Swarm intelligence, exact and matheuristic approaches for minimum weight directed dominating set problem

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ABSTRACT

There exist numerous heuristic and exact approaches in the literature for addressing minimum weight dominating set problem (MWDS) on vertex-weighted undirected graphs which is a well known $\mathcal{NP}$ -hard problem. However, little attention has been paid to its counterpart in vertex-weighted directed graphs called minimum weight directed dominating set problem (MWDDS) despite its use in modeling real-world applications involving directed interactions. As directed graphs can model undirected graphs, MWDDS can be considered as a generalization of MWDS, and hence, MWDDS is also $\mathcal{NP}$ -hard. In this paper, we present two approaches based on swarm intelligence, one approach based on integer linear programming (ILP) and one matheuristic approach to address the MWDDS. These approaches are the first approaches for MWDDS in their respective categories. One of our swarm intelligence approach is based on artificial bee colony (ABC) algorithm, whereas the other is based on invasive weed optimization (IWO) algorithm. Both these approaches are hybridized with problem specific heuristics and a local search mechanism. We have evaluated the performance of our approaches on benchmark instances derived from the standard benchmark instances of MWDS. Computational results show the effectiveness of our approaches.

1. Introduction

Given a vertex-weighted directed graph $G = (V, A, w)$, where V is the set of vertices, A is the set of arcs, and $w : V \rightarrow \mathbb{R}_{\geq 0}$ is the weight function that assigns a positive weight to each vertex of V . Each arc $(u, v) \in A$ is directed from vertex u to vertex v . In the context of the arc (u, v) , the vertex u is referred to as parent vertex or initial vertex or predecessor vertex, and the vertex v is referred to as child vertex or terminal vertex or successor vertex. A directed dominating set on this graph is a subset of vertices such that each vertex of the graph is either a member of this subset or the terminal vertex of at least one arc whose initial vertex is a member of this subset. In other words, a directed dominating set $S \subseteq V$ is such that each vertex $v \notin S$ is child of at least one vertex belonging to S . A vertex that belongs to S is referred to as a dominating vertex or a dominator, and a vertex that is not a member of S is referred to as a dominated or a dominee or a non-dominating vertex. The minimum directed dominating set problem (MDDS) consists in finding a directed dominating set on G having minimum number of vertices. The minimum weight directed dominating set problem (MWDDS) is a generalization of MDDS and seeks a directed dominating set on G with minimum sum of weights of its constituent vertices. As directed graphs

can model undirected graphs, MDDS and MWDDS can be considered as generalizations of corresponding problems in undirected graphs, viz. minimum dominating set problem (MDS) and minimum weight dominating set problem (MWDDS) respectively, which are well known $\mathcal{NP}$ -hard problems (Garey and Johnson, 1979). Hence, MDDS and MWDDS are also $\mathcal{NP}$ -hard. Throughout this paper, we will use vertex and node, and, directed graph and digraph interchangeably.

The in-degree of a vertex v is the number of its parent vertices, and the out-degree of a vertex v is number of its child vertices. A source vertex is a vertex with in-degree zero, and a leaf vertex is a vertex with out-degree zero. An isolated vertex is a vertex with both in-degree and out-degree zero, i.e., an isolated vertex is both a source vertex and a leaf vertex. Obviously, all source vertices (including isolated vertices) must be part of each and every directed dominating sets.

