocks and commodity prices contain information to predict the business cycle troughs and peaks (see Hamilton [2003], Hamilton [2011] and Kilian and Vigfusson [2013] on the specific role of oil prices). In this context, the MIDAS structure is useful, since these indicators are available at a higher frequency than macroeconomic variables. In this specification, it is not necessary to aggregate the financial indicators at a lower frequency, which may lead to a loss of information and therefore to inefficient and/or biased estimates (Andreou, Ghysels and Kourtellis [2010]).

¹According to the NBER, the US recession began in December 2007 and ended in June 2009. US Real GDP fell by 0.3% in 2008 and 2.8% in 2009.

²Markov-Switching models with time-varying probabilities have been recently reconsidered by Kim, Piger and Startz [2008] and Bazzi, Blasques, Koopman and Lucas [2014].

This paper is related to the literature showing with MIDAS regressions that financial variables are useful predictors of GDP growth. Andreou, Ghysels and Kourtellis [2013], Galvão [2013] and Ferrara, Marsilli and Ortega [2014] show a statistical improvement of GDP forecast accuracy in the euro-area, UK and US with models incorporating the forward-looking information contained in high-frequency financial data. Besides, Guérin and Marcellino [2013], Bessec and Bouabdallah [2015] and Barsoum and Stankiewicz [2015] use a MIDAS approach to show that financial variables help to predict turning points in the United Kingdom and in the United States. From a methodological point of view, this paper also contributes to the recent literature introducing time-variation in MIDAS models: Galvão [2013] with the Smooth Transition MIDAS regressions, Guérin and Marcellino [2013] with a MS-MIDAS model and Schumacher [2014] with time-varying parameters (TVP-MIDAS model).

The MSV-MIDAS model is estimated via maximum likelihood methods. The estimation relies on a slightly modified version of the filter in Hamilton [1989]. We use Monte Carlo simulations to investigate the small sample properties of the maximum likelihood estimators of the MSV-MIDAS parameters, as well as related test-statistics. The simulations are conducted for various parameterizations and sample sizes. Monte Carlo evidence shows that maximum likelihood method provides accurate estimates. The average bias of the estimates is small and decreases with the size of the sample, as well as their volatility. However, as shown by Psaradakis, Sola, Spagnolo and Spagnolo [2010] in Markov-switching models with variables sampled at the same frequency, the t-statistics of the parameters involved in the transition probabilities may not be reliable. The significance tests of these parameters may lack power in small samples, especially in the shorter regime.

We apply the MSV-MIDAS model to US and UK data. As leading indicators for the inference on the future state, we consider monthly indicators from financial and energy markets: interest rate, term spread, stock return and oil price. These variables, widely recognized as business cycle predictors, are available without any publication lags and are not subject to revisions. The evaluation of the model is based both on an in-sample and out-of-sample analysis. The new specification appears to provide better signals of economic downturn and recovery than the usual model with constant probabilities, especially for the United States. The results are more mixed for the UK. Among the economic indicators used to improve the transition mechanism, the slope of the yield curve turns out to be the best candidate for the United States in line with the previous literature.

Stock return is more helpful for the detection of British recessions. These results hold both in-sample and out-of-sample.

The remainder of this paper proceeds as follows. In section 2, we present the MSV-MIDAS specification and describe the estimation techniques. In section 3, we use Monte Carlo simulations to assess the robustness of the estimation procedure and related test statistics for the inference. Section 4 is devoted to the empirical application to US and UK data. The last section offers some concluding remarks.

2 The MSV-MIDAS model

Let y_t be a variable which dynamics differs according to the state of the economy. The unobserved state variable follows a first-order Markov chain whose transition probabilities depend on a higher frequency indicator $z_t^{(m)}$. In the following, the time index t denotes the time unit of the low frequency variable y_t (a quarter in our application). The high frequency indicator $z_t^{(m)}$ is sampled m times between two time units of y , e.g. t and $t-1$ ($m = 3$ for monthly indicators as in our application). The lag operator $L^{1/m}$ operates at the higher frequency, e.g. $L^{s/m} z_t^{(m)} = z_{t-s/m}^{(m)}$.

The low frequency variable y_t follows an AR(p) process with a switching mean, as motivated by Hamilton [1989]. The dynamics of a MSM(M)-AR(p) model is described by the following equation:

$$y_t = \mu_{s_t} + \phi_1(y_{t-1} - \mu_{s_{t-1}}) + \dots + \phi_p(y_{t-p} - \mu_{s_{t-p}}) + \sigma \varepsilon_t \quad (1)$$

where $\varepsilon_t \rightarrow NID(0, 1)$. The variable $s_t = \{1, 2, \dots, M\}$ represents the state that the process is in at time t .

Following Diebold et al. [1994] and Filardo [1994], the variable s_t is assumed to follow a first-order Markov chain defined with time-varying transition probabilities. In the case of two regimes ($M = 2$), the four transition probabilities are expressed as follows:

$$\begin{aligned} P(s_t = 1 | s_{t-1} = 1, z_{t-1}^{(m)}) &= \Gamma[\alpha_1 + \beta_1 B(L^{1/m}, \Theta_1) z_{t-1}^{(m)}] \\ P(s_t = 2 | s_{t-1} = 2, z_{t-1}^{(m)}) &= \Gamma[\alpha_2 + \beta_2 B(L^{1/m}, \Theta_2) z_{t-1}^{(m)}] \\ P(s_t = 2 | s_{t-1} = 1, z_{t-1}^{(m)}) &= 1 - \Gamma[\alpha_1 + \beta_1 B(L^{1/m}, \Theta_1) z_{t-1}^{(m)}] \\ P(s_t = 1 | s_{t-1} = 2, z_{t-1}^{(m)}) &= 1 - \Gamma[\alpha_2 + \beta_2 B(L^{1/m}, \Theta_2) z_{t-1}^{(m)}] \end{aligned} \quad (2)$$

where Γ is the logistic function $\Gamma(x) = 1/(1 + \exp(-x))$, α_i and β_i are unknown parameters for regime $s_t = i$ and $z_{t-1}^{(m)}$ is an exogenous variable. In this model, the transition probabilities are not time invariant. Instead, they depend on an exogenous variable and its lags. When β_{s_t} is positive (negative), an increase in $z_{t-1}^{(m)}$ increases (decreases) the probability of staying in regime s_t . If $\beta_1 = \beta_2 = 0$, the specification simplifies to the usual model with constant transition probabilities.

The exogenous variable $z_t^{(m)}$ is sampled at a higher frequency. To keep the specification parsimonious, functional polynomial lags are employed. The function $B(L^{1/m}, \Theta_{s_t})$ is the exponential Almon lag³ with:

$$B(L^{1/m}, \Theta_{s_t}) = \sum_{j=1}^K b(j, \Theta_{s_t}) L^{(j-1)/m}, \quad b(j, \Theta_{s_t}) = \frac{\exp(\theta_{1,s_t}j + \theta_{2,s_t}j^2)}{\sum_{j=1}^K \exp(\theta_{1,s_t}j + \theta_{2,s_t}j^2)} \quad (3)$$

with $s_t = \{1, 2\}$ in the case of two states. The weights defined by $b(j, \Theta_{s_t})$ are positive and sum up to one. Note that we allow a switch in the parameters of the weighting function θ_1 and θ_2 . It means that the weighting scheme may not be the same in the two regimes. The coefficient $\Theta_{s_t} = \{\theta_{1,s_t}, \theta_{2,s_t}\}$ defines the lag structure in each regime and the coefficient β_{s_t} in equation (2) gives the final impact of z on the probability. If $\theta_{2,s_t} < 0$, the weight decreases with the lag j in regime s_t . In the particular case where $\Theta_i = \{0, 0\}$, we obtain the standard equal weighting aggregation scheme in state i (the high frequency variable is simply aggregated to the low frequency with an arithmetic average).

The lag function $B(L^{1/m}, \Theta_{s_t})$ allows a parsimonious specification since only two coefficients are needed for the K lags. This is particularly interesting in regime-switching models, where the number of coefficients is large. As indicated by Ghysels et al. [2007], the use of distributed lag polynomials also avoids the lag-length selection for the variable in the probabilities. The decay rate of the weights estimated from the data determines the number of lags of the high frequency indicator in the transition probabilities. Hence, more or less persistent impact of $z_t^{(m)}$ can be captured according to the shape of the function. This feature is attractive in our context since the inference on the transition parameters is fragile as shown by Psaradakis et al. [2010] in the case of data sampled at the same frequency. The next section of this study will confirm this fragility for models involving data sampled at different frequencies.⁴

³Other possible specifications of the MIDAS polynomials are based on beta or step functions. See Ghysels et al. [2007] for a presentation of the various parameterizations of $B(L^{1/m}, \Theta_{s_t})$.

⁴In a linear context, Foroni, Marcellino and Schumacher [2015] compare MIDAS models with functional distributed lags to MIDAS models with unconstrained weights estimated by least squares. The

The model is estimated by the maximum likelihood method.⁵ The likelihood is derived in a modified version of Hamilton’s filter to account for the variation of the transition probabilities. In the first step of the filter, the fixed transition probabilities are replaced by time-varying probabilities related to the high frequency variable $z_t^{(m)}$, as specified in equations (2) and (3) and the rest of the estimation procedure is similar.⁶ A Newton’s search method is applied to find the vector of parameters maximizing the function. The estimation algorithm is initialized with several sets of parameters to avoid local optima. A smoothing algorithm is then applied to obtain a better estimation of the states (Kim [1994]). The standard errors of the parameters are obtained from the inverse of information matrix at the optimum. In the estimation procedure, the parameter $\theta_{2,i}$, $i = \{1, 2\}$ of the Almon function is constrained to be negative, which guarantees in the two regimes a declining weight of $z_t^{(m)}$ as the lag length increases (see for instance Ghysels et al. [2007] for a further discussion of this point).

