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monopoly and redistribution: a note***

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Two-part pricing, public discriminating monopoly and redistribution : a note*

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Résumé

This note analyzes some properties of optional two-part pricing in a two type economy. First, the optimal contracts along the Paretian frontier are described. Then, the duality relation between the Rawlsian program and the discriminating monopoly is demonstrated. Last, this property is used to build a mutualist mechanism implementing the constrained Pareto optima.

Classification J.E.L. : D42, D61, D63

1 Introduction

Optional two-part pricing, extensively used by many public utilities (electricity, water, railways ...), gives, as shown by Sharkey and Sibley (1993)[4], some freedom to redistribute the social surplus.¹ These authors show in a partial equilibrium framework that, when a monopoly proposes a menu of contracts, each specifying the fee and the charge price, it is possible for a social planner controlling this monopoly to redistribute towards the weak demand consumer. Our contribution is not to extend their study to a new framework or to other pricings. Our ambition is, first, to emphasize the redistributive mechanism of optional two-part pricing, notably with the help of some graphical presentations, and, second, to propose a simple incentive mechanism implementing the more redistributive optima. As we will see, this mechanism takes advantage of the dual relationship between the program of the discriminating monopoly and the social planner's program.

This note is organized as follows. The economy is described in section 2. The constrained Pareto optima are characterized in section 3 even though an implementing mechanism is proposed and studied in section 4. Limits and possible extensions of this work are discussed in the last section.

2 The economy

There are two goods, the produced good and the numéraire one. Their quantities are respectively noted q and w . The economy is composed of two types of agents, indexed $i = 1, 2$, defined by their quasi-linear utility functions:

$$u_i(q_i, w_i) = V_i(q_i) + w_i$$

The functions V_i verify the following properties :

Assumption 1 V_i is continuously twice differentiable with :

$$V'_i(q) := \frac{\partial V_i(q)}{\partial q} > 0, \quad V''_i(q) := \frac{\partial^2 V_i(q)}{\partial q^2} < 0, \quad V_i(0) = 0 \quad (1)$$

and :

$$V'_2(q) > V'_1(q) \quad (2)$$

¹See also Roberts (1979) [3] for the case of non-linear pricing.

Relations (1) state that the inverse demand is positive and strictly decreasing. Relation (2) implies that type 2's demand is higher than type 1's; for quasi-linear utility functions this relation is also the standard single crossing assumption.

The cost function C of the monopoly which produces the good q verifies the following assumption :

Assumption 2 C is a convex function on $]0, +\infty[$ ²:

$$C'(q) \geq 0, C''(q) \geq 0 \quad (3)$$

The monopoly using this technology proposes two contracts (p_1, E_1) and (p_2, E_2) , where p_1, p_2 are the usage charges, E_1 and E_2 the fees. As asymmetric information prevents perfect discrimination, the contracts must be incentive-compatible. Moreover, in order to eliminate trivial cases, we will suppose that First-best optima are characterized by strictly positive consumptions.

3 Constrained Pareto optima

In this section, the constraints regimes of the Paretian program are specified. Then, the Pareto frontier is outlined and the main properties of optimal contracts are discussed. This section doesn't propose any new results, its aim is simply to clarify the characterization of the Pareto frontier and above all to present, with the help of a graphic, a pedagogical analysis of the redistributive mechanism of the optional two-part pricing.

In our economy, the constrained Pareto program is :

$$P_p(\bar{s}_2) : \begin{cases} \max_{(p_i, E_i)_{i=1,2}} S_1(p_1) - E_1 \\ s.t. : \\ (SC_2) : S_2(p_2) - E_2 \geq \bar{s}_2 \\ (IC_i) : S_i(p_i) - E_i \geq S_i(p_j) - E_j, \quad i, j = 1, 2 \\ (B) : \sum_{i=1,2} n_i [p_i D_i(p_i) + E_i] = C(D(p_1, p_2)) \end{cases}$$

where $D_i(p_i) = V'_i{}^{-1}(p_i)$, $S_i(p_i) = V_i(D_i(p_i)) - p_i D_i(p_i)$, $s_i = S_i(p_i) - E_i$, $D(p_1, p_2) = n_1 D_1(p_1) + n_2 D_2(p_2)$ and n_i is the number of agents of type i .