MDDS and MWDDS are the smallest set of vertices and the smallest weighted set of vertices respectively that have directed edges to all other vertices of the digraph. Hence, they can be used at minimum cost to monitor and control the directed interactions in complex networks arising in diverse areas such as spread of infectious diseases (Takaguchi et al., 2014), power generation and transmission (Yang et al., 2012), chemical reaction and metabolic regulation (Liu et al., 2013), genetic

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regulation (Wuchty, 2014; Wang et al., 2014). Concept of domination in digraphs has also been used for routing in ad hoc wireless networks with unidirectional links (Wu, 2002) and processing skyline queries in databases (Koltun and Papadimitriou, 2005). MDDS is used in case of homogeneous networks. On the other hand, MWDDS is used in case of networks composed of heterogeneous vertices where vertices differ in their capability/cost to monitor and control the directed interaction. Ghoshal et al. (1998) cited the application of MWDDS in facility location where a set of locations is given and each location has a positive weight representing the cost of opening the facility at that location. There is an arc from location u to location v if location v can be reached from location u . Identical facilities need to be opened in a subset of these locations in such a manner that those locations which are not in this subset should be reachable from at least one member of the subset and total cost of opening all these facilities is minimized. Facilities in this example can be radio or TV stations. Obviously, this is an instance of MWDDS.

Minimum weight dominating set problem (MWDS) on undirected graphs is a widely-studied problem, and many approaches (Albuquerque and Vidal, 2018; Lin and Guan, 2018; Wang et al., 2018; Lin et al., 2016; Chaurasia and Singh, 2015a; Nitash and Singh, 2014; Potluri and Singh, 2013; Jovanovic et al., 2010; Shetgaonkar and Singh, 2020) have been proposed in the literature for this problem. However, the corresponding problem in directed graphs, viz. MWDDS is an understudied problem, though several approaches exist in the literature for MDDS (Pang et al., 2010; Molnár Jr. et al., 2013; Takaguchi et al., 2014; Habibulla et al., 2015). Nakkala and Singh (2020) presented five greedy heuristics to solve MWDDS. A local search mechanism is also presented to further improve the quality of solutions obtained through these greedy heuristics. These heuristics are the first heuristic approaches for MWDDS in the literature. A brief overview of these heuristics has been provided in Section 2. This problem has not been studied in the literature from the perspective of metaheuristics, exact approaches or matheuristics.

Motivated by the absence of any metaheuristic, integer linear programming (ILP) or matheuristic approach in the literature for solving MWDDS, we have developed two swarm intelligence based metaheuristic approaches, an exact approach based on ILP and a matheuristic approach for MWDDS. Our first swarm intelligence approach is based on artificial bee colony (ABC) algorithm, whereas the second approach is based on invasive weed optimization (IWO) algorithm. The ABC algorithm proposed by Karaboga (2005) upon getting inspiration from intelligent foraging behavior of honey bees has been successfully used to solve numerous optimization problems in various domains, e.g. Singh (2009), Singh and Sundar (2011), Chaurasia and Singh (2015b), Alshamiri et al. (2016), Jayalakshmi and Singh (2017), Pandiri and Singh (2018, 2019), Pandiri et al. (2020). Likewise, the invasive weed optimization (IWO) proposed by Mehrabian and Lucas (2006) is inspired by colonizing behavior exhibited by weeds in an ecosystem. IWO algorithm too has been applied to solve a number of optimization problems (Venkatesh and Singh, 2015; Alshamiri et al., 2018; Safari et al., 2020; Liu and Nie, 2021). Motivated by the effectiveness demonstrated by ABC and IWO algorithms in solving numerous optimization problems, we have developed ABC and IWO algorithms based approaches to solve the MWDDS problem. For small and medium size instances, an exact approach based on ILP may be used to address $\mathcal{NP}$ -hard optimization problems like MWDDS. It can also serve the purpose of assessing the quality of the solutions returned by metaheuristics for the instances that can be solved to optimality. This served as the motivation for developing an ILP based approach for MWDDS. In addition, a matheuristic algorithm that takes an approximate solution as an input and tries to improve it by exploring its neighborhood using ILP is also proposed for those instances that are out of the range of an exact approach. We have evaluated the performance of our approaches on benchmark instances derived from the standard benchmark instances available in the literature for MWDS. Computational results on these benchmark instances show the effectiveness of our approaches.