3 Monte Carlo simulations

In this section, we set up several Monte Carlo experiments to assess the robustness of the estimation procedure and explore the reliability of usual test-statistics for the inference on the model parameters. A similar exercise has been conducted by Psaradakis and Sola [1998] in MSM models with constant probabilities and by Psaradakis et al. [2010] in MSM models with time-varying probabilities. We extend their analysis to the case of mixed-frequency data.

3.1 Design of the Monte Carlo study

We use Monte Carlo experiments to investigate the small-sample properties of the ML estimators and related test-statistics.

The Monte Carlo study involves the following steps. In a first step, we simulate the high frequency variable $z_\tau^{(m)}$ according to an autoregressive process:

$$z_\tau^{(m)} = c + \rho z_{\tau-1}^{(m)} + \omega u_\tau^{(m)} \quad \tau = 1, \dots, T \times m \quad (4)$$

unconstrained specification performs well for small differences in sampling frequencies.

⁵We use Matlab for all simulations and estimations.

⁶See the appendix for a presentation of the filter and the derivation of the log-likelihood in the MSV-MIDAS model.

Second, we generate a first-order Markov chain s_t , $t = 1 \dots, T$ with time-varying transition probabilities as defined in equations (2) and (3). We consider $K = 12$ lags in the polynomial $B(L^{1/m}, \Theta_{s_t})$. Finally, we simulate the low frequency variable y_t , $t = 1 \dots, T$ as a first-order autoregressive process subject to Markov shifts in mean as described in equation (1). The residuals u_t and ε_t are i.i.d. standard normal and independent. They are generated via a pseudo-random number generator. The first 100 simulated observations of s_t and y_t and the first $100 \times m$ observations of $z_\tau^{(m)}$ are discarded to reduce the effect of initial conditions. We assume that $m = 3$ which corresponds to a model mixing quarterly and monthly data. We consider various sample sizes $T = \{200, 400, 800\}$ and we use 1,000 Monte Carlo replications for each experiment.

[INSERT TABLE 1 HERE]

The values of the parameters in equations (1)-(4) are given in Table 1. The benchmark configuration (DGP1) is close to the empirical setting obtained for US data in section 4. The low frequency variable follows an AR(1) process with a switching mean. The mean parameter is negative in the least persistent regime. The high frequency indicator affects positively the transition probability of the favorable state and negatively the probability of staying in the recession state. In DGP1, the weights are time-invariant. We consider alternatively in DGP2 a model with regime-switching weights. In DGP3 and DGP4, the high frequency indicator z_t is less persistent and more volatile. In DGP5, the difference between the mean parameters is smaller in the two regimes, which may affect the classification of the observations in the two regimes. Finally, DGP6 is used to investigate the sensitivity of the results to the shape of the weighting function. In this last DGP, the profile is flatter with lower values of θ_1 and θ_2 . A typical realization of $\{Z_t^{(m)}\}$, $\{S_t\}$ and $\{Y_t\}$ from DGP2 is shown in Figure 1.

[INSERT FIGURE 1 HERE]

3.2 Robustness of the ML estimates

In a first step, we explore the finite sample performance of the maximum likelihood estimator for the data generating processes considered in Table 1.

The parameter of the models are estimated via a numerical optimization of the log-likelihood of the model. As starting values, we use the true vector of parameters used

to generate the data, plus random values drawn from a normal distribution with a standard deviation equal to 0.1.⁷ To gauge the robustness of ML estimates, we examine the average bias on the estimated coefficients of the model and the standard deviations of the estimates in the 1,000 replications. To measure the quality of the estimated parameters involved in the transition probabilities, we report additional criteria. For parameters $\Theta_{s_t} = \{\theta_{1,s_t}, \theta_{2,s_t}\}$, we provide an average measure of the error on the weights given by:

$$err_b_j(s_t) = \frac{\sum_{j=1}^K [b(j, \hat{\Theta}_{s_t}) - b(j, \Theta_{s_t})]^2}{\sum_{j=1}^K b(j, \Theta_{s_t})^2} \quad (5)$$

Second, we compare the simulated transition probabilities with the estimated ones with the mean absolute error statistics:

$$\begin{aligned} err_p_{11} &= \frac{1}{T} \sum_{t=1}^T \left| \Gamma(\alpha_1 + \beta_1 B(L^{1/m}, \Theta_1) z_{t-1}^{(m)}) - \Gamma(\hat{\alpha}_1 + \hat{\beta}_1 B(L^{1/m}, \hat{\Theta}_1) z_{t-1}^{(m)}) \right| \\ err_p_{22} &= \frac{1}{T} \sum_{t=1}^T \left| \Gamma(\alpha_2 + \beta_2 B(L^{1/m}, \Theta_2) z_{t-1}^{(m)}) - \Gamma(\hat{\alpha}_2 + \hat{\beta}_2 B(L^{1/m}, \hat{\Theta}_2) z_{t-1}^{(m)}) \right| \end{aligned} \quad (6)$$

This last measure allows to gauge the effect of the error on the parameters α , β and θ on the time-varying transition probabilities. We report the average value of these criteria in the 1,000 Monte Carlo simulations.

[INSERT TABLE 2 HERE]

The results in Table 2 show that the estimation procedure provides accurate estimates of the parameters present in the equation of y_t . The average bias is generally very close to zero and the dispersion is low. The bias is slightly larger for ϕ in small samples. The estimated parameters of the transition probabilities, α_i and β_i , $i = \{1, 2\}$, are less accurate, especially for small values of T , and the estimates of these four parameters show a higher dispersion. The error on the weights is also larger for small samples, even though it is rather limited as shown by the rather low values of err_b_j . However, the approximate error on the probabilities remains moderate, even for small values of T (inferior to 5 points for $p_{11,t}$ and 10 points for $p_{22,t}$ for $T = 200$ and to 3 and 5 points respectively in the largest sample).

⁷Note that we do not assess the effect of possible model misspecifications on the estimation accuracy. The model is estimated with the true number of autoregressive parameters and with the same parameters subject to changes of regime.

Comparing results across DGPs, the quality of parameter estimates increases with a less persistent or more volatile dynamics of z_t (DGP3 and DGP4) or with more uniformly distributed weights (DGP6). By contrast, decreasing the difference between the parameters of the two states (DGP5) has an adverse effect on the estimation accuracy. In particular, the parameters entering the probabilities show a higher bias for $T = 200$ and are more volatile. The error on the weights is larger too in the less persistent state with regime-switching weights (DGP2). However, in both cases, the impact on the identification of the regimes is rather limited in comparison to the benchmark.

3.3 Robustness of the tests

We now turn to the reliability of the t-statistics related to the parameters of the model. The t-statistics are computed as the ratio of the estimation error to the estimated standard error. The estimated standard errors are based on the Hessian matrix of the estimated log-likelihood function.

Table 3 reports some characteristics of the sampling distribution of the t-statistics of the parameters obtained in the Monte Carlo simulations: the mean, the standard deviation, the skewness, the excess kurtosis, the p-value of the Jarque Bera test for normality. The t-statistics associated with the estimated parameters of the model are expected to be approximately distributed as $N(0, 1)$. The standard deviations of the 1,000 simulated t-statistics are generally close to one. However, the average t-statistics associated with $\hat{\phi}$ in the equation for y_t and with $\hat{\alpha}_i$ and $\hat{\beta}_i$, $i = \{1, 2\}$ in the transition probabilities depart from zero⁸. Besides, the skewness coefficient shows some asymmetry in the distributions of the t-statistics for $\hat{\alpha}_i$ and $\hat{\beta}_i$, $i = \{1, 2\}$. The distributions of $\hat{\beta}_1$ and $\hat{\beta}_2$ are also highly leptokurtic for small values of T . As a consequence, the null of normality is strongly rejected for $\hat{\alpha}_i$ and $\hat{\beta}_i$, $i = \{1, 2\}$, even in large samples.

[INSERT TABLE 3 HERE]

To assess the potential effect of the non-normality on the inference, we investigate the performance of the t-statistics over the 1,000 Monte Carlo simulations, when we use standard normal critical values. Table 3 provides the empirical size of the two-sided tests of equality of each parameter to its true value, as well as the empirical power of the signif-

⁸Psaradakis and Sola [1998] obtain similar results in MS models with fixed transition probabilities and Psaradakis et al. [2010] in a MS model with time-varying probabilities.

ificance tests for each parameter at a 5 and 10% significance level (lines size and power).⁹ The empirical sizes are found close to the nominal levels, 5% and 10%. However, like Psaradakis et al. [2010], we find that the parameters entering the transition probabilities are more likely to be found insignificant in small samples. Indeed, the frequency of rejection of the nullity of α_2 and β_2 in the shorter state is lower for $T = 200$ and $T = 400$. For instance in DGP3, the nullity of α_2 is rejected in 35% cases and that of β_2 in 53% cases for $T = 200$, while the frequency of rejection is around 100% for the other parameters.

Again, the departure from normality is lower with a less persistent or more volatile dynamics of z_t or with more uniformly distributed weights. By contrast, it is more pronounced when μ_1 and μ_2 are closer or for state-dependent weights. This departure has an adverse effect of the power of the significance tests in DGP2 and DGP5. In sum, the t-statistics of the parameters α_i and β_i should be used with caution, especially in the shorter regime.

