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Use of multi-criteria decision-aids for risk zoning and management of large area subjected to mining-induced hazards

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Abstract

The end of iron-mining operations in France's Lorraine region has raised the problem of post-mining management. Collapses and subsidence have recently occurred in this region, which focused on the necessity to develop a specific methodology for risk zoning. The proposed methodology is based on the multi-criteria decision-aid approach called 'ELECTRE TRI', with aim to assign zones at risk into predefined classes. It allows experts opinions, qualitative and quantitative criteria and uncertainties to be taken into account.

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1. Introduction

Mine subsidence events have recently occurred in the Lorraine iron-mining basin (North-eastern France) causing major damages to surface buildings and infrastructures (Deck et al., 2003). These events have raised widespread awareness of the severity of human, technical, financial and legal risks due to the end of mining activities. The Lorraine iron-mining basin, which covers a surface area of 1150 km², had been mined for more than a century (Varoquaux, 1995) up until 1997 by reliance upon the 'room and pillar' caving method under urban areas in order to prevent the ground from subsidence. This 'technical' prevention strategy, however, has failed several times throughout history and, consequently, it has now become necessary to establish a 'managerial' prevention strategy based on risk analysis. According to natural hazard theory, risk appears wherever and whenever assets are subjected to hazards; it is usually defined as 'the expected potential loss due to a particular hazard for a given area and reference period' (UN, 1992) and can be mathematically defined as the combination of hazard and vulnerability (UN, 1992;

Varnes, 1984). In France, Risk Prevention Plans (RPPs) have been adopted to control risks due to ground movement (MATE, 1999). Such plans consist of producing four principal outputs: an informative report about the natural phenomena; a hazard map (gives the contours of the area subjected to given levels of hazards); an assets evaluation (whether formalized or not on a map); and a risk map (superimposition of hazard and assets maps). In practice, RPPs are commonly prepared using the 'typical configurations' method (Didier, 1999), which entails identifying within the study area a limited number of rupture scenarios, called 'configurations'. A hazard level is then ascribed to each configuration. Afterwards, homogeneous zones are established according to these configurations in order to produce a hazard map. Despite the advantages associated with the 'typical configurations' method, which serves to formalize expert approach, some imperfections still remain. Firstly, when this method is placed into practice, it only provides a hazard map, not a risk map. Secondly, the hazard map established is extremely dependent upon the knowledge and experience held by the engineer in charge. Thirdly, the method has been developed for low extended area (quarries) rather than large extended area (mining basin). Lastly, uncertainties, inaccuracies and

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Fig. 1. Risk induced by the presence of a mine and assets on the ground surface.

Fig. 2. Surface impacts due to mining collapse.

Fig. 3. Example of a house damaged by mining subsidence (Auboué, France, 1996).

incomplete knowledge are not explicitly identified nor taken into account. In order to overcome these shortcomings, we are proposing herein another method, based on a multi-criteria decision-aid approach, for prioritising zones at risk.

2. Multi-criteria, decision-aid methods for risk analysis

The zones at risk studied lie in urbanized areas within France's Lorraine region, which had been mined using the 'rooms and pillars' method. These zones span a land area of over 20 km². The 'rooms and pillars' method consists of extracting ore in the rooms and then leaving pillars to support the weight of the overlying ground (see Fig. 1).

Mining works, however, can present a danger to people and buildings when pillars fail to withstand ground loads. Such a rupture can induce a gradual settlement of the surface, called subsidence (Fig. 2a),

Fig. 4. General principle underlying the chosen risk analysis method.

or a more brutal settlement called collapse (Fig. 2b), which consists in an instantaneous drop of the entire overlying ground (Whittaker and Reddish, 1989). These phenomena may, in certain instances, produce dramatic consequences (ruin) on surface buildings (Fig. 3) and infrastructures.

Within such a context, risk analysis proves to be a complex decision-making problem where the available information is uncertain (missing information, such as geological data) and imprecise (mining works maps) and where knowledge is incomplete (e.g. soil–structure interaction). Multi-criteria, decision-aid methods can help overcome these limitations because they basically include mathematical mechanisms to manage these aspects. One such method called ELECTRE (Roy, 1985; Roy and Bouyssou, 1993) is widely used in France and Europe for resolving decision-type problems involving institutions and experts (Roy and Bouyssou, 1993). We would like to emphasize the feasibility, advantages and difficulties encountered when adapting the ELECTRE method to the prioritisation of zones subjected to mining subsidence or collapse. The application of the proposed

methodology requires the assistance of a committee of experts to handle complexity issues. The general principle underlying this methodology has been depicted in Fig. 4.