To discuss the constraints regimes of $P_p(\bar{s}_2)$, it is useful to consider the first-best optima which verify the incentive constraints. Actually, as for every first-best optimum, prices are equal to the marginal cost, incentive constraints

²Fixed costs are thus allowed.

require equality of fees. Hence, there is a unique first-best optimum which verify the incentive constraints, the so-called Coase two-part pricing. The types surplus at the Coase solution are noted s_1^{co} and s_2^{co} .³

In the following, only the domain $\bar{s}_2 < s_2^{co}$ is studied⁴. In this domain the binding incentive constraint is (IC_2) .⁵ To know if (SC_2) binds, it is useful to introduce the Rawlsian solution defined by the maximization of the type 1 surplus subject to incentive and budget constraints. As these constraints always bind, the Rawlsian objective function, after some substitutions, can be rewritten:

$$W_R(p_1, p_2) := \frac{1}{n_1 + n_2} (S_s(p_1, p_2) - n_2 (S_2(p_1) - S_1(p_1))) \quad (4)$$

where the social surplus $S_s(p_1, p_2) = \sum_i n_i V_i(D_i(p_i)) - C(D(p_1, p_2))$.

As usual in this literature, we assume the concavity of this function, and hence the unicity of the Rawlsian solution (p_1^R, p_2^R) . If s_i^R is the surplus of type i at this Rawlsian optimum⁶, two cases must be distinguished depending on whether $\bar{s}_2$ is above or below s_2^R . For $s_2^R \leq \bar{s}_2 < s_2^{co}$ ⁷, the constrained Pareto program is equivalent to maximize $W_R(p_1, p_2)$ subject to (SC_2) . The (assumed) strict concavity of W_R implies two results. First, (SC_2) is binding, second, the second-best frontier, in the surplus space (s_1, s_2) , is continuous and strictly monotonic (see figure 1).

The remaining question is what are the properties of the optimal contracts along the second-best frontier. As, for $s_2^R \leq \bar{s}_2 < s_2^{co}$, only incentive and participation constraints of type 2 bind, the program $P_p(\bar{s}_2)$ is reduced to the following :

$$P_{pr}(\bar{s}_2) : \begin{cases} \max_{p_1, p_2} \frac{1}{n_1 + n_2} [S_s(p_1, p_2) - n_2 (S_2(p_1) - S_1(p_1))] \\ s.c : \\ (SC_2) : \sum_{i=1,2} n_i V_i(D_i(p_i)) = C(D(p_1, p_2)) - \\ n_1 (S_2(p_1) - S_1(p_1)) + (n_1 + n_2) . \bar{s}_2 \end{cases}$$

³Of course, the Coase solution only exists if the fixed cost is not too big with respect to the demand.

⁴The other case is symmetrical. To extend our results to the domain $\bar{s}_2 > s_2^{co}$, one only needs to rewrite program $P_p(\bar{s}_2)$ by permuting indices, *i.e.* one needs to maximize s_2 subject to the participation constraint of type 1.

⁵The space being limited here and the proof being classical, it is not reproduced in this note.

⁶As by assumption s_1^{co} is strictly positive, $s_1^R > 0$. Then, from (IC_2) and eq. (2), it can be shown that $s_2^R > s_1^R > 0$.

⁷Of course, for $\bar{s}_2 < s_2^R$, as s_2^R is the lower value that the Paretian social planner can actually assign to type 2, (SC_2) of program $P_p(\bar{s}_2)$ is always released.