The rest of this paper is organized as follows: Section 2 provides a brief overview of five existing greedy heuristics for MWDDS. Our ABC and IWO approaches for the MWDDS problem have been described in Section 3 and Section 4 respectively. Section 5 presents our integer linear programming approach, whereas our matheuristic approach is described in Section 6. Section 7 provides the computational results and their analysis. Finally, Section 8 outlines some concluding remarks about contributions of this paper and possible directions for future research.

2. Greedy heuristics

There are only five greedy heuristics available for MWDDS in the literature (Nakkala and Singh, 2020). These heuristics are called H1, H2, H3, H4 and H5 in Nakkala and Singh (2020). As these heuristics have been utilized in our swarm intelligence approaches, we have provided a brief overview of their working in this section. One may refer to Nakkala and Singh (2020) for motivation behind each of these heuristics. These heuristics have been utilized in two ways in our swarm intelligence approaches, viz. to generate 5 initial solutions each corresponding to one heuristic and to generate neighboring solutions. For generating an initial solution, each of these heuristics begins with an empty solution, and for generating a neighboring solution begins with an incomplete solution containing some dominating nodes. Then each of these heuristics iteratively builds a directed dominating set where during each iteration a vertex is added to the incomplete solution by following a greedy policy which varies from heuristic to heuristic. These iterations continue till a directed dominating set is obtained, and then, an improve procedure is applied to further reduce the cardinality of the obtained directed dominating set. Algorithm 1 provides the pseudo-code for our heuristics where *greedy_policy()* is the only function whose implementation differ for each of H1, H2, H3, H4 and H5. For generating an initial solution, a heuristic H_i ($i = 1 \dots 5$) is called with $H_i(G, \emptyset)$, and for generating a neighboring solution with $H_i(G, S)$, where S is the incomplete solution containing some dominating nodes.

Algorithm 1: Pseudo-code of our greedy heuristics

function $H_i(G, S)$

Input: A vertex-weighted directed graph $G := (V, A, w)$ and a partial solution S

Output: Solution constructed

begin

```

    while  $S$  is infeasible do
         $v \leftarrow \text{greedy\_policy}(V \setminus S)$ ;
         $S_D \leftarrow S_D \cup \{v\}$ ;
     $S_D \leftarrow \text{improve}(S)$ ;
    return  $S$ ;

```

While constructing the directed dominating set, all those vertices which are either part of the partially constructed directed dominating set or have at least one parent vertex as part of the partially constructed directed dominating set are said to be covered, whereas all other vertices are said to be uncovered. As is the usual practice with dominating set problems, we have followed a coloring scheme to aid in description of working of these heuristics. Vertices with color white represent uncovered vertices, whereas the vertices colored black or gray are covered (dominating or dominated). Dominating vertices are represented with black, and the dominated vertices are represented with gray. Our heuristics work iteratively to construct a feasible solution. Initially, all vertices in the graph are assumed to be of white color. A non-black vertex is selected and colored black in every iteration, and all its white children are colored gray. This iterative process continues till no white vertex left in the graph.

Table 1

Notations and their explanation.

Notation	Explanation
$CW(u)$	1 if vertex u is of white color, 0 otherwise
$W_{wc}(u)$	Sum of weights of white children of vertex u , i.e., $\sum_{(u,v) \in A} w(v) \times CW(v)$
$N_{wc}(u)$	Number of white children of vertex u , i.e., $\sum_{(u,v) \in A} CW(v)$
$N_p(u)$	Number of parents of vertex u

Before describing the greedy policy used by each of these heuristics, we will introduce few notational conventions in Table 1 and two equations based on these notational conventions.

$$P_{score}(u) \leftarrow \frac{W_{wc}(u) + CW(u) \times w(u)}{w(u)} \quad (1)$$

$$T_{score}(u) \leftarrow \frac{N_{wc}(u) + CW(u)}{w(u)} \quad (2)$$

Now, the greedy policy of each heuristic is described below:

- **Heuristic H1:** In every iteration, H1 tries to select a non-black vertex with the highest score as per (1) (highest P_{score}), a dominating vertex. If two or more vertices have the highest P_{score} , then from among these vertices, a vertex having the highest score as per (2) (highest T_{score}) is selected. If still there are more than one vertex having the highest T_{score} , then one such vertex is selected arbitrarily.
- **Heuristic H2:** H2 begins by making all source vertices as dominating vertices, and, then it proceeds exactly like H1
- **Heuristic H3:** H3 is same as H2 except in case of a tie as per (1) (highest P_{score}), it selects among the tied vertices, a white vertex with the minimum number of parents (minimum N_p). If still there are more than one white candidate then ties are broken arbitrarily. In case there is no white vertex among the vertices having the tie, then a gray node with the highest score is selected. If two or more gray nodes have the same highest score, then one of them is chosen uniformly at random.
- **Heuristic H4:** Like H2 and H3, this heuristic also selects all source vertices as dominating vertices at the beginning. After that, in every iteration, H4 makes use of following two rules while selecting a dominating vertex.
Rule 1: It is an extension of rule 2 in Habibulla et al. (2015) for computing minimum directed dominating set on unweighted directed graphs. It searches for a white vertex with only one non-black parent and without any white child in increasing order of vertex ids and the search stops when first such vertex is found. If such a white vertex is found, then the white vertex or its non-black parent will be selected as a dominating vertex depending on who among the two have the highest score as per (1). If they have the same score, then the non-black parent becomes the dominating vertex.
Rule 2: If rule 1 is unable to find a suitable vertex, then the dominating vertex is selected in the same way as in H1.
- **Heuristic H5:** H5 is the same as H4 except for rule 2. A vertex is selected exactly in the same way as in H3 when there is no source node.

3. The ABC approach to MWDDS problem

This section is devoted to our ABC algorithm based approach for MWDDS. We have combined our ABC approach with the problem-specific heuristics and a local search mechanism with the intention of providing high quality solutions. We will not provide any introduction to ABC algorithm as it is a well-established metaheuristic technique. However, those requiring such an introduction may refer to this recent survey (Akay et al., 2021).

Salient features of our ABC approach for MWDDS are described in the following subsections.

3.1. Solution encoding and fitness

The solution is represented directly by the set of dominating nodes. The objective function is used as the fitness function. As the MWDDS problem is a minimization problem, lower the objective function value of a solution, more fit it is considered.

3.2. Generation of initial employed bee solutions

The initial employed bee population has been generated in two ways. Initial five population members have been generated by each of the five heuristics discussed in the previous section. The remaining population members are generated by following an iterative procedure that is a mix of greediness and randomness. During each iteration of this procedure, with the probability P_r , a non-black vertex is randomly selected and added to the candidate solution. Otherwise, a node is added greedily. Here in the greedy procedure, to select a dominating node, every time one of the five heuristics is selected uniformly at random. This procedure continues until the candidate solution becomes a directed dominating set. We have utilized 5 greedy heuristics with the intention of generating superior quality diverse solutions.

3.3. Procedure used by onlooker bees to select a food source

The method of binary tournament selection (BTS) has been employed to select a food source for an onlooker bee. In this method, two food sources are selected randomly and their fitness values are compared. The better food source between the two is chosen with the probability P_{ol} and the worst with probability $1 - P_{ol}$.

3.4. Neighboring solution generation

During the employed bee phase, for every employed bee solution (population member), a neighboring solution is generated, and its fitness is compared with the corresponding employed bee solution. During onlooker bee phase, a neighboring solution is generated around only selected employed bee solutions and its fitness is compared with the corresponding employed bee solution. In both the phases (employed bee phase and onlooker bee phase), if the newly generated neighbor is more fit than the corresponding employed bee solution, then neighbor replaces the employed bee solution immediately, otherwise the employed bee solution is retained. To generate a neighboring solution for an employed bee solution, we have used a destroy-and-repair procedure (discussed in Section 3.4.1) and an improve procedure (discussed in Section 3.4.3). Destroy-and-repair procedure takes an employed bee solution S as input and copies some vertices randomly from the solution S to a solution S' which is initially empty. Obviously, S' will be infeasible. S' is then repaired, and the resulting feasible solution is passed to the improve procedure which further improves the solution quality. Pseudo-code for generating a neighboring solution has been given in Algorithm 2.