The committee of experts called upon for the present purposes has defined four risk classes, given that an appropriate monitoring device must be set up for the identified zones as a function of estimated risk level. By convention, Class 1 corresponds to the highest risk level and Class 4 to the lowest. A permanent monitoring system must be introduced for all Class 1 zones, while only reference levelling (topographic surveys) is required for Class 4 zones (see Table 1). The aim of this method is to identify and categorize homogeneous zones into these four predefined risk classes.

The first step consists in selecting the studied risk (collapse or subsidence) among those available (collapse, subsidence, pollution, sinkholes, slope instabilities, etc.). This choice will then influence the second step, which entails choosing the criteria applied to provide information on the risk and, similarly, to identify homogeneous zones over the study area; homogeneous

Table 1
Monitoring measures for the four risk classes

Risk classes	Monitoring measures
1	Long term continuous monitoring based on the recording of underground microseismic activity.
2	In depth investigation including geological boring if necessary.
3	Reference and annual levelling.
4	Reference levelling.

zones are those whose intrinsic characteristics (i.e. values of the selected criteria) are identical (see Figs. 5 and 6). The last step consists of choosing a prioritisation method for analysing these homogeneous zones, i.e. moving from a partial evaluation of the zone on each criterion to a global evaluation of the entire zone (level of risk) (Fig. 7) based on all criteria.

In the following sections, we will examine in greater detail how risk zones are selected, how a set (family) of criteria and sets of weights are defined, and how the ‘ELECTRE TRI’ method is integrated for prioritising zones at risk.

3. Selection of zones at risk and choice of evaluation criteria and weighting sets

The procedure used to select zones at risk is illustrated

in Fig. 8. In addition to this schematic approach, the zones must satisfy two selection tests defined by the committee of experts: (1) the stress applied on pillars must be greater than or equal to 7.5 MPa (no pillar rupture is expected below this value); (2) zone width must exceed $0.9 \times H$, where H represents depth from the operating wall (no significant effects on surface below this value). If these conditions are not fulfilled, it is considered that the selected zone presents no danger and monitoring is not necessary in such cases. The same conclusions are valid for zones with no surface assets.

Let Z be the set of zones at risk and Z_i be a zone such that $Z_i \in Z$, with $i = 1, \dots, m$, where m represents the number of zones.

The data available for each zone can be either quantitative (stress applied to pillars, depth, zone extent, etc.) or qualitative (presence of faults, superposition of

Fig. 5. Mining works map divided into homogeneous zones (Z₁ to Z₈).

Fig. 6. Aerial photograph of assets above mining works shown on Fig. 5, with the contours of zones Z_1 to Z_8 .

pillars, sensitivity to flooding, etc.). The risk due to mining subsidence is therefore represented in a multi-criteria form. Due to the extent of the basin studied and

the objectives inherent in risk mapping, relevant data are synthesized through a family of criteria that may be defined on each zone selected for the prioritisation study.

Fig. 7. Results of the risk analysis by zone.

Fig. 8. Identification and selection of zones for prioritisation.

These criteria are implemented in conjunction with and under the supervision of a committee of experts. These criteria taken as a whole form a consistent family (Roy, 1985), meaning that:

- Two zones with identical criteria values are assigned to the same risk class: the criteria family is said to be *exhaustive*;
- A zone on which the value on one criterion is reduced (in terms of decreasing risk) cannot be assigned to a lower class (higher risk) than that initially assigned: the criteria family is said to be *consistent*;
- A zone cannot be assigned to a class when one criterion is removed from the criteria family: the criteria family is said to be *non-redundant*.

The various criteria used have been grouped into two headings: ‘susceptibility of the mine to collapse’ and ‘surface sensitivity’ (see Table 1). The first refers to the ‘probability of rupture’ and the second to both the ‘intensity of the rupture’ and the ‘value and vulnerability of assets’.