4 Application to US and UK GDP

4.1 The data

The empirical relevance of the MSV-MIDAS model will now be illustrated with the business cycle analysis of two countries: United States and United Kingdom.

The database consists of the quarterly growth rate of real GDP and a set of monthly financial indicators in US and in UK. The dataset was collected in July 2014. The data on GDP cover the period from 1959Q1 to 2013Q4 (220 quarters) for the United States and 1975Q1 to 2013Q4 (156 quarters) for the United Kingdom. These samples include 8 recessions for the United States and 4 recessions for the United Kingdom. The business cycle chronology is taken from the NBER for the US and from ECRI for the UK.¹⁰ We will assess if monthly financial indicators can help to detect in real-time the recessions for the two countries, when they are incorporated in the transition probabilities of the MSV-MIDAS model.

It might be more challenging to identify a recession with GDP data available at the time given the revision of GDP data, especially in time of recessions. Over the period

⁹In contrast to the linear case, the particular test of the nullity of β_1 in $P(s_t = 1 | s_{t-1} = 1, z_{t-1}^{(m)})$ in the model with time-invariant weights does not involve non-identified parameters under the null hypothesis, since the vector Θ is still present in the other transition probability $P(s_t = 2 | s_{t-1} = 2, z_{t-1}^{(m)})$. The same applies to β_2 .

¹⁰<https://www.businesscycle.com/ecri-business-cycles/international-business-cycle-dates-chronologies>

1990-2010, quarterly growth rate of US GDP was revised by an average of 0.26 point three years after its first publication. This revision reaches up to 0.37 point for the recession quarters, as opposed to 0.21 point in expansion. Over the same period, the average revision of the UK GDP growth rate stands at 0.25 point: 0.43 point for recession quarters and 0.21 point in expansion. To take into account the revision of the data in the out-of-sample evaluation, we use vintages of output growth from the real-time datasets constructed by Croushore and Stark [2001] and available on the website of the Federal Reserve Bank of Philadelphia¹¹ for the United States and the real-time database available on the Bank of England’s website for the United Kingdom.¹² We use the vintages in January 1990 till March 2014.¹³ We assume that the financial variables are not revised.

The set of monthly indicators includes a short-term interest rate, term spread, stocks and oil prices. Interest rates are considered in difference and term spread in level. Stock and oil prices are taken in log difference. US interest rates are released by the Federal Reserve Bank of Saint Louis¹⁴. We consider the effective federal funds rate and the slope of the yield curve measured as the difference between the 10-Year Treasury bond and the 3-month Treasury-bill. The stock market index SP500 is provided by yahoo finance. UK interest rates are taken from the Bank of England’s website. We use the Bank of England base rate (Bank of England’s website, end of month), the term spread between the 10 year interest rate (datastream) and 3-month Treasury-bill (Bank of England’s website, end of month). As a stock price, we use FTSE all shares obtained from datastream. Finally, we consider the Brent oil price in London (datastream).

4.2 Estimation results

In a first step, we investigate the in-sample performance of the MSV-MIDAS models to track the GDP dynamics and identify the business cycle turning points in the entire sample.

We consider MSV-MIDAS models for GDP growth whose transition probabilities depend on a monthly financial variable (interest rate, term spread, stock or oil price). To ensure exogeneity with respect to the dependent variable, we lag the financial indicators by one quarter in the transition probabilities. We retain $K = 12$ lags in the almon function, so that the probabilities may depend on the monthly indicators over the entire past year.

¹¹<http://www.philadelphiafed.org/research-and-data/real-time-center/real-time-data/>

¹²<http://www.bankofengland.co.uk/statistics/Pages/gdpdatabase/>

¹³For the UK, the last vintages available from July 1993 to March 2014 were obtained from OECD.

¹⁴<http://research.stlouisfed.org/fred2/>

We use two-state MSV-MIDAS specifications with constant and time-varying weights (denoted by MSV1 and MSV2 respectively). In order to select the number of autoregressive terms, we use the AIC criterion in the linear specification. Tests for omitted autocorrelation are implemented to check whether these autoregressive orders are sufficient. We apply Ljung-Box tests either to the standardized generalized residuals (Gourieroux, Monfort, Renault and Trognon [1987]) or to standard-normal residuals constructed with the Rosenblatt transformation (Smith [2008]).¹⁵

To assess the gain due to time-variation and high frequency indicators in the transition probabilities, two particular cases of the MSV-MIDAS model are considered. First, the MSV-MIDAS model is compared to the usual model with fixed probabilities (FTP) as considered in (Hamilton [1989]). This model is obtained from equations (1)-(3) when β_1 and β_2 are constrained to zero. This restriction can be tested with a LR statistics but the distribution of the statistics is unknown (the parameters of the almon function Θ_{s_t} are not identified under the null of non-variation). As suggested by Andreou et al. [2010] in the linear case, we also consider the case when the high frequency indicator is converted to the low frequency data with a simple average to assess the gain due to mixed frequency data. This particular case is obtained for $\Theta = \{0, 0\}$ in the model with time-invariant weights and $\Theta_i = \{0, 0\}$, $i = \{1, 2\}$ in the model with regime specific weighting schemes. The relevance of these restrictions is tested with a standard LR test.

We estimate the MSV-MIDAS models and their constrained counterparts on the whole sample, from 1959 to 2013 for the US GDP and 1975 to 2013 for the UK GDP. The estimation of the models is done for a large set of initial conditions.¹⁶ Tables 4a and 4b provide the estimates and tests of significance of the models. The second part of the tables reports criteria assessing the quality of the inference on the state: the quadratic probability score (QPS) and the proportion of turning points accurately detected or with

¹⁵In a MSM(M)-AR(p) model with $p = 1$ lag, the standardized generalized residuals are obtained as: $\sum_{i=1}^M \sum_{j=1}^M P(s_t = i, s_{t-1} = j \mid I_{t-1}; \Theta) \times \sigma^{-1}(\varepsilon_t - \mu_i - \phi(y_{t-1} - \mu_j))$, while the Rosenblatt's residuals are defined as: $\Phi^{-1}\left(\sum_{i=1}^M \sum_{j=1}^M P(s_t = i, s_{t-1} = j \mid I_{t-1}; \Theta) \times \Phi[\sigma^{-1}(\varepsilon_t - \mu_i - \phi(y_{t-1} - \mu_j))]\right)$, with Φ the cumulative distribution function of the standard normal distribution and I_{t-1} the observed information on y and z available at time $t - 1$. Using Monte Carlo experiments, Smith [2008] shows that the test applied to the Rosenblatt's transformation of standardized residuals performs well to detect autocorrelation in Markov-Switching models.

¹⁶The optimization algorithm is initiated from 1,000 sets of initial values given by the estimates of the models with fixed probabilities plus random draws in the normal distribution. For the model with variable weights, we estimate in a first step the vectors Θ_1 and Θ_2 in a constrained model with the other parameters fixed to their value in the time-varying model with fixed weights. In a second step, we use the whole set of parameters to initiate the algorithm.

a lead/lag of τ quarters, $\tau = \{0, 1, 2\}$.¹⁷ The test for flat aggregation and the residual diagnostic tests are given in the last part of the tables.

[INSERT TABLES 4a-4b HERE]

The MSM models are estimated with two lags in the US and three lags in the UK, as found in the linear specification. The Ljung Box tests applied to generalized or Rosenblatt residuals do not show sign of remaining autocorrelation. In the two countries, GDP exhibits a positive mean growth rate in the first regime and declines in the second state. The coefficients β_1 and β_2 generally have opposite signs, showing that a variation of z_t leads to opposite movements of $p_{11,t}$ and $p_{22,t}$. Even in the case the two coefficients have the same sign, the size of the coefficients is clearly different. These two coefficients are often non-significant, especially in the shorter regime. However, the t-statistics must be interpreted with caution given the possible lack of power of the significance tests, as shown in the previous section. This caveat is particularly strong for the shorter regime.

The estimated parameters in the transition probabilities have expected signs. Lower stock returns increase the risk of recession ($\beta_1 > 0$) while making a recovery less likely ($\beta_2 < 0$) in the two countries. A similar pattern holds with term spread for the US. The positive coefficient in the expansion probability is consistent with the sharp decline in the slope of the yield curve and sometimes the inversion of the yield curve observed before economic downturns.¹⁸ The coefficients for the central bank rates are negative in the expansion probabilities and positive in the recession probabilities. In particular, a policy tightening increases the probability of recession ($\beta_1 < 0$). Finally, the rise in oil price increases the probability of entering in recession ($\beta_1 < 0$). As noted in Hamilton [2011], a majority of U.S. recessions had been preceded by a sharp rise in the price of crude petroleum.

The results reported in the second part of the table show that the model with time-varying probabilities provides a better fit than the FTP model. The significance of time-variation in the transition probabilities is not assessed, since the distribution of

¹⁷The quadratic probability score is defined as $\frac{2}{T} \sum_{t=1}^T (P(s_t = i | I_T; \Theta) - r_t)^2$ with r_t a dummy variable equal to one if the regime i is the true regime in t and zero otherwise. The QPS value lies in $[0, 2]$. The lower the QPS, the better the state is estimated. The second criterion focuses on the quarters with turning points and gives the proportion of turning points correctly detected in the sample. A signal is considered as correct when the smoothed probability of the realized state exceeds 0.5. The higher the index is, the better the model detects the turning points in recessions and recoveries.