Algorithm 2: Generating a neighboring solution

function New_Neighbour(S , $Iter$)

Input: A solution S , current iteration number $Iter$

Output: A neighboring solution S'

begin

$S' \leftarrow \text{Destroy_and_Repair}(S, Iter);$

$S' \leftarrow \text{Improve}(S');$

return S' ;

3.4.1. Destroy-and-repair procedure

This procedure takes an employed bee solution S as input and iteratively copies $\lfloor R \times |S| \rfloor$ vertices randomly from the solution S to

an empty solution S' , where $0 < R < 1$ is a parameter to be determined empirically. Clearly, S' will be infeasible. S' is then repaired into a feasible solution. Then this procedure tries to select the most appropriate heuristic among H1–H5 as per the procedure described in Section 3.4.2, so that infeasible solution can be transformed into a high quality solution. The selected heuristic iteratively adds vertices from $V \setminus S'$ to S' as per its greedy policy. Iterations of the heuristic continue till the solution becomes feasible.

Algorithm 3 provides the pseudo-code of our destroy-and-repair procedure where $Random(S)$ is a function that returns an element of set S randomly, and, $Random_Heuristic()$ is a function that selects a heuristic among H1, H2, H3, H4, and H5 uniformly at random and returns the index of the heuristic selected. $Select_Heuristic()$ is another function that selects a heuristic based on their selection probabilities according to the procedure described in Section 3.4.2 and returns the index of the heuristic selected. The last function $Update_Heuristic_Probabilities()$ updates the selection probabilities of five heuristics as per the description in Section 3.4.2.

Algorithm 3: Pseudo-code of Destroy-and-Repair method

```

function Destroy_and_Repair( $S, Iter$ )
Input: A solution  $S$ , current iteration number  $Iter$ 
Output: A new solution  $S'$ 
begin
   $S' \leftarrow \emptyset$ ;
  for  $i \leftarrow 1$  to  $\lfloor R \times |S| \rfloor$  do
     $v \leftarrow Random(S \setminus S')$ ;
     $S' \leftarrow S' \cup \{v\}$ ;
  if  $Iter < I_{update}$  then
     $i \leftarrow Random\_Heuristic()$ ;
  else
    if  $Iter \% I_{update} == 0$  then
       $Update\_Heuristic\_Probabilities()$ ;
     $i \leftarrow Select\_Heuristic()$ ;
   $S' \leftarrow Hi(G, S')$ ;
  return  $S'$ ;

```

3.4.2. Heuristic selection

As the efficacy of different heuristics vary from one instance to another and even on the same instance from one phase in search process to another, we have developed an adaptive mechanism for selecting a heuristic. In this mechanism, each heuristic has a certain probability of selection that depends on its performance in the previous I_{update} number of iterations. This selection probability is updated after every I_{update} iterations.

In this mechanism, each heuristic has equal probability of selection up to I_{update} initial iterations, and hence, one of the heuristics is selected uniformly at random. After I_{update} initial iterations, each heuristic is selected according to its own probability of selection. Each heuristic Hi has a count Hi_used to keep track of the number of times that heuristic Hi got used, and, another count $Hi_improved$ to keep track of the number of times, it has improved the corresponding employed bee solution. Heuristic probabilities are recomputed by using these two counts every I_{update} number of iterations. Hi_used and $Hi_improved$ are initialized to zero at the beginning and after every I_{update} number of iterations after heuristic probabilities have been recomputed. Heuristic probabilities are computed using Eqs. (3) and (4). Eq. (3) computes X_i , which refers to the success rate of heuristic Hi . The probability P_{Hi} of using a particular heuristic Hi is determined by using Eq. (4). Based on these probabilities, the five heuristics get selected for the next I_{update} number of iterations after which these probabilities are computed again.