Let g_j be a criterion within the family of criteria, where $j=1,\dots,n$ and n is the number of criteria. A qualitative or quantitative scale is then defined for each criterion g_j . The quantitative criteria $g_{1.1}$, $g_{2.1}$, $g_{2.2}$, $g_{2.3}$ and $g_{2.4}$ (Table 2) are evaluated in their own units. For example, $g_{1.1}$, which denotes the stress applied on pillars, is given in MPa and $g_{2.2}$, representing depth, in m. The remaining criteria are qualitative and expressed in various appropriate scales. The ‘superimposition of pillars’ criterion can take the following status: (0) a mined layer; (10) two layers with well superimposed pillars or a thick intermediate layer (≥ 7 m); (20) two layers with poorly-superimposed pillars or a thin intermediate layer (< 7 m). This codification (0, 10, 20) does not connote any numerical significance (Roy, 1985) and other coding systems, such as (6, 7, 8), could also be used.

Two types of surface assets are considered for each zone Z_i : buildings and infrastructures. This distinction makes it necessary to apply different criteria to evaluate the risk for Z_i according to surface occupancy. If Z_i is a zone with buildings, then the criteria used would be

Table 2
Family of criteria

No.	Criterion	Nature	Unit	Codes	Direction of risk increase
G1. Susceptibility of the mine to collapse					
$g_{1.1}$	Corrected mean stress applied on pillars	Quanti.	MPa	$k \times \frac{0.25 \times H}{1 - \tau}$	+
$g_{1.2}$	Existence of fault	Quali.	(code)	0: no; 10: yes.	+
$g_{1.3}$	Superimposition of pillars	Quali.	(code)	0: only one mined layer; 10: two well superimposed layers or thick intermediate layer (≥ 7 m); 40: two bad superimposed layers or thin intermediate layer (< 7 m).	+
$g_{1.4}$	Size and regularity of pillars	Quali.	(code)	0: large pillars; 10: small regular pillars; 20: small irregular pillars.	+
$g_{1.5}$	Sensitivity of rock to flooding (depending on the rock type)	Quali.	(code)	0: no sensitivity; 10: sensitive; 20: very sensitive; 30: highly sensitive.	+
G2. Surface sensitivity					
$g_{2.1}$	Depth of the top mined layer	Quanti.	m	Given on maps. Called H.	–
$g_{2.2}$	Maximum expected subsidence	Quanti.	m	Deduced from subsidence models. Called A_m .	+
$g_{2.3}$	Expected surface deformation (deduced from subsidence models)	Quanti.	mm/m	$\varepsilon_{\max} = 1.5 \times \frac{A_m}{H}$	+
$g_{2.4}$	Zone extent	Quanti.	km ²	Given on maps.	+
$g_{2.5.1}$	Vulnerability of building	Quali.	(code)	5: commercial zones; 10: isolated houses; 20: grouped houses; 30: long buildings; 40: urban area.	+
$g_{2.5.2}$	Vulnerability of roads	Quali.	(code)	0: no road; 10: local road; 20: national road; 30: highway.	+
$g_{2.5.3}$	Vulnerability of railways	Quali.	(code)	0: no line; 10: standard line; 20: speed line; 30: high speed line.	+
$g_{2.5.4}$	Vulnerability of bridges	Quali.	(code)	0: no bridge; 10: presence of bridge.	+
$g_{2.5.5}$	Vulnerability of various networks	Quali.	(code)	0: no network; 10: electricity network; 20: water network; 30: gas network.	+

$g_{1.1}$ to $g_{1.5}$ and $g_{2.1}$ to $g_{2.5.1}$; if, however, Z_i is a zone with infrastructures, the criteria would be $g_{1.1}$ to $g_{1.5}$, $g_{2.1}$ to $g_{2.4}$ and $g_{2.5.2}$ to $g_{2.5.5}$. Table 3 gives a detailed example of the evaluation of a zone with buildings.

Note that due to the ultimate ‘Public safety’ purpose of this risk analysis, buildings have been considered more sensitive than infrastructures. If both buildings and infrastructures are present on a given surface zone Z_i ,

then the criteria for zones with buildings are to be employed.

4. From partial to global risk evaluation

4.1. Choice of a method for prioritising zones at risk

Several multi-criteria, decision-aid methods can be used to progress from a partial evaluation of the risk on

Table 3:
Example of information provided on the zone Z9 (see table 5) for all prioritisation criteria