Figure~1: The frontier of the constrained Pareto optima with optional two-part pricing

For each $\bar{s}_2$, first-order conditions give optimal prices:

$$p_2 = C' (D(p_1, p_2)) \quad (5)$$

$$p_1 = (1 + \lambda)C' (D(p_1, p_2)) - \lambda Rm_1 (D_1(p_1)) + \frac{\lambda}{D'_1(p_1)} D_2(p_1) + \frac{n_2}{n_1 D'_1(p_1)} (D_2(p_1) - D_1(p_1)) \quad (6)$$

where $Rm_1(q_1) = V'_1(q_1) + q_1 V''_1(q_1)$ is the marginal revenue upon type 1 and λ the Lagrangian multiplier.⁸

⁸Equation (6) can be rewritten as follows :

$$p_1 = C' (D(p_1, p_2)) + \frac{n_2 - \lambda n_1}{1 + \lambda} \frac{D_1(p_1) - D_2(p_1)}{n_1 D'_1(p_1)}$$

We note that for $\lambda = n_2/n_1$, we obtain the Coasian prices: $p_1 = p_2 = C' (D(p_1, p_2))$. Otherwise, one could show that for $\lambda = 0$, $p_2 = p_2^R$. So, intuitively, we could interpret λ as a relative weight of type 2 in a linear social welfare function; but, this interpretation assumes the concavity of the Pareto frontier (in the surplus space).

of each type when the fee is E and the price equal to the marginal cost c ¹⁰. The curves passing through A and B are the iso-surplus curves corresponding respectively to types 1 and 2.

At the Coase optimum, and in fact at each constrained Pareto optimum, the only way to increase s_1 is obviously to decrease s_2 . Nevertheless, for this, one needs to release the incentive constraint of type 2, *i.e.* to decrease the surplus of the dishonest type 2. Starting from the Coasian point A, the only way to proceed is to raise p_1 (with an appropriate adjustment of E_1 leaving s_1 constant)¹¹. As the surplus of the dishonest type 2, reached at point F, is now only s'_2 ($< s_2$), the social planner can extract at most $d\pi_2$ with the new contract (E', c) . As we can see in figure 2, the increase of budget surplus $d\pi_2$ over type 2 exceeds the budget loss $d\pi_1$ over type 1.¹² So, starting from the Coase equilibrium, such an adjustment leaves a positive net budget surplus which, equally redistributed to check incentive constraints, increases type 1's surplus¹³.

Since it permits more redistributive surplus allocation than the Coase solution, optional two-part pricing is a useful tool for the social planner. But, if he doesn't directly control the monopoly, implementation of the constrained optima is questionable: how can he induce the monopoly to select the right two-part pricing?

4 Implementation by discriminating monopoly

In this section we build mechanisms which implement the more redistributive optima. To reach this aim we study a regulated monopoly, the so-called monopoly *à la* Edgeworth.¹⁴ This monopoly is supposed to use optional two-part pricing and is subject to an additional constraint to leave a minimal surplus to type 1. The implementing mechanisms are then deduced from the duality relation between the discriminating program of this monopoly and the Rawlsian program; this duality relation was incidentally noticed by Roberts (1979) ([3], p. 80-81) in a continuous types economy but for a *non linear* pricing.

¹⁰For simplicity, we supposed in this graphic that the marginal cost is constant.

¹¹Indeed, it is easy to see that a decrease of p_1 (leaving s_1 constant) incites type 2 to lie, increases s_2 , and breaks the budget constraint.

¹²In fact, at first order, $d\pi_1$ is negligible which is not the case of $d\pi_2$.

¹³Those adjustments can be reproduced for all constrained Pareto optima but the Rawlsian one.

¹⁴This solution deserves to be called monopoly *à la* Edgeworth with reference to Edgeworth's contributions to the regulated monopoly literature (e.g. Edgeworth (1910) [2]).