$$X_i = \frac{Hi_improved}{Hi_used} \quad (3)$$

$$P_{Hi} = \frac{X_i}{\sum_{j=1}^5 X_j} \quad (4)$$

3.4.3. Improve method

The solution obtained by destroy-and-repair procedure is further improved by using a local search mechanism. For each vertex, whether dominating or non-dominating, a reference count has been maintained that keeps track of number of dominating vertices covering it and this reference count is updated for all affected vertices whenever a vertex has been added to/removed from the set of dominating vertices. Note that a dominating vertex also cover itself. If the reference count of a dominating vertex and all its children is more than one, then that vertex can be removed from the solution without affecting the feasibility of the solution. Such dominating vertices are termed as redundant vertices. Our improve method begins by computing the set of redundant vertices and then iteratively removes the redundant vertices one-by-one till there is no redundant vertex. In each iteration, the redundant vertex having the highest ratio of its weight to out-degree is removed (ties are broken arbitrarily) and set of redundant vertices is computed again. Algorithm 4 provides the pseudo-code of our improve method where $Ref(v)$ is the reference count of vertex v .

Algorithm 4: Procedure of Improve method

```

function Improve( $S'$ )
Input: A solution  $S'$  with redundant nodes
Output: A solution  $S'$  without redundant nodes
begin
   $Rdd \leftarrow \{v : (v \in S') \wedge (Ref(v) > 1) \wedge (Ref(u) > 1 \forall u \text{ such that } (v, u) \in A)\}$ ;
  while  $Rdd \neq \emptyset$  do
     $v' \leftarrow \underset{v \in Rdd}{\operatorname{argmax}} \frac{w(v)}{out\_deg(v)}$ ;
     $S' \leftarrow S' \setminus \{v'\}$ ;
    recalculate  $Rdd$ ;
  return  $S'$ ;

```

3.5. Other features

If an employed bee solution has not improved over a fixed number ($limit_{non_imp}$) of trials, where $limit_{non_imp}$ is a parameter to be determined empirically, then that employee bee solution is replaced with a new solution which is generated in the same manner as an initial solution other than first 5 initial solutions. Note that the number of trials for an employed bee solution means the number of times the employed bee solution is called for generating a neighboring solution. A solution may be called more than once in an iteration based on the number of onlookers choosing this solution.

Algorithm 5 provides the pseudo-code of our ABC approach where E_n and O_n are number of employed bees and onlooker bees. $GR_initial_solution()$ is a function that generates an initial solution as per mixed greedy-random procedure described in Section 3.2 and $New_Neighbour(S_i)$ is a function that generates a solution in the neighborhood of solution S_i as per procedure described in Section 3.4. $BTS(S_1, S_2, \dots, S_{E_n})$ is another function that selects and returns a solution from among solutions $S_1, S_2, \dots, S_{E_n}$ as per binary tournament selection (Section 3.3).

4. The IWO approach to MWDDS problem

This section presents our IWO based approach for MWDDS. Like our ABC approach, our IWO approach too makes use of the problem-specific heuristics and a local search mechanism. Our IWO approach has been designed by borrowing several features from ABC approach described in the previous section (Section 3). Solution encoding, fitness

Algorithm 5: Pseudo-code of our ABC approach

Input: Set of parameters for the ABC Algorithm and an MWDDS instance $G = (V, A, w)$