	Mine surface	Calculation formula	Definition of criteria
	<p>Area extent = 2.5 km² Presence of a building, isolated houses and road.</p>		$g_{2,4}(Z_g) : 2.5 \text{ km}^2$ $g_{2,5,1}(Z_g) : 10$
	Mine bottom	Calculation formula	Definition of criteria
	<p>Mined area = 35 %. Min depth = 115 m. Max depth = 120 m. No fault. Small and irregular pillars. Two layers. No sensitive to flooding. Neighbour dropremoval. Span : 4 m.</p>	<p>Corrected stress : $1.2 \cdot 0.25 \cdot \text{max depth} / (1 - \text{extraction ratio})$.</p> <p>Max expected subsidence: deduced from subsidence curves.</p> <p>Max deformation : Parameter $1.5 \cdot \text{Subsidence} / \text{Depth}$.</p>	$g_{1,1}(Z_g) : 11.3 \text{ MPa}$ $g_{1,2}(Z_g) : 0$ $g_{1,3}(Z_g) : 40$ $g_{1,4}(Z_g) : 20$ $g_{1,5}(Z_g) : 0$ $g_{2,1}(Z_g) : 115 \text{ m}$ $g_{2,2}(Z_g) : 2.18 \text{ m}$ $g_{2,3}(Z_g) : 1.89 \text{ mm/m}$

each criterion g_i to a global risk evaluation based on all criteria (Roy, 1985; Roy and Bouyssou, 1993). One of the most frequently used methods is the ‘weighted mean

method’, which provides a complete order and enables differentiating the weight of criteria. It is possible, for example, to ascribe a particular criterion g_j with a greater weight, hence a greater role in determining the class assignment. This method, however, imposes certain strict constraints, such as the numerical evaluation on each criterion g_j and leads to compensation effects since equal evaluation differences correspond to equal risk variation measurements (Roy, 1985). Furthermore, the weighted mean method does not easily accommodate uncertainties and inaccuracies. These elements make ‘ELECTRE’-type methods and ‘ELECTRE TRI’ in particular, applicable. ELECTRE TRI is appropriated to sort zones Z_i into four predefined, mutually exclusive and ordered risk classes (Roy, 1985; Yu, 1992).

Fig. 9. Four risk classes separated by three low profiles Pr_h .

Fig. 10. Principal position of class profiles Pr_h on each criterion g_j .

Fig. 11. Three comparison set-ups between Z_i and Pr_h .

Each risk class h ($h \in \{1, 2, 3, 4\}$) is delimited by a threshold profile Pr_h (see Fig. 9). A class profile Pr_h is a virtual zone representing an entry standard into a risk class. Each profile Pr_h takes a value on each criterion g_j (Fig. 10). Since Class 1 is the highest risk class, profile Pr_1 'displays a risk at least as high as' Pr_2 for every criterion. This is written $Pr_1 S Pr_2$. In turn, Pr_2 'displays a risk at least as high as' Pr_3 for every criterion ($Pr_2 S Pr_3$).

For each criterion g_j it is possible to compare a zone Z_i with a profile Pr_h . This is called a partial comparison. It is thus partially possible to determine if zone Z_i lies within risk class h . In this comparison process, uncertainty on criteria values can be taken into account (with ELECTRE TRI) through two thresholds: the indifference threshold q_j and the preference threshold p_j (see Appendix A and Fig. 11).

In order to assign zones to risk classes (to go from a partial assignment to a global assignment), we have chosen the 'ELECTRE TRI' pseudo-disjunctive procedure (see Appendix A). This procedure is based on the calculation of a credibility index σ and its comparison with a parameter λ . Intuitively, λ represents a desired majority of decision-making criteria (Roy and Bouyssou, 1993). There are only three possible situations (Yu, 1992):

1. Indifference: $\sigma(Z_i, Pr_h) \geq \lambda$ and $\sigma(Pr_h, Z_i) \geq \lambda$, which is denoted $Z_i I Pr_h$ (with Z_i and Pr_h being equivalent);
2. Strict preference (risk strictly higher than):
 - 2.1 $\sigma(Z_i, Pr_h) \geq \lambda$ and $\sigma(Pr_h, Z_i) < \lambda$, which is denoted $Z_i P Pr_h$ (Z_i exhibits greater risk than Pr_h);
 - 2.2 $\sigma(Pr_h, Z_i) \geq \lambda$ and $\sigma(Z_i, Pr_h) < \lambda$, which is denoted $Pr_h P Z_i$ (Pr_h exhibits greater risk than Z_i);
3. Incomparability: $\sigma(Z_i, Pr_h) < \lambda$ and $\sigma(Pr_h, Z_i) < \lambda$, which is denoted $Z_i R Pr_h$, meaning that Z_i and Pr_h are incomparable.