Before introducing the monopoly *à la* Edgeworth, let us first introduce the program of the simple discriminating monopoly using optional two-part pricing :

$$P_m : \begin{cases} \max_{(p_i, E_i)_{i=1,2}} \sum_{i=1,2} n_i (p_i D_i(p_i) + E_i) - C(D(p_1, p_2)) \\ s.t. : \\ (PC_i) : S_i(p_i) - E_i \geq 0, \quad i = 1, 2 \\ (IC_i) : S_i(p_i) - E_i \geq S_i(p_j) - E_j, \quad i, j = 1, 2 \end{cases}$$

and begin to show that the Rawlsian prices are also the monopolistic ones.

Proposition 1 *The Rawlsian prices (p_1^R, p_2^R) are the solutions of the monopoly program.*

Proof. Under assumption 1, (IC_2) and (PC_1) are the only active constraints and one gets :

$$E_1 = S_1(p_1), \quad E_2 = S_1(p_1) + (S_2(p_2) - S_2(p_1))$$

Hence, after substitutions, the objective function becomes :

$$\begin{aligned} \Pi(p_1, p_2) &= \sum_{i=1,2} n_i \cdot V_i(D_i(p_i)) - C(D(p_1, p_2)) - n_2(S_2(p_1) - S_1(p_1)) \\ &= (n_1 + n_2) \cdot W_R(p_1, p_2) \end{aligned}$$

The end of the proof is now obvious. ■

Consequently, the monopoly equilibrium and the Rawlsian solution differ only by E_1 and E_2 . In fact, this result hides a fundamental link between them: they are the two polar solutions of the monopoly *à la* Edgeworth. The latter is a discriminating monopoly with an additional surplus constraint for type 1 :

$$P_{em}(\bar{s}_1) : \begin{cases} \max_{(p_i, E_i)_{i=1,2}} \sum_{i=1,2} n_i (p_i D_i(p_i) + E_i) - C(D(p_1, p_2)) \\ s.t. : \\ (SC_1) : S_1(p_1) - E_1 \geq \bar{s}_1 \\ (PC_2) : S_2(p_2) - E_2 \geq 0 \\ (IC_i) : S_i(p_i) - E_i \geq S_i(p_j) - E_j, \quad i, j = 1, 2 \end{cases}$$

As one can easily demonstrate using classical arguments, equation 2 of assumption 1 implies that (SC_1) and (IC_2) are the only binding constraints. After manipulations, the program $P_{em}(\bar{s}_1)$ is reduced to the subsequent free maximization :

$$\max_{p_1, p_2} (n_1 + n_2) [W_R(p_1, p_2) - \bar{s}_1]$$

Optimal prices and quantities are independent of $\bar{s}_1$ level and equal to the monopoly ones. By raising $\bar{s}_1$ (from 0 to s_1^R), all surplus distributions between the monopoly equilibrium and the Rawlsian solution can be achieved. Actually, for $\bar{s}_1 = s_1^R$, the program of the monopoly à la Edgeworth is the dual of the Rawlsian program. So, naturally, it gives not only the same prices but also the same fees. Of course, the monopoly à la Edgeworth is an abstract mechanism since $\bar{s}_1$ is exogenous.

A way to make the mechanism more realistic is to consider a *mutualist mechanism*, i.e. a profit sharing device. In our framework, one can view a mutualist monopoly as a firm which redistributes all its profit to its members¹⁵ according to a sharing key. If this key is contingent upon the chosen contracts, membership guarantees a part of the profit even if the member doesn't consume. Furthermore, we will suppose that this sharing key is fixed ex ante and the profits are redistributed ex post. The mutualist monopoly's customers are thus considered just as shareholders. Therefore, it is natural to suppose that the aim of the mutualist monopoly is to maximize profit. Finally, for a sharing key $(\theta_1, \theta_2, \bar{\theta})$ the program of this monopoly is then :