Output: Best solution found

```

for  $i \leftarrow 1$  to  $E_n$  do
  if  $i < 6$  then
     $S_i \leftarrow Hi(G, \emptyset)$ ;
  else
     $S_i \leftarrow GR\_Initial\_Solution()$ ;
  if  $((i==1) \vee (S_i \text{ is better than } best))$  then
     $best \leftarrow S_i$ ;
while Termination condition not satisfied do
  for  $i \leftarrow 1$  to  $E_n$  do
     $S'_i \leftarrow New\_Neighbour(S_i)$ ;
    if  $S'_i$  is better than  $S_i$  then
       $S_i \leftarrow S'_i$ ;
      if  $S_i$  is better than  $best$  then  $best \leftarrow S_i$ ;
    else if  $S_i$  has not improved consecutively for  $limit_{non\_imp}$  trials then
       $S_i \leftarrow GR\_Initial\_Solution()$ ;
  for  $i \leftarrow 1$  to  $O_n$  do
     $S_j \leftarrow BTS(S_1, S_2, \dots, S_{E_n})$ ;
     $S'_j \leftarrow New\_Neighbour(S_j)$ ;
    if  $S'_j$  is better than  $S_j$  then
       $S_j \leftarrow S'_j$ ;
      if  $S_j$  is better than  $best$  then  $best \leftarrow S_j$ ;
    else if  $S_j$  has not improved consecutively for  $limit_{non\_imp}$  trials then
       $S_j \leftarrow GR\_Initial\_Solution()$ ;
return  $best$ ;

```

function, destroy-and-repair procedure, and improve method are the same as ABC. As IWO is not as well-established metaheuristic technique as ABC algorithm, we will provide a brief introduction to IWO prior to describing salient features of our IWO approach for MWDDS.

4.1. A brief introduction to invasive weed optimization (IWO)

Invasive weed optimization (IWO) is a recently proposed swarm intelligence algorithm inspired by the colonizing behavior of weeds. As the weeds have the ability to grow and multiply in an intrusive and rapid manner, they are considered detrimental to the growth of desirable, cultivated plants. Weeds have immense survival capacity owing to their ability to adopt as per their environment. Hence, weeds are always considered a serious threat to farming. IWO algorithm is proposed by [Mehrabian and Lucas \(2006\)](#).

In the IWO algorithm, each weed/weed plant represents a potential solution to the problem at hand and the fitness of a weed means the fitness of the solution that this weed represents. The IWO algorithm begins by creating an initial population of n_{init} solutions (weed plants or weeds in short). These solutions are usually generated in a random manner. Then an iterative process ensues. In each iteration, certain number of seeds (child solutions) are created by each weed as per its own fitness. The number of seed solutions, a weed solution can create is usually mapped linearly from a given minimum value x_{min} to a given maximum value x_{max} as per its fitness. Each weed creates child seeds randomly in its vicinity by following a normal distribution with mean equal to zero, but varying variance, thereby yielding some sort of randomized local search in the vicinity of each weed plant. Exact method for creating seed solutions depends on the problem at hand. However, the standard deviation (SD), σ , of the normal distribution is

reduced from a given initial value (σ_{iv}) to a given final value (σ_{fv}) over the iteration. Usually, the SD at an iteration $iter$ is determined by using the following expression.

$$\sigma_{iter} := \left(\frac{iter_{mx} - iter}{iter_{mx}} \right)^{nlm} (\sigma_{iv} - \sigma_{fv}) + \sigma_{fv} \quad (5)$$

Where σ_{iter} is the standard deviation at the current iteration, $iter_{mx}$ is the maximum permissible number of iterations and nlm is non-linear modulation index. This distribution guarantees that the probability of dropping a seed in a distant area reduces nonlinearly over iterations.

Once all weeds have created seed solutions, a phase called competitive exclusion phase begins. All newly created seeds become the weeds for this phase. All these newly created weeds are added to the weed population in case the number of weeds in the population is less than the maximum number of weeds permitted in the population n_{max} . When the number of weeds in the population becomes n_{max} , only the best n_{max} weeds according to their fitness among the existing ones and newly created ones are kept in the population and remaining weeds are discarded. After the competitive exclusion phase finishes, the next iteration of IWO algorithm starts. This entire process repeats as long as the termination condition remains unsatisfied. Please note that number of weeds in the initial population is n_{init} and maximum possible number of weeds in the population is n_{max} . Hence, we start with a smaller initial population and wait for the population to reach its maximum permissible limit through insertion of seed solutions over few initial iterations before competitive exclusion phase can start.