In cases where $Z_i P Pr_h$ or $Z_i I Pr_h$, Z_i is assigned to the class with Pr_h representing the lower profile of the

class h . The use of the pseudo-disjunctive procedure within the ELECTRE TRI method implies that when comparison between a zone and a profile proves impossible (due to incomparability), the zone also gets assigned to the class where Pr_h is the lower profile.

For each zone, the assignment procedure starts by a comparison with the lower profile (Pr_3) and ends when a profile is strictly preferred to the zone ($Pr_h P Z_i$), in which case Z_i is assigned to class $h + 1$.

4.2. Choice of weighting sets for the criteria family

The choice of criteria-specific weightings requires the input of expert opinion. Two methods were used to determine such weighting sets. The first consists of holding a meeting of the expert committee and discussing the criteria until a consensus is reached regarding the weight of each criterion. The second, known as the 'cards method' (Arondel and Girardin, 2000; Roy and Figuera, 2002), is well-adapted to ELECTRE-type methods and facilitates communication and discussion between experts. The operating approach consists of associating a card with each criterion. Experts are then individually asked to: sort the criteria from the least important to the most important, with the possibility of placing criteria at the same rank; insert blank cards to indicate the relative importance among criteria from one rank to the next; and specify the ratio of the highest weight to the least weight assigned the criteria. This revised Simos method (Roy and Figuera, 2002) has many advantages, namely preventing criteria from being eliminated by a zero-weight assignment and incorporating the various weightings ascribed by participants as a function of their interests and skills. This method was used to obtain one weighting set for zones with buildings and another set for zones with infrastructures (see Table 4). The weight of criterion j is denoted k_j .

The weights listed in Table 4 demonstrate that the

Table 4

Weighting sets selected by the experts committee, for the prioritisation of zones at risk under buildings and under infrastructures

Criterion	$g_{1,1}$	$g_{1,2}$	$g_{1,3}$	$g_{1,4}$	$g_{1,5}$	$g_{2,1}$	$g_{2,2}$	$g_{2,3}$	$g_{2,4}$	$g_{2,5,1}$	$g_{2,5,2}$	$g_{2,5,3}$	$g_{2,5,4}$	$g_{2,5,5}$
Weights for zones with buildings	5	1	1	1	5	1	1	20	1	10	–	–	–	–
Weights for zones with infrastructures	5	1	1	1	5	1	1	20	4	–	4	4	4	2

Table 5
Evaluations of zones on each criterion

Zone	Criterion									
	$g_{1.1}$	$g_{1.2}$	$g_{1.3}$	$g_{1.4}$	$g_{1.5}$	$g_{2.1}$	$g_{2.2}$	$g_{2.3}$	$g_{2.4}$	$g_{2.5.1}$
Z_1	5.8	10	0	20	0	35	2.37	6.8	3.6	20
Z_2	4.8	0	40	0	0	70	1.28	1.83	0.2	10
Z_3	9.7	10	10	0	30	200	1.67	0.84	7.4	30
Z_4	10.4	10	10	10	30	203	1.68	0.83	9.0	20
Z_5	9.7	0	10	0	10	222	1.2	0.54	1.8	20
Z_6	9.8	10	0	20	0	50	1.27	2.54	6.7	20
Z_7	12.3	0	0	0	30	155	0.96	0.61	14.1	10
Z_8	11.2	10	0	0	30	180	0.71	0.39	6.4	20
Z_9	11.3	0	40	20	0	115	2.18	1.89	2.5	10
Z_{10}	11.0	10	0	10	30	170	0.31	0.18	2.6	20
Profile										
Pr_1	14	10	40	20	20	110	1.8	1	35	30
Pr_2	10	10	10	10	10	150	1.4	0.82	20	20
Pr_3	8	0	10	10	10	190	1	0.63	6	20
Threshold										
Q	0.05	0	0	0	0	10	0.1	0.05	0.5	0
P	0.1	0	0	0	0	20	0.2	0.09	1	0
Weights	5	1	1	1	5	1	1	20	1	10

expert committee has assigned considerable importance to both the expected surface deformation ($g_{2.3}$) and building vulnerability ($g_{2.5.1}$) criteria, by ascribing weights of 20 and 10, respectively, which represent 43% and 22% of total criteria family weight. Such a decision suggests that these two criteria play a determinant role in the choice of the risk class.

Note that a single criterion with a low weight may be sufficient to change the decision made by the ‘strongest’ criterion in the family, provided it is contradictory and imposes a ‘veto’ (Roy, 1985; Yu, 1992), as shown below.