¹⁸See Wheelock and Wohar [2009] for a survey on the usefulness of the term spread for predicting economic activity.

the likelihood-ratio statistics comparing the FTP and MSV models is unknown. However, there is a large improvement of the log-likelihood in the models with fixed weights. The improvement is particularly strong when the probabilities are related to stock return and federal funds rate in the United States and to oil prices in the United Kingdom. When taking into account the number of parameters, the MSV models with fixed weights still provides substantially smaller AIC values. By contrast, the difference between the likelihoods of the models with time-invariant and variable weights is smaller and the AIC criterion favors the models with invariant weights. The only exceptions are the models estimated with stock return for the United States and with oil prices for the United Kingdom. Overall, the models with fixed weights look more appropriate in our application.

We find a gain in incorporating monthly indicators in the probabilities rather than converting them to the quarterly frequency by the use of the simple average. In most cases, the LR test reported in the third part of the table (line LR flat) shows at the 5 or 10% significance level that the likelihood of the model is significantly improved in comparison to a model estimated with $\Theta_i = \{0, 0\}$. High-frequency information in the transition probabilities also helps to track the state of the US economy. In Table 4a, QPS values are generally inferior in MSV-MIDAS models (around 0.06 with the spread and federal rate against 0.08 in the benchmark) and the proportion of detected turning points (TPI) is much higher, except for the model with variable weights estimated with oil price. Over the last five decades, 56% of turning points are identified without any delay with the stock return as opposed to 31% in Hamilton’s model. The proportions range from 75 to 100% against 62.5% in the benchmark for $\tau = 2$.

The good performance of the MSV-MIDAS model for identifying the past recessions in the United States is also evident in Figure 2a. This graph plots the smoothed probabilities of being in the low-growth state obtained in the FTP and MSV models, together with the NBER recession periods. We focus in Figure 2a on the results obtained with term spread in the transition probabilities. The results obtained with the three other variables are provided in Figure 3. Interestingly, spread and stock return improve the signals of the last three recessions, driven by financial factors (see Ng and Wright [2013]). In particular, the dot-com bubble in 2000-01 is well detected by the new specification, while the signal was almost nonexistent in the FTP model. The signals obtained for the four episodes in the 70s and 80s are also much clearer and a fake signal in the fourth quarter of 1977 disappears.

[INSERT FIGURES 2a-2b HERE]

In the case of the United Kingdom, the results reported in Table 4b and in Figure 2b-3 are less supportive of the new specification. In Figure 2b-3, FTP and MSV-MIDAS models perform equally well in detecting the first three recessions (except the model with term spread missing the economic downturn in the 90s). Both classes of models fail to detect the latest recession in 2010-2012. QPS and TPI criteria given in Table 4b are better in the benchmark, except in the model with fixed weights depending on stock returns.¹⁹ In this model, we note a slight improvement over the model with constant probabilities with a QPS equal to 0.075 versus 0.082 in the benchmark. The proportion of recessions detected with a delay/lag of one or two quarters is higher too: 63 and 75% against 50 and 63% in the FTP model. Moreover, the MSV-MIDAS model estimated with stock returns provides a clearer signal for the economic downturn in the early 1990s, as shown in Figure 2b. The exit of the 2008-09 recession is pointed properly, while it is identified too early by the benchmark. However, the MSV-MIDAS model estimated with stock returns produces a false signal of recession in the 70s.

[INSERT FIGURE 4 HERE]

Finally, Figure 4 plots the estimated weight functions obtained with term spread in the United States and oil price in the United Kingdom. We report the results for the model with time-invariant and with regime-switching weights. In the models with fixed weights, the functions decrease slowly and flatten for a lag equal to 12. According to this chart, the financial variables contain useful information for predicting the US and UK business cycles up to one year ahead. This result is consistent with the literature on the predictive content of financial variables (see Stock and Watson [2003] for a survey). In the models with regime-specific functions, a distinct pattern appears in the two states. The weights are more persistent in expansion (state 1), while they vanish quickly in recession (state 2). It means that the financial indicators contain more advanced information to identify the peaks than the troughs. In recessions which are shorter-lived episodes, financial signals have a much shorter scope.

4.3 Out-of-sample results

Incorporating monthly indicators in the transition probabilities might help improving the signal of future recessions. In this last part, we check the ability of the MSV-MIDAS

¹⁹The bad performance of the base rate to detect the last recession (2010-2012) is not surprising. After the cut from 5% to 0.5% in the months following the collapse of Lehman Brothers in September 2008, the Bank of England kept rates on hold at 0.5% after march 2009. Similarly, the oil traded in a narrow band around 100 dollar per barrel after the beginning of 2011 (till the second half of 2014).

model to infer in real-time the current and future state of the economy.

We conduct an out-of-sample study with a recursive window scheme. The last observations of the sample are discarded for the forecasting exercise. For the United States, the forecasting window spans from 1990Q1 to 2013Q4 and includes three recessions. In the case of the United Kingdom, the out-of-sample period running from 2000Q1 to 2013Q4 contains two recessions. The forecasted chain is sampled at a quarterly frequency but the forecast can be updated every month after the release of the monthly indicator incorporated in the transition probabilities. In this study, we focus on the forecast of each quarter done 6 months to a few days before the GDP release, about one month after the end of the reference quarter. We expand recursively the estimation period. The parameters of the models are estimated using the only information available at the time of the forecast. The evaluation is conducted in real-time conditions. The models are estimated from the observations available at the time of the forecast. At this level, we use the vintages of output growth provided by the Federal Reserve Bank for the US and the Bank of England for the UK (the financial variables are supposed not to be subject to data revisions).

We focus on models with time-invariant weights. Models with variable weights are not considered further since allowing a variation does not lead to a large improvement in the likelihood in our application and since the estimation is more tricky and time-consuming. Tables (5a-5b) show the QPS and TPI criteria for the forecast states in the two countries.

[INSERT TABLES 5a-5b HERE]

Again, the MSV-MIDAS model displays better forecasts of the recessionary state than the model with constant probabilities in the United States. The QPS criteria in Table 5a are lower especially in the model using the slope of the yield curve as a leading indicator of recession (0.021 against 0.059 in the FTP model). The term spread is found yet again to be a good predictor of the US business cycle turning points. The results are positive too with the stock return at the shortest horizons. The proportion of detected turning points is also higher in these two models. A few days before the GDP release, the switch in the state is detected in 83% of cases with the MSV-MIDAS model estimated with the SP500 and in 67% with the spread against 33% with the FTP model.

Turning to the United Kingdom, the results are again less supportive of the model with time-varying probabilities. In Table 5b, the FTP and MSV-MIDAS models display similar QPS values, although slightly smaller in the model using the bank rate as a leading indicator at the shorter horizons (0.140 versus 0.144 for $h = 0$ and 0.132 against 0.142

one month before the GDP release). The proportions of detected turning points are also similar in the FTP model and in the model with probabilities depending on the stock returns and the bank rate: 25% of switches without any delay and 75% otherwise just before the GDP release. The FTP model outperforms the MSV models estimated with the two other indicators, especially the model with oil price.

5 Concluding remarks

In this paper, we introduce the MSV-MIDAS model. This specification incorporates higher frequency information in the transition mechanism of Markov-switching models.

The MSV-MIDAS model is estimated via maximum likelihood methods. Monte Carlo evidence suggests that our estimation procedure provides robust estimates of the parameters of the model. The Monte Carlo experiments also show that the t-statistics associated to the coefficients in the time-varying probabilities should be used with caution. In the empirical part, the new specification is applied to the detection and forecast of business cycle turning points. We find that the MSV-MIDAS model detects recessions more successfully than the specification with invariant transition probabilities in the United States. The slope of the yield curve provides particularly useful signals for the identification and forecast of business cycle turning points. The results are less clear-cut for the United Kingdom, where models with fixed and variable transition probabilities have rather similar performance. These findings hold both in-sample and out-of-sample.

There are a number of potential extensions to this paper. In particular, it would be interesting to introduce high frequency regressors in the equation for GDP. Exploiting the information provided by weekly or daily data is also on our research agenda. Finally, this model could be applied to other areas of macroeconomics and finance.

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APPENDIX

Filter and derivation of the log-likelihood in the MSV-MIDAS model

Let $\{y_t\}_{t=1}^T$ be a time series following a MSM(M)-AR(p) process with transition probabilities depending on a high-frequency indicator $z_t^{(m)}$, as described in section 2. The conditional log-likelihood function of the observed data is given by:

$$L(\theta) = \sum_{t=p+1}^T \ln f(y_t | \underline{y_{t-1}}, \underline{z_{t-1}^{(m)}}; \lambda)$$

with $\underline{y_{t-1}} = \{y_{t-1}, \dots, y_1\}$, $\underline{z_{t-1}^{(m)}} = \{z_{t-1}^{(m)}, \dots, z_1^{(m)}\}$ the past of y_t and $z_t^{(m)}$ and λ the vector of parameters of the model.