$$p_{mu}(\theta_1, \theta_2, \bar{\theta}) : \begin{cases} \max_{(p_i, E_i)_{i=1,2}} \sum_{i=1,2} n_i (p_i D_i(p_i) + E_i) - C(D(p_1, p_2)) \\ s.t. : \\ (PC_1) : S_1(p_1) - E_1 + \theta_1 \Pi \geq \bar{\theta} \Pi \\ (PC_2) : S_2(p_2) - E_2 + \theta_2 \Pi \geq \bar{\theta} \Pi \\ (IC_i) : S_i(p_i) - E_i + \theta_i \Pi \geq S_i(p_j) - E_j + \theta_j \Pi, \quad i, j = 1, 2 \end{cases}$$

where $\bar{\theta} \Pi$ is the guarantee share profit and $\theta_i \Pi$ is the profit share of type i .

Proposition 2 *If the monopoly profit is uniformly distributed, $\theta_1 = \theta_2 = \bar{\theta}$, the mutualist monopoly equilibrium gives the Rawlsian surplus to each type.*

Proof. With the uniform sharing key, the program is reduced to the program P_m . So the optimal quantities and fees are the Rawlsian ones. ■

The intuition of the previous proposition can be easily grasped graphically (see figure 3). Points A and B correspond to the private monopoly equilibrium (where $s_1 = 0$)¹⁶. Because of the quasi-linearity of preferences, a uniform monetary transfer (to both types) implies a vertical translation of the Edgeworthian monopoly equilibrium: when a surplus $\bar{s}_1$ is granted to type 1, the private equilibrium is translated to the new equilibrium represented by A' and B' .

¹⁵If the good produced is a public utility, all agents are potential consumers and can be viewed as members of the mutualist monopoly.

¹⁶As it is depicted, a further rise of p_1 doesn't increase the net profit ($0 < d\pi_2 = -d\pi_1$).

is active and the program $P_{mpc}(\bar{p})$ is reduced to :

$$P_{mpc}(\bar{p}) : \begin{cases} \max_{p_1, p_2} S_s(p_1, p_2) - n_2 (S_2(p_1) - S_1(p_1)) \\ s.t. : \\ (PCC_i) : p_i \leq \bar{p}, i = 1, 2 \end{cases}$$

For right values of $\bar{p}$ ($p_1^{co} \leq \bar{p} \leq p_1^R$), this program implies, for every value of p_1 , $p_2 = C'$. And by strict concavity, $p_1 = \bar{p}$. To implement constrained Pareto optima (in quantities and prices) for $s_2^R < \bar{s}_2 \leq s_2^{co}$, it is sufficient to set $\bar{p} = p_1^p(\bar{s}_2)$, where $p_1^p(\bar{s}_2)$ is the optimal price p_1 of program $P_p(\bar{s}_2)$.¹⁷ ■

5 Conclusion

This note explores the redistributive properties of optional two-part pricing in a two type economy. It shows that a monopolistic structure market augmented by a uniform profit sharing allows one to implement the most redistributive optimum, the Rawlsian solution. If a price-cap is added, this mutualist mechanism allows one to achieve less redistributive constrained optima. However, there are three caveats to bear in mind.

First, in a pure mutualist mechanism, only customers share profit, even though, in the proposed mechanism, each agent receives profit independently of his consumption decision. However, as here each agent is a customer, the difference is blurred. So, this mechanism can only be applied to a subset of quasi-universally consumed goods, such as electricity, water, public transport.

Second, the efficiency of this mechanism requires of course the social planner to have such precise information as to prevent managers and employees from capturing profits. So, the mechanism supposes a strict monitoring of the managers.

Last, a strong implicit assumption of this paper is the fact that the social planner has a unique redistribution tool : public pricing. Of course, in a more general framework he can also use income taxation. So, a natural extension would be to study the complementarity between discriminating public pricing and income taxation.¹⁸

Références

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¹⁷To understand that the optimal price p_1 of program $P_p(\bar{s}_2)$ is a function of $\bar{s}_2$, it is useful to notice that the equation (8) implicitly depends of $\bar{s}_2$ (through λ).

¹⁸See for example Boadway and Marchand (1995) [1].

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