5. Implementation of the ‘ELECTRE TRI’ method

This section will describe the prioritisation method for zones at risk through a ten-zone sample (Table 5). In practical terms, the method is capable of handling many more zones; up until now, we have actually applied it on over 300 zones.

All ten zones included in Table 5 contain buildings. Parametric studies have allowed identifying indifference and preference thresholds for each quantitative criterion.

In order to specify the appropriate monitoring system for each zone in Table 5, it is necessary to define entry conditions for risk classes. A statistical data analysis of an assignment sample of 52 zones under buildings and 69 zones under infrastructures helped establish these conditions which have then been validated by the committee of experts.

The ELECTRE TRI method was used in conjunction with the ELECTRE TRI software, Version 2.0 (Mousseau et al., 1999). This software relies upon two procedures for risk class assignment: the pessimistic procedure and the optimistic procedure. The pseudo-disjunctive procedure mentioned above (Section 4.1) corresponds to the optimistic procedure in the software.

The two credibility indices $\sigma(Z_i, Pr_h)$ and $\sigma(Pr_h, Z_i)$ were calculated for each of the ten zones and are presented in Table 6.

These credibility indices are then compared to parameter λ (see Section 4.1) which must be greater than 0.5. This value of λ means that at least 50% of the criteria family accredits the entry of the zone Z_i into the considered risk class. In practice, the value of λ has to be greater than (1-highest weight/total weight), i.e. above 0.57 in our case. With this minimum λ value, assignment of a zone to a risk class needs the agreement of more than one criterion within the criteria family (this value prevents the most weighted criterion from being the only one to decide the assignment class). No value of parameter λ higher than 0.57 is better than another. In this example, we have tested six values: 0.60, 0.65, 0.70, 0.75, 0.80 and 0.85.

For purposes of illustration, let's consider zone Z_8 and $\lambda=0.65$. We first calculate the credibility indices and then proceed with risk class assignment. The comparison procedure begins with the low profile Pr_3 of Class 3. Z_8 and Pr_3 assume the values 11.2 MPa and 8 MPa on criterion $g_{1.1}$, respectively. From Table 5, we

Table 6
Credibility indices $\sigma(Z_i, Pr_h)$ and $\sigma(Pr_h, Z_i)$ for the ten study zones

	Pr_3	Pr_2	Pr_1	Assignment of zones to risk classes $\lambda=0.65$
$\sigma(Z_i, Pr_h)$				
$\sigma(Pr_h, Z_i)$				
Z_1	0.717	0.717	0.500	Class 1
	0.500	0.522	0.544	
Z_2	0.500	0.478	0.478	Class 1
	0.522	0.522	0.544	
Z_3	0.956	0.913	0.348	Class 2
	0.087	0.674	0.891	
Z_4	0.972	0.935	0.130	Class 2
	0.283	0.891	0.891	
Z_5	0.565	0.456	0.000	Class 4
	0.891	1.000	1.000	
Z_6	0.848	0.826	0.500	Class 1
	0.383	0.522	0.544	
Z_7	0.717	0.239	0.109	Class 3
	0.739	0.783	0.891	
Z_8	0.500	0.456	0.130	Class 4
	0.739	0.783	0.891	
Z_9	0.630	0.609	0.489	Class 1
	0.391	0.391	0.565	
Z_{10}	0.500	0.478	0.130	Class 4
	0.717	0.783	0.891	

Table 7

Agreement and disagreement indices for the comparison between Z_8 and Pr_3 on each criterion

Criterion	Weight of criterion k_j	Z_8 S Pr_3		Pr_3 S Z_8		Relation between Z_8 and Pr_3
		Agreement index c_j	Disagreement index d_j	Agreement index c_j	Disagreement index d_j	
$g_{1.1}$	5	1	0	0	0	$Z_8 > Pr_3$
$g_{1.2}$	1	1	0	0	0	$Z_8 > Pr_3$
$g_{1.3}$	1	0	0	1	0	$Z_8 < Pr_3$
$g_{1.4}$	1	0	0	1	0	$Z_8 < Pr_3$
$g_{1.5}$	5	1	0	0	0	$Z_8 > Pr_3$
$g_{2.1}$	1	1	0	0	0	$Z_8 > Pr_3$
$g_{2.2}$	1	0	0	1	0	$Z_8 < Pr_3$
$g_{2.3}$	20	0	0	1	0	$Z_8 < Pr_3$
$g_{2.4}$	1	1	0	1	0	$Z_8 \text{ I } Pr_3$
$g_{2.5.1}$	10	1	0	1	0	$Z_8 \text{ I } Pr_3$