The conditional log-likelihood function is derived from the following computations iterated for $t = p + 1, \dots, T$. In a first step, we derive the joint probability:

$$\begin{aligned} P(s_t = i, s_{t-1} = j, \dots, s_{t-p} = k | \underline{y_{t-1}}, \underline{z_{t-1}^{(m)}}; \lambda) = \\ P(s_t = i | s_{t-1} = j, z_{t-1}^{(m)}) \times P(s_{t-1} = j, \dots, s_{t-p} = k | \underline{y_{t-1}}, \underline{z_{t-2}^{(m)}}; \lambda) \end{aligned}$$

with $i, j, k = \{1, 2, \dots, M\}$ and where the time-varying transition probabilities are expressed as follows in the case of two regimes ($M = 2$):

$$\begin{aligned} P(s_t = 1 | s_{t-1} = 1, z_{t-1}^{(m)}) &= \Gamma[\alpha_1 + \beta_1 B(L^{1/m}, \Theta_1) z_{t-1}^{(m)}] \\ P(s_t = 2 | s_{t-1} = 2, z_{t-1}^{(m)}) &= \Gamma[\alpha_2 + \beta_2 B(L^{1/m}, \Theta_2) z_{t-1}^{(m)}] \\ P(s_t = 2 | s_{t-1} = 1, z_{t-1}^{(m)}) &= 1 - \Gamma[\alpha_1 + \beta_1 B(L^{1/m}, \Theta_1) z_{t-1}^{(m)}] \\ P(s_t = 1 | s_{t-1} = 2, z_{t-1}^{(m)}) &= 1 - \Gamma[\alpha_2 + \beta_2 B(L^{1/m}, \Theta_2) z_{t-1}^{(m)}] \end{aligned}$$

with the function $B(L^{1/m}, \Theta_{s_t})$ specified as:

$$B(L^{1/m}, \Theta_{s_t}) = \sum_{j=1}^K b(j, \Theta_{s_t}) L^{(j-1)/m}, \quad b(j, \Theta_{s_t}) = \frac{\exp(\theta_{1,s_t} j + \theta_{2,s_t} j^2)}{\sum_{j=1}^K \exp(\theta_{1,s_t} j + \theta_{2,s_t} j^2)}$$

In a second step, the joint density is derived as follows:

$$\begin{aligned} f(y_t, s_t = i, s_{t-1} = j, \dots, s_{t-p} = k | \underline{y_{t-1}}, \underline{z_{t-1}^{(m)}}; \lambda) = \\ f(y_t | s_t = i, s_{t-1} = j, \dots, s_{t-p} = k, y_{t-1}, \dots, y_{t-p}, \underline{z_{t-1}^{(m)}}; \lambda) \times P(s_t = i, s_{t-1} = j, \dots, s_{t-p} = k | \underline{y_{t-1}}, \underline{z_{t-1}^{(m)}}; \lambda) \end{aligned}$$

where the conditional density of y_t conditional upon the present state s_t and the past states

$s_{t-1}, \dots, s_{t-p}$ is given by:

$$f(y_t | s_t, s_{t-1}, \dots, s_{t-p}, y_{t-1}, \dots, y_{t-p}, \underline{z_{t-1}^{(m)}}; \lambda) = \frac{1}{\sqrt{2\pi}\sigma} \exp \left\{ \frac{y_t - \mu_{s_t} - \phi_1(y_{t-1} - \mu_{s_{t-1}}) - \dots - \phi_p(y_{t-p} - \mu_{s_{t-p}})}{2\sigma^2} \right\}$$

In a third step, the conditional density $f(y_t | \underline{y_{t-1}}, \underline{z_{t-1}^{(m)}}; \lambda)$ is derived by summing over all possible state sequences:

$$f(y_t | \underline{y_{t-1}}, \underline{z_{t-1}^{(m)}}; \lambda) = \sum_{i=1}^M \sum_{j=1}^M \dots \sum_{k=1}^M f(y_t, s_t = i, s_{t-1} = j, \dots, s_{t-p} = k | \underline{y_{t-1}}, \underline{z_{t-1}^{(m)}}; \lambda)$$

Finally, we derive the joint probability of the p states conditional upon to $\underline{y_t}$ and $\underline{z_{t-1}^{(m)}}$ from:

$$\begin{aligned} P(s_t = j, \dots, s_{t-p+1} = k | \underline{y_t}, \underline{z_{t-1}^{(m)}}; \lambda) &= \sum_{k=1}^M P(s_t = j, \dots, s_{t-p+1} = k, s_{t-p} = l | \underline{y_t}, \underline{z_{t-1}^{(m)}}; \lambda) \\ &= \sum_{k=1}^M \frac{f(y_t, s_t = i, s_{t-1} = j, \dots, s_{t-p+1} = k, s_{t-p} = l | \underline{y_{t-1}}, \underline{z_{t-1}^{(m)}}; \lambda)}{f(y_t | \underline{y_{t-1}}, \underline{z_{t-1}^{(m)}}; \lambda)} \end{aligned}$$

The initialization of the filter relies on the ergodic probabilities of the state in the FTP model.

Table 1: Monte Carlo experiment - DGP

	DGP1	DGP2	DGP3	DGP4	DGP5	DGP6
c	0.1	0.1	0.1	0.1	0.1	0.1
ρ	0.8	0.8	0.5	0.8	0.8	0.8
ω	1.0	1.0	1.0	1.5	1.0	1.0
μ_1	1.0	1.0	1.0	1.0	0.5	1.0
μ_2	-0.8	-0.8	-0.8	-0.8	-0.4	-0.8
ϕ	0.5	0.5	0.5	0.5	0.5	0.5
σ	0.3	0.3	0.3	0.3	0.3	0.3
α_1	2.0	2.0	0.5	0.5	0.5	0.5
α_2	0.5	0.5	1.0	1.0	1.0	1.0
β_1	2.0	2.0	2.0	2.0	2.0	0.6
β_2	-1.0	-1.0	-1.0	-1.0	-1.0	-0.5
$\theta_{1,1}$	2.0	2.0	2.0	2.0	2.0	0.2
$\theta_{2,1}$	-0.15	-0.15	-0.15	-0.15	-0.15	-0.015
$\theta_{1,2}$	2.0	1.0	2.0	2.0	2.0	2.0
$\theta_{2,2}$	-0.15	-0.5	-0.15	-0.15	-0.15	-0.15

Note: This table details the parameterizations of the MSV-MIDAS models used to simulate the Monte Carlo samples.

Table 2: Monte Carlo results - Estimate accuracy of the MSV-MIDAS model

	T	μ_1	μ_2	ϕ	σ	α_1	α_2	β_1	β_2	err_lbj	err_p11	err_p22
DGP1	200	0.000 (0.05)	0.001 (0.06)	-0.017 (0.06)	-0.003 (0.02)	0.115 (0.62)	-0.044 (0.44)	0.309 (0.75)	-0.214 (0.56)	0.20	0.04	0.09
	400	0.000 (0.03)	0.000 (0.04)	-0.010 (0.04)	0.000 (0.01)	0.060 (0.33)	-0.010 (0.28)	0.140 (0.40)	-0.070 (0.29)	0.09	0.03	0.06
	800	0.000 (0.02)	0.000 (0.03)	0.000 (0.03)	0.000 (0.01)	0.030 (0.21)	-0.010 (0.18)	0.060 (0.26)	-0.050 (0.18)	0.03	0.02	0.04
DGP2	200	0.000 (0.04)	0.001 (0.06)	-0.015 (0.06)	-0.002 (0.02)	0.162 (0.59)	-0.034 (0.56)	0.289 (0.74)	-0.277 (0.67)	0.29\0.52	0.04	0.10
	400	0.002 (0.03)	0.002 (0.05)	-0.010 (0.05)	-0.001 (0.01)	0.091 (0.33)	-0.031 (0.31)	0.145 (0.42)	-0.117 (0.32)	0.11\0.31	0.03	0.07
	800	0.000 (0.02)	0.001 (0.03)	-0.004 (0.03)	-0.001 (0.01)	0.050 (0.20)	-0.027 (0.21)	0.064 (0.24)	-0.052 (0.19)	0.04\0.17	0.02	0.05
DGP3	200	-0.002 (0.05)	-0.002 (0.06)	-0.016 (0.06)	-0.003 (0.02)	0.090 (0.36)	-0.012 (0.38)	0.248 (0.71)	-0.215 (0.71)	0.21	0.04	0.09
	400	0.000 (0.03)	0.000 (0.04)	-0.005 (0.04)	-0.001 (0.01)	0.044 (0.24)	-0.006 (0.24)	0.107 (0.45)	-0.064 (0.41)	0.09	0.03	0.06
	800	0.001 (0.02)	0.001 (0.03)	-0.003 (0.03)	-0.001 (0.01)	0.017 (0.16)	-0.001 (0.16)	0.042 (0.29)	-0.029 (0.27)	0.04	0.02	0.04
DGP4	200	-0.002 (0.05)	-0.001 (0.06)	-0.016 (0.06)	-0.002 (0.01)	0.157 (0.62)	0.013 (0.41)	0.266 (0.68)	-0.166 (0.42)	0.10	0.04	0.07
	400	0.000 (0.03)	0.000 (0.04)	-0.009 (0.04)	-0.002 (0.01)	0.085 (0.39)	-0.021 (0.29)	0.146 (0.40)	-0.063 (0.24)	0.05	0.03	0.05
	800	0.000 (0.02)	0.000 (0.03)	-0.003 (0.03)	-0.001 (0.01)	0.018 (0.26)	-0.016 (0.19)	0.053 (0.26)	-0.032 (0.15)	0.02	0.02	0.04
DGP5	200	0.000 (0.05)	0.001 (0.07)	-0.023 (0.08)	-0.002 (0.02)	0.272 (0.99)	-0.097 (0.87)	0.636 (1.32)	-0.380 (1.18)	0.25	0.05	0.11
	400	0.001 (0.03)	0.002 (0.05)	-0.007 (0.05)	-0.001 (0.01)	0.096 (0.44)	-0.006 (0.34)	0.177 (0.53)	-0.114 (0.39)	0.13	0.04	0.07
	800	0.000 (0.02)	-0.001 (0.03)	-0.005 (0.04)	-0.001 (0.01)	0.052 (0.27)	-0.015 (0.22)	0.095 (0.33)	-0.054 (0.24)	0.05	0.03	0.05
DGP6	200	-0.001 (0.05)	0.000 (0.07)	-0.016 (0.06)	-0.003 (0.01)	0.133 (0.48)	-0.044 (0.41)	0.246 (0.67)	-0.150 (0.57)	0.14	0.04	0.09
	400	0.001 (0.03)	-0.001 (0.05)	-0.008 (0.04)	-0.001 (0.01)	0.053 (0.28)	-0.044 (0.29)	0.099 (0.40)	-0.097 (0.35)	0.06	0.03	0.07
	800	0.001 (0.02)	0.001 (0.03)	-0.004 (0.03)	-0.001 (0.01)	0.033 (0.19)	-0.007 (0.20)	0.068 (0.26)	-0.038 (0.23)	0.03	0.02	0.05

Note: This table provides the average bias and in brackets the standard deviation of the parameter estimates of the MSV-MIDAS models for sample sizes $T = \{200, 400, 800\}$ over 1,000 replications. The last three columns show the approximation error on the weights and on the transition probabilities. We report the average value of these three criteria in the 1,000 Monte Carlo simulations.