observe that Z_8 is preferred to Pr_3 as far as criterion $g_{1.1}$ is concerned. Thus, the agreement index $c_{1.1}(Z_8, Pr_3) = 1$ and the disagreement index $d_{1.1}(Z_8, Pr_3) = 0$ (see Appendix A). We now proceed in the same manner for all criteria and for the two complementary assertions ‘ Z_8 displays a risk at least as high as Pr_3 ’ (Z_8 S Pr_3) and ‘ Pr_3 displays a risk at least as high as Z_8 ’ (Pr_3 S Z_8) as shown on Table 7.

Since no veto has been used in the present study (see Appendix A), all disagreements are equal to 0 (Table 7). The credibility indices $\sigma(Z_8, Pr_3)$ and $\sigma(Pr_3, Z_8)$ thereby have the following values:

$$\sigma(Z_8, Pr_3) = c(Z_8, Pr_3) = \frac{\sum_{j \in F} k_j c_j(Z_8, Pr_3)}{\sum_{j \in F} k_j} = 0.50 < 0.65$$

$$\sigma(Pr_3, Z_8) = c(Pr_3, Z_8) = \frac{\sum_{j \in F} k_j c_j(Pr_3, Z_8)}{\sum_{j \in F} k_j} = 0.74 > 0.65$$

Consequently, Pr_3 is strictly preferred to Z_8 , the procedure is stopped and Z_8 is then assigned to Class 4.

To understand how this comparison procedure may proceed with other profiles, let's now consider the example of zone Z_1 . It has been determined that when this zone is compared with the low profile Pr_3 of Class 3, the credibility index $\sigma(Z_1, Pr_3) = 0.72 > 0.65$, while $\sigma(Pr_3, Z_1) = 0.50 < 0.65$. Consequently, Z_1 can be placed in Class 3. If we were to continue the comparison with the low profile Pr_2 of Class 2, the credibility indices would be: $\sigma(Z_1, Pr_2) = 0.72 > 0.65$ and $\sigma(Pr_2, Z_1) = 0.52 < 0.65$. Z_1 can thus be placed in Class 2 as well. A comparison between Z_1 and the low profile Pr_1 of Class 1 reveals that: $\sigma(Z_1, Pr_1) = 0.50 < 0.65$ and $\sigma(Pr_1, Z_1) =$

$0.54 < 0.65$. Z_1 and Pr_1 are thereby incomparable and Z_1 can then be assigned to Class 1 (according to the pseudo-disjunctive procedure).

By proceeding in the same manner for all zones with $\lambda = 0.65$, zones Z_1, Z_2, Z_6 and Z_9 are assigned to Class 1, zones Z_3 and Z_4 to Class 2, Z_7 to Class 3, and zones Z_5, Z_8 and Z_{10} to Class 4.

The robustness of the classification must, however, be demonstrated by analysing the sensitivity to the change of parameter λ , the weighting sets, the thresholds and profiles (Roy and Bouyssou, 1993). The results from such a study are summarized in Table 8, which reveals that the classification of zones Z_3, Z_4 and Z_{10} remains insensitive to parameter changes. The classification of other zones oscillates between two successive classes (see Table 8).

We have only shown results for $\lambda = 0.65$. If λ were ascribed too high a value (e.g. 0.85), the principle of maximum caution would become exacerbated. Under such conditions, most comparisons conducted between zones and profiles would conclude with their incomparability and would assign zones to Class 1 by virtue of the pseudo-disjunctive procedure.

Table 8
Sensitivity analysis for all 10 study zones

Zone	Assignment of zone to risk Class ($\lambda = 0.65$)	Sensitivity analysis
Z_1	Class 1	Classes 1 and 2
Z_2	Class 1	Classes 1 and 2
Z_3	Class 2	Stable
Z_4	Class 2	Stable
Z_5	Class 4	Classes 3 and 4
Z_6	Class 1	Classes 1 and 2
Z_7	Class 3	Classes 3 and 4
Z_8	Class 4	Classes 3 and 4
Z_9	Class 1	Classes 1 and 2
Z_{10}	Class 4	Stable

This index is built with all criteria and used to estimate the credibility of the assertion: ‘the risk level of Z_i is at least as high as that of Pr_h ’ ($Z_i \leq Pr_h$) (or ‘the risk level of Pr_h is at least as high as that of Z_i ’ ($Pr_h \leq Z_i$)). It is denoted $\sigma(Z_i, Pr_h)$ (or $\sigma(Pr_h, Z_i)$) and may be equal to values lying between 0 and 1 (Roy, 1985). When this index equals 0, the assertion is completely rejected either because no criterion satisfies it or because a criterion is in a position to veto. If the

Fig. 12. Partial agreement index c_j and disagreement index d_j .

index equals 1, the assertion is completely accepted, which means that no criterion is in opposition. Between 0 and 1, the credibility index is based on two other component indices: an agreement index and a disagreement index (see Fig. 12).

The agreement index is a weighted mean of the partial agreement indices and is calculated as follows:

$$c(Z_i, Pr_h) = \frac{\sum_{j \in F} k_j c_j(Z_i, Pr_h)}{\sum_{j \in F} k_j}$$

where F is the set of criteria numbers and c_j the partial agreement index on criterion j (Fig. 12).

When the assertion ‘the risk for Z_i is at least as high as that for Pr_h ’ is rejected for a criterion, the partial agreement index for this criterion is equal to 0. The agreement index therefore only takes account of those criteria for which this assertion is not rejected.

The partial disagreement index $d_j(Z_i, Pr_h)$ is calculated for criteria that reject the assertion ‘the risk for Z_i is at least as high as that for Pr_h ’ and is defined as follows:

$$d_j(Z_i, Pr_h) = 1 - \frac{v_j(g_j(Pr_h)) - \min\{g_j(Z_i) - g_j(Pr_h), v_j(g_j(Pr_h))\}}{v_j(g_j(Pr_h)) - \min\{g_j(Z_i) - g_j(Pr_h), p_j(Pr_h)\}}$$

where $v_j(g_j(Pr_h))$ denotes a veto threshold for criterion g_j , related to profile Pr_h . When the risk level of zone Z_i for criterion g_j is significantly lower than that of profile Pr_h for the same criterion ($g_j(Pr_h) - g_j(Z_i) > v_j(g_j(Pr_h))$), the assertion ‘the risk level for Z_i is at least as high as that for Pr_h ’ must be rejected and $d_j(Z_i, Pr_h) = 1$ (Roy, 1985). When the assertion is validated on

criterion g_j , this index is equal to 0. Such a case arises when the difference between the risk level of Z_i and that of Pr_h on the considered criterion is less than the preference threshold $p_j(Pr_h)$. Between these two values, the index increases as a function of the value of the difference between the risk level of Z_i and that of Pr_h (Yu, 1992) (Fig. 12).

In light of the above discussion, the credibility index is determined as follows:

- $\sigma(Z_i, Pr_h) = c(Z_i, Pr_h)$, in the absence of disagreeing criteria;
- $\sigma(Z_i, Pr_h) = 0$, if a disagreeing criterion imposes its veto on the assertion: ‘the risk level of Z_i is at least as high as that of Pr_h ’;
- $\sigma(Z_i, Pr_h) < c(Z_i, Pr_h)$, if $\exists j / d_j(Z_i, Pr_h) > c(Z_i, Pr_h)$.

The credibility index is thus defined as follows:

$$\sigma(Z_i, Pr_h) = c(Z_i, Pr_h) \prod_{j \in \bar{F}} \frac{1 - d_j(Z_i, Pr_h)}{1 - c(Z_i, Pr_h)},$$

where:

$$\bar{F} = \{j \in F / d_j(Z_i, Pr_h) > c(Z_i, Pr_h)\}$$

A.3. ‘ELECTRE TRI’ assignment procedures

Two extreme rationales may be called upon when assigning a zone to a particular risk class: either Z_i is assigned to the lowest class given by a criterion or Z_i is assigned to the highest class given by a criterion. These two rationales correspond to a principle of caution, which is minimized in the first case and maximized in the second. ELECTRE TRI offers two assignment procedures to attenuate these two extremes, namely a

pseudo-conjunctive procedure and a pseudo-disjunctive procedure. The pseudo-disjunctive procedure, inspired from the maximum caution rationale, is therefore well adapted to risk prioritisation. According to this procedure, the entry of a zone into a class is accepted if the zone is preferred (higher risk level), indifferent (equivalent risk level) or incomparable (risk level not comparable) to the low profile of the class. With the pseudo-conjunctive procedure, the zone may enter into a class only if it is preferred or indifferent to the low profile of the class. The treatment of incomparability differs from one procedure to the other.

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