Table 3: Monte Carlo results - Distribution of the t-statistics

	T	μ_1	μ_2	ϕ	σ	α_1	α_2	β_1	β_2	μ_1	μ_2	ϕ	σ	α_1	α_2	β_1	β_2	μ_1	μ_2	ϕ	σ	α_1	α_2	β_1	β_2
mean	DGP1																								
	200	-0.01	0.00	-0.22	-0.22	0.03	-0.05	0.23	-0.20	0.00	0.02	-0.18	-0.18	0.13	-0.08	0.27	-0.28	-0.05	-0.04	-0.21	-0.24	0.13	-0.02	0.20	-0.14
	400	0.02	0.01	-0.12	-0.12	0.08	-0.02	0.21	-0.10	0.05	0.03	-0.19	-0.13	0.16	-0.11	0.21	-0.24	-0.01	0.00	-0.07	-0.16	0.10	-0.01	0.11	-0.03
	800	-0.04	-0.04	-0.12	-0.07	0.07	-0.05	0.14	-0.16	0.02	0.02	-0.11	-0.11	0.17	-0.14	0.15	-0.17	0.05	0.03	-0.07	-0.13	0.05	0.00	0.07	-0.01
	200	1.08	1.01	1.02	1.04	0.98	0.98	0.95	0.96	1.03	1.03	0.99	1.01	1.01	1.01	0.98	0.97	1.06	1.05	0.98	1.07	0.98	1.01	1.02	1.03
	400	0.99	0.99	0.96	1.00	1.00	1.01	0.98	0.98	1.02	1.01	1.01	0.99	1.02	1.00	1.02	1.03	1.05	1.03	1.02	1.05	1.00	1.00	1.05	1.03
	800	1.00	1.00	1.02	1.02	0.99	0.99	0.98	0.95	1.02	0.99	0.98	1.04	0.98	1.02	0.94	1.06	1.02	1.03	1.05	0.97	0.99	0.99	0.96	1.00
	200	-0.02	-0.04	-0.05	-0.32	-0.30	0.05	-0.44	0.42	0.04	0.49	-0.01	-0.41	-0.26	0.37	-1.29	0.50	0.04	-0.03	-0.08	-0.14	-0.30	-0.05	-0.50	0.54
	400	0.05	-0.05	0.04	-0.08	-0.15	0.05	-0.37	0.33	-0.04	0.08	0.08	-0.14	-0.19	-0.16	-0.21	0.23	0.01	0.03	-0.04	-0.03	-0.06	-0.20	-0.29	0.34
kurt	800	-0.09	0.04	0.06	-0.07	-0.20	0.02	-0.26	0.40	0.01	0.00	0.03	-0.07	-0.11	-0.10	-0.33	0.34	0.11	0.01	-0.06	-0.17	-0.12	0.02	-0.07	0.26
	200	0.10	0.19	0.22	0.22	-0.16	-0.43	-0.13	0.06	0.36	4.40	-0.01	0.65	1.22	2.77	8.38	0.96	0.32	0.25	0.06	-0.26	-0.19	-0.24	0.58	0.27
	400	-0.02	0.09	0.46	0.04	0.10	-0.26	0.41	0.21	0.44	0.44	0.25	0.19	-0.13	0.08	-0.04	0.55	0.08	-0.11	-0.09	0.11	-0.18	-0.06	0.20	-0.07
JB	800	-0.03	0.10	-0.10	0.22	-0.30	-0.24	-0.08	0.28	0.06	-0.09	-0.09	0.27	-0.04	0.01	0.67	1.03	0.45	-0.02	-0.17	-0.07	-0.02	-0.24	-0.10	0.05
	200	0.78	0.41	0.29	0.00	0.00	0.02	0.00	0.00	0.06	0.00	0.98	0.00	0.00	0.00	0.00	0.00	0.10	0.25	0.53	0.05	0.00	0.24	0.00	0.00
	400	0.83	0.67	0.01	0.59	0.11	0.18	0.00	0.00	0.02	0.01	0.16	0.09	0.04	0.10	0.02	0.00	0.86	0.70	0.75	0.73	0.36	0.04	0.00	0.00
size 5%	800	0.54	0.69	0.61	0.23	0.00	0.30	0.00	0.00	0.92	0.86	0.79	0.15	0.37	0.41	0.00	0.00	0.01	0.98	0.39	0.07	0.29	0.29	0.51	0.00
	200	0.07	0.05	0.06	0.07	0.04	0.03	0.03	0.03	0.06	0.05	0.05	0.06	0.05	0.04	0.04	0.04	0.06	0.07	0.06	0.07	0.04	0.05	0.06	0.05
	400	0.04	0.05	0.04	0.06	0.06	0.05	0.05	0.05	0.06	0.06	0.06	0.05	0.05	0.05	0.06	0.07	0.06	0.06	0.05	0.07	0.05	0.05	0.07	0.06
size10%	800	0.05	0.05	0.05	0.06	0.04	0.04	0.05	0.04	0.06	0.04	0.06	0.04	0.06	0.04	0.06	0.06	0.06	0.06	0.07	0.05	0.05	0.05	0.05	0.05
	200	0.12	0.10	0.11	0.11	0.09	0.10	0.08	0.09	0.11	0.10	0.11	0.10	0.09	0.10	0.09	0.09	0.12	0.11	0.11	0.13	0.10	0.11	0.11	0.11
	400	0.10	0.09	0.08	0.10	0.09	0.10	0.10	0.08	0.10	0.10	0.11	0.10	0.11	0.11	0.11	0.13	0.11	0.12	0.10	0.12	0.10	0.09	0.12	0.10
power5%	800	0.11	0.10	0.11	0.10	0.10	0.10	0.09	0.09	0.10	0.09	0.10	0.12	0.10	0.11	0.08	0.11	0.10	0.11	0.09	0.10	0.10	0.10	0.10	0.11
	200	1.00	1.00	1.00	1.00	1.00	0.28	1.00	0.91	0.95	0.95	0.95	0.95	0.95	0.21	0.95	0.89	1.00	1.00	1.00	1.00	0.35	0.99	0.53	
	400	1.00	1.00	1.00	1.00	1.00	0.50	1.00	1.00	0.97	0.97	0.97	0.97	0.97	0.39	0.97	0.97	1.00	1.00	1.00	1.00	0.60	1.00	0.85	
power10%	800	1.00	1.00	1.00	1.00	1.00	0.76	1.00	1.00	0.99	0.99	0.99	0.99	0.99	0.66	0.99	0.99	1.00	1.00	1.00	1.00	0.87	1.00	0.99	
	200	1.00	1.00	1.00	1.00	1.00	0.37	1.00	0.96	0.95	0.95	0.95	0.95	0.95	0.30	0.95	0.93	1.00	1.00	1.00	1.00	0.45	0.99	0.70	
	400	1.00	1.00	1.00	1.00	1.00	0.60	1.00	1.00	0.97	0.97	0.97	0.97	0.97	0.50	0.97	0.97	1.00	1.00	1.00	1.00	0.71	1.00	0.92	
	800	1.00	1.00	1.00	1.00	1.00	0.84	1.00	1.00	0.99	0.99	0.99	0.99	0.99	0.75	0.99	0.99	1.00	1.00	1.00	1.00	0.92	1.00	1.00	
mean	DGP4																								
	200	-0.04	-0.02	-0.21	-0.15	0.08	0.06	0.14	-0.19	0.01	0.01	-0.25	-0.21	0.08	-0.06	0.28	-0.17	-0.03	0.01	-0.21	-0.27	0.13	-0.04	0.18	-0.07
	400	0.01	0.00	-0.18	-0.20	0.09	-0.04	0.19	-0.10	0.05	0.03	-0.11	-0.17	0.08	0.03	0.12	-0.08	0.02	-0.01	-0.15	-0.18	0.08	-0.10	0.10	-0.13
	800	0.02	-0.01	-0.07	-0.15	-0.02	-0.06	0.07	-0.09	0.01	-0.04	-0.11	-0.12	0.09	-0.03	0.13	-0.08	0.06	0.04	-0.12	-0.09	0.10	0.00	0.15	-0.05
	200	1.05	1.07	1.03	1.01	0.95	0.94	0.96	0.92	1.04	1.07	1.06	1.08	1.01	0.94	0.99	1.01	1.07	1.04	0.99	0.99	0.98	0.90	0.94	0.98
	400	1.07	1.04	1.02	1.06	0.97	1.00	0.93	1.01	1.03	1.00	0.94	1.05	1.00	0.99	0.98	1.02	0.97	1.06	1.00	1.04	0.97	1.00	0.96	1.01
	800	1.03	1.03	0.95	1.04	1.00	1.01	0.97	0.97	0.99	1.00	1.03	1.00	1.00	0.99	0.98	1.03	1.04	0.96	1.05	0.99	1.00	1.03	0.97	1.05
	200	0.10	-0.15	-0.07	-0.36	-0.42	0.10	-0.70	0.67	0.14	0.18	0.05	-0.23	-0.56	0.14	-0.71	0.76	-0.05	0.01	-0.09	-0.17	-0.32	-0.05	-0.64	0.52
	400	0.12	0.11	0.06	-0.23	-0.28	0.00	-0.31	0.37	0.00	0.05	0.03	-0.08	-0.25	0.19	-0.60	0.52	-0.01	0.00	-0.04	-0.18	-0.17	-0.05	-0.32	0.25
kurt	800	-0.09	0.01	-0.06	-0.33	-0.25	-0.02	-0.08	0.21	-0.04	0.15	0.04	-0.10	-0.36	-0.10	-0.40	0.41	0.02	-0.04	0.04	-0.19	-0.16	-0.06	-0.32	0.28
	200	0.07	-0.21	0.01	0.05	0.02	-0.28	0.57	0.87	0.16	-0.07	-0.03	0.20	0.15	-0.49	0.15	0.57	0.31	-0.29	-0.04	-0.23	-0.26	-0.28	0.60	0.33
	400	-0.07	0.11	-0.14	-0.08	-0.01	-0.07	0.01	-0.14	0.03	0.04	0.07	0.28	-0.23	-0.15	0.24	0.06	-0.04	-0.12	-0.07	0.01	-0.25	-0.17	0.29	-0.20
JB	800	-0.16	-0.09	-0.24	0.31	0.19	-0.27	0.03	0.15	-0.15	-0.07	-0.06	-0.29	0.00	-0.25	0.18	0.08	-0.07	0.03	-0.08	-0.02	0.11	-0.06	0.08	-0.10
	200	0.42	0.06	0.65	0.00	0.00	0.08	0.00	0.00	0.10	0.06	0.78	0.01	0.00	0.00	0.00	0.00	0.10	0.18	0.53	0.03	0.00	0.16	0.00	0.00
	400	0.27	0.30	0.48	0.01	0.00	0.91	0.00	0.00	0.98	0.78	0.83	0.12	0.00	0.03	0.00	0.00	0.97	0.75	0.80	0.06	0.03	0.47	0.00	0.00
size 5%	800	0.31	0.82	0.23	0.00	0.00	0.20	0.60	0.02	0.57	0.12	0.81	0.07	0.00	0.11	0.00	0.00	0.87	0.86	0.76	0.06	0.09	0.66	0.00	0.00
	200	0.06	0.07	0.07	0.06	0.03	0.04	0.03	0.03	0.07	0.06	0.07	0.08	0.06	0.02	0.03	0.04	0.07	0.06	0.06	0.06	0.04	0.03	0.03	0.04
	400	0.06	0.06	0.06	0.07	0.04	0.06	0.04	0.05	0.06	0.05	0.04	0.06	0.05	0.06	0.04	0.04	0.04	0.07	0.05	0.07	0.04	0.04	0.05	0.05
size10%	800	0.06	0.06	0.04	0.07	0.05	0.05	0.04	0.04	0.05	0.05	0.06	0.05	0.04	0.04	0.05	0.06	0.06	0.04	0.07	0.06	0.05	0.06	0.04	0.06
	200	0.12	0.13	0.13	0.11	0.08	0.08	0.08	0.07	0.11	0.13	0.12	0.1												

Table 4a: Estimation results of the MSV-MIDAS on US data

	STOCK			SPREAD		RATE		POIL	
	FTP	MSV1	MSV2	MSV1	MSV2	MSV1	MSV2	MSV1	MSV2
μ_1	0.932*** (0.10)	1.045*** (0.07)	0.959*** (0.077)	0.999*** (0.084)	1.019*** (0.068)	0.978*** (0.096)	0.972*** (0.096)	0.914*** (0.089)	0.914*** (0.088)
μ_2	-0.864*** (0.237)	-0.093 (0.119)	-0.273* (0.162)	-0.232 (0.236)	-0.17 (0.15)	-0.785*** (0.188)	-0.805*** (0.189)	-0.804*** (0.216)	-0.789*** (0.225)
ϕ_1	0.281*** (0.081)	0.106 (0.068)	0.211*** (0.075)	0.138 (0.085)	0.126* (0.075)	0.261*** (0.071)	0.263*** (0.071)	0.273*** (0.083)	0.267*** (0.084)
ϕ_2	0.247*** (0.08)	0.127* (0.069)	0.151** (0.075)	0.098 (0.085)	0.057 (0.075)	0.277*** (0.072)	0.275*** (0.072)	0.194** (0.082)	0.197** (0.083)
σ	0.639*** (0.038)	0.685*** (0.034)	0.67*** (0.035)	0.692*** (0.039)	0.686*** (0.037)	0.625*** (0.034)	0.625*** (0.034)	0.647*** (0.043)	0.648*** (0.044)
α_1	2.895*** (0.381)	8.125** (3.837)	2.122** (0.962)	1.453 (1.058)	-0.833 (2.001)	3.964*** (0.648)	3.944*** (0.675)	3.047*** (0.452)	3.027*** (0.463)
α_2	0.177 (0.621)	4.380** (1.765)	-3.511 (2.236)	3.085* (1.761)	2.001 (1.311)	1.231* (0.732)	1.320 (0.831)	-1.653 (1.517)	-1.622 (1.417)
β_1	-	3.401** (1.693)	3.627*** (1.368)	2.969* (1.689)	8.669 (6.578)	-5.768*** (1.871)	-5.216*** (1.712)	-0.110* (0.059)	-0.095 (0.088)
β_2	-	-1.600*** (0.621)	-1.509* (0.903)	-1.367 (0.921)	-0.742 (0.699)	2.729* (1.435)	3.056 (1.861)	-2.325 (1.815)	-1.991 (1.325)
k	7	11	13	11	13	11	13	11	13
LL	-253.07	-238.49	-235.66	-242.71	-241.65	-235.87	-235.6	-247.74	-246.99
AIC	520.15	498.98	497.33	507.42	509.3	493.74	497.21	517.48	519.98
QPS	0.161	0.208	0.178	0.121	0.154	0.117	0.117	0.159	0.158
TPI0	0.313	0.438	0.688	0.313	0.438	0.50	0.50	0.438	0.438
TPI1	0.563	0.688	1	0.625	0.688	0.75	0.75	0.563	0.563
TPI2	0.625	0.875	1	0.938	0.875	0.75	0.75	0.563	0.563
LR flat	-	0.01	0.00	0.10	0.15	0.00	0.00	0.06	0.13
LB1(4)	0.70	0.13	0.23	0.70	0.77	0.26	0.25	0.75	0.74
LB1(12)	0.48	0.10	0.20	0.43	0.49	0.65	0.63	0.12	0.14
LB1(20)	0.35	0.03	0.15	0.21	0.31	0.37	0.35	0.13	0.15
LB2(4)	0.82	0.15	0.3	0.67	0.68	0.6	0.58	0.71	0.73
LB2(12)	0.56	0.12	0.32	0.31	0.37	0.77	0.77	0.16	0.18
LB2(20)	0.31	0.03	0.22	0.13	0.19	0.5	0.50	0.12	0.14

Notes: This table shows the estimation results for US over 1959Q1-2013Q4 of the model with fixed transition probabilities (column FTP), the MSV-MIDAS models with fixed weights (MSV1) and variable weights (MSV2). The first part of the table gives the parameter estimations and the associated standard errors in brackets. Significance levels: *** if the coefficient is significant at 1%, ** at 5%, * at 10%. The second part of the table provides the number of estimated parameters (k), the estimated log-likelihood (LL), the Akaike information criterion (AIC), the QPS and TPI criteria. The TPI is computed with $\lambda = 0.5$ and with a lead/lag parameter τ equal to 0, 1 and 2 quarters. The last part of the table shows the p-value of the LR test of the null hypothesis of equal weights (LR flat), the p-values of the Ljung-Box test for omitted autocorrelation of order 1 to p in the generalized LB1(p) and Rosenblatt's residuals LB2(p).

Table 4b: Estimation results of the MSV-MIDAS on UK data

	STOCK			SPREAD		RATE		POIL	
	FTP	MSV1	MSV2	MSV1	MSV2	MSV1	MSV2	MSV1	MSV2
μ_1	0.777*** (0.075)	0.809*** (0.065)	0.826*** (0.067)	0.774*** (0.107)	0.779*** (0.107)	0.825*** (0.081)	0.803*** (0.078)	0.737*** (0.066)	0.746*** (0.068)
μ_2	-0.917*** (0.268)	-0.782*** (0.166)	-0.848*** (0.177)	-1.211*** (0.236)	-1.191*** (0.231)	-0.896*** (0.194)	-0.882*** (0.184)	-1.303*** (0.233)	-1.216*** (0.222)
ϕ_1	-0.037 (0.112)	-0.038 (0.083)	0.068 (0.101)	0.077 (0.091)	0.07 (0.089)	-0.018 (0.097)	-0.051 (0.089)	-0.026 (0.081)	-0.041 (0.081)
ϕ_2	0.081 (0.1)	0.128 (0.084)	0.049 (0.09)	0.265*** (0.096)	0.276*** (0.093)	0.243*** (0.093)	0.243*** (0.087)	-0.005 (0.08)	0.064 (0.078)
ϕ_3	0.105 (0.091)	0.018 (0.082)	0.02 (0.085)	0.168** (0.081)	0.17** (0.081)	0.099 (0.082)	0.103 (0.082)	0.177** (0.077)	0.151** (0.077)
σ	0.675*** (0.045)	0.664*** (0.039)	0.644*** (0.040)	0.634*** (0.04)	0.631*** (0.039)	0.639*** (0.038)	0.641*** (0.038)	0.681*** (0.039)	0.676*** (0.039)
α_1	3.646*** (0.669)	3.576*** (0.602)	3.147*** (0.502)	3.603*** (0.646)	3.741*** (0.63)	4.253*** (0.832)	3.886*** (0.692)	8.518** (3.627)	10.538** (4.409)
α_2	1.223* (0.74)	35.113 (36.03)	42.456 (119.48)	8.886 (13.09)	9.631 (18.82)	14.101 (10.05)	16.003 (11.67)	6.414 (4.36)	6.052 (5.24)
β_1	-	0.337** (0.165)	0.093* (0.051)	-0.293 (0.241)	-0.388* (0.208)	-5.073*** (1.718)	-4.858** (1.955)	-1.054* (0.565)	-1.488** (0.67)
β_2	-	-9.664 (9.70)	-22.921 (67.00)	-37.297 (108.21)	-48.944 (610.11)	57.142 (38.91)	65.532 (46.09)	-4.158 (2.59)	-3.686 (3.59)
k	8	12	14	12	14	12	14	12	14
LL	-174.05	-168.22	-167.83	-165.53	-164.84	-162.44	-161.96	-164.76	-161.54
AIC	364.11	360.44	363.67	355.05	357.68	348.88	351.92	353.51	351.08
QPS	0.164	0.151	0.173	0.272	0.276	0.197	0.197	0.235	0.222
TPI0	0.25	0.25	0.25	0.125	0.125	0.25	0.25	0.25	0.25
TPI1	0.50	0.625	0.50	0.375	0.375	0.375	0.375	0.375	0.50
TPI2	0.625	0.75	0.75	0.375	0.375	0.625	0.625	0.50	0.50
LR flat	-	0.02	0.07	0.45	0.57	0.01	0.04	0.13	0.03
LB1(4)	0.65	0.53	0.35	0.43	0.49	0.64	0.71	0.71	0.75
LB1(12)	0.76	0.30	0.44	0.37	0.36	0.46	0.46	0.78	0.53
LB1(20)	0.71	0.26	0.53	0.47	0.45	0.35	0.32	0.57	0.50
LB2(4)	0.94	0.76	0.60	0.68	0.73	0.87	0.86	0.84	0.93
LB2(12)	0.85	0.63	0.49	0.36	0.36	0.50	0.48	0.73	0.53
LB2(20)	0.76	0.42	0.48	0.42	0.41	0.24	0.22	0.58	0.56

Notes: This table shows the estimation results for UK over 1975Q1-2013Q4 of the model with fixed transition probabilities (column FTP), the MSV-MIDAS models with fixed weights (MSV1) and variable weights (MSV2). The first part of the table gives the parameter estimations and the associated standard errors in brackets. Significance levels: *** if the coefficient is significant at 1%, ** at 5%, * at 10%. The second part of the table provides the number of estimated parameters (k), the estimated log-likelihood (LL), the Akaike information criterion (AIC), the QPS and TPI criteria. The TPI is computed with $\lambda = 0.5$ and with a lead/lag parameter τ equal to 0, 1 and 2 quarters. The last part of the table shows the p-value of the LR test of the null hypothesis of equal weights (LR flat), the p-values of the Ljung-Box test for omitted autocorrelation of order 1 to p in the generalized LB1(p) and Rosenblatt's residuals LB2(p).

Table 5a: Out-of-sample results of the MSV-MIDAS on US data

	h	2	5/3	4/3	1	2/3	1/3	0
QPS	FTP	0.2170	0.2114	0.2114	0.1912	0.1804	0.1806	0.1180
	STOCK	0.1942	0.2424	0.2090	0.1586	0.1206	0.1038	0.0794
	SPREAD	0.1840	0.1834	0.1752	0.1338	0.1406	0.1516	0.0424
	RATE	0.2304	0.2362	0.2710	0.1898	0.1910	0.2062	0.1430
	POIL	0.2106	0.2022	0.1964	0.1670	0.1668	0.1490	0.0678
TPI0	FTP	0	0	0	0.167	0.167	0.167	0.333
	STOCK	0.167	0	0.167	0.500	0.667	0.833	0.833
	SPREAD	0.333	0.333	0.333	0.333	0.333	0.167	0.667
	RATE	0	0	0	0.167	0.333	0.167	0.333
	POIL	0	0	0.167	0.333	0.333	0.167	0.667
TPI1	FTP	0	0	0	0.333	0.167	0.167	0.667
	STOCK	0.667	0.667	0.667	1	1	1	1
	SPREAD	0.667	0.500	0.667	1	1	0.500	1
	RATE	0	0.167	0	0.500	0.667	0.500	0.500
	POIL	0.167	0.167	0.167	0.500	0.500	0.500	0.667
TPI2	FTP	0	0	0	0.500	0.333	0.333	0.833
	STOCK	0.833	0.833	0.833	0.833	0.833	0.833	1
	SPREAD	0.667	0.500	0.667	0.833	0.833	0.667	1
	RATE	0.167	0.500	0.167	0.500	0.833	0.500	0.500
	POIL	0.167	0.167	0.167	0.333	0.333	0.333	0.833

Notes: This table shows the QPS and TPI criteria obtained on the forecast states for US over 1990Q1-2013Q4 for the model with fixed transition probabilities (line FTP) and the MSV-MIDAS models with fixed weights. The TPI is computed with $\lambda = 0.5$ and with a lead/lag parameter τ equal to 0, 1 and 2 quarters. For each criterion, entries in bold indicate the best performing model.

Table 5b: Out-of-sample results of the MSV-MIDAS on UK data

	h	2	5/3	4/3	1	2/3	1/3	0
QPS	FTP	0.3070	0.2988	0.2962	0.2748	0.2754	0.2842	0.2878
	STOCK	0.3274	0.3152	0.31	0.3036	0.3178	0.3258	0.2840
	SPREAD	0.3354	0.3554	0.3576	0.3482	0.3146	0.3466	0.2818
	RATE	0.3546	0.3372	0.2924	0.2910	0.2756	0.2634	0.2794
	POIL	0.3576	0.3468	0.3122	0.3554	0.3258	0.3246	0.3706
TPI0	FTP	0.250	0.250	0.250	0.250	0	0	0.250
	STOCK	0	0.250	0.250	0.250	0	0	0.250
	SPREAD	0	0	0	0	0	0	0.250
	RATE	0	0	0.250	0.250	0.250	0.250	0
	POIL	0	0.250	0	0	0	0	0
TPI1	FTP	0.250	0.250	0.250	0.500	0.500	0.500	0.750
	STOCK	0.250	0.250	0.250	0.500	0.500	0.500	0.750
	SPREAD	0	0	0	0.500	0.500	0.250	0.250
	RATE	0.250	0.250	0.250	0.250	0.250	0.500	0.750
	POIL	0.250	0.250	0.250	0	0.250	0.250	0.250
TPI2	FTP	0.500	0.500	0.500	0.750	0.750	0.750	0.750
	STOCK	0.500	0.500	0.500	0.750	0.750	0.750	0.750
	SPREAD	0.500	0.250	0.250	0.500	0.500	0.500	0.500
	RATE	0.250	0.250	0.500	0.500	0.250	0.500	0.750
	POIL	0.250	0.250	0.500	0.500	0.500	0.750	0.250

Notes: This table shows the QPS and TPI criteria obtained on the forecast states for UK over 2000Q1-2013Q4 for the model with fixed transition probabilities (line FTP) and the MSV-MIDAS models with fixed weights. The TPI is computed with $\lambda = 0.5$ and with a lead/lag parameter τ equal to 0, 1 and 2 quarters. For each criterion, entries in bold indicate the best performing model.

Figure 1: Simulated data for DGP2

Notes: This figure depicts the simulated high- and low-frequency variables from DGP2, the weights of the high-frequency variable in each regime and the time-varying transition probabilities $p_{11}(t)$ and $p_{22}(t)$.

Figure 2a: United States: Smoothed probabilities of being in recession

(a) Fixed transition probabilities

(b) Variable transition probabilities (spread with fixed weights)

Notes: This graph plots the US smoothed recession probabilities of the FTP and MSV-MIDAS models. The shaded areas represent recessions according to the NBER business cycle classification.

Figure 2b: United Kingdom: Smoothed probabilities of being in recession

(a) Fixed transition probabilities

(b) Variable transition probabilities (stock returns with fixed weights)

Notes: This graph plots the UK smoothed recession probabilities of the MSV-MIDAS model. The shaded areas represent recessions according to the ECRI business cycle classification.

Figure 3a: United States: Smoothed probabilities of being in recession

(a) Stock returns (fixed weights)

(b) Spread (fixed weights)

(c) Federal Funds (fixed weights)

(d) Oil price (fixed weights)

(e) Stock returns (variable weights)

(f) Spread (variable weights)

(g) Federal Funds (variable weights)

(h) Oil price (variable weights)

Figure 3b: United Kingdom: Smoothed probabilities of being in recession

(a) Stock returns (fixed weights)

(b) Spread (fixed weights)

(c) Base rate (fixed weights)

(d) Oil price (fixed weights)

(e) Stock returns (variable weights)

(f) Spread (variable weights)

(g) Base rate (variable weights)

(h) Oil price (variable weights)

Figure 4: Estimated weights of the MSV-MIDAS models

Notes: This graph depicts the estimated weights of the MSV-MIDAS model on US and UK data. The first column reports the results for the model with fixed weights and the last two columns the estimated functions in the model with state-dependent weights.