4.2. Solution encoding and fitness

IWO uses the same solution encoding and fitness function as our ABC approach (Section 3.1).

4.3. Generation of initial weed solutions

n_{init} initial weed solutions are generated in the exactly same manner as the ABC initial employed bee solutions (Section 3.2).

4.3.1. Determination of number of seeds a weed can produce

Each weed solution in the population will produce a certain number of seed solutions (neighboring solutions). The number of seeds produced by a weed depends on its objective function value. The number of seeds produced by a weed is restricted to the closed interval of $\{x_{min}, x_{max}\}$. Weeds in the colony will be sorted in non-decreasing order of their objective function values. The colony is partitioned into $(x_{max} - x_{min}) + 1$ number of groups. Weeds in the first group will produce x_{max} number of seeds, and the second group of weeds will produce $x_{max} - 1$ number of seeds. Like this, the last group will produce x_{min} seeds. Algorithm 6 presents the procedure to determine the number of seeds a weed can produce where n_c is the current population size. Please remember that MWDDS is a minimization problem and that is why weed solutions are sorted in non-decreasing order of their objective function values.

Algorithm 6:

Procedure to determine the number of seeds

function Number_of_Seeds(S_i)

Input: Index i of a solution S_i in the sorted weed solutions list

Output: Number of seeds that a weed S_i can produce

begin

```

  if  $(i \leq \frac{n_c}{(x_{max}-x_{min})+1})$  then return  $x_{max}$  ;
  else if  $(i \leq \frac{2*n_c}{(x_{max}-x_{min})+1})$  then return  $x_{max} - 1$  ;
  :
  else if  $(i \leq \frac{(x_{max}-x_{min})*n_c}{(x_{max}-x_{min})+1})$  then return  $x_{min} + 1$  ;
  else return  $x_{min}$  ;

```

4.3.2. Production of a seed by a weed

A seed solution is generated in the same manner as a neighboring solution in our ABC approach, i.e., each seed solution is generated utilizing the destroy-and-repair procedure (discussed in Section 3.4.1) and the improve procedure (discussed in Section 3.4.3). Each seed solution produced by a weed is placed at the end of the population.

4.4. Competitive exclusion

This process has been used to eliminate inferior quality weed solutions so that only the superior quality weed solutions remain to produce seeds. In every iteration, when each weed solution in the population has produced its own seed solutions, the competitive exclusion phase begins. For this phase, all the newly produced seed solutions become weed solutions. Now, all the weed solutions (existing as well as newly produced) are sorted in non-decreasing order according to their objective function values. If the number of weed solutions in the population has exceeded the maximum allowed limit n_{max} , then the excess weed solutions lying at the end of population are discarded.

Algorithm 7 provides the pseudo-code of our IWO approach where we have used the same function names as used in pseudo-code of our ABC approach for whatever components that we have borrowed from our ABC approach. In addition to these functions, *Number_of_Seeds*(S_i) is a function that determines the number of seed solutions, a weed solution S_i can produce and *Sort_The_Weeds*(n_c) is a function that sorts all the weed solutions in population as per non-decreasing order of their objective function values.

Algorithm 7: Pseudo code of IWO Algorithm for MWDDS

Input: Set of parameters for IWO Algorithm and an MWDDS instance $G = (V, A, w)$

Output: Best solution found

```

for  $i \leftarrow 1$  to  $n_{init}$  do
  if  $i < 6$  then
     $S_i \leftarrow Hi(G, \emptyset)$ ;
  else
     $S_i \leftarrow GR\_Initial\_Solution()$ ;
  if  $((i==1) \vee (S_i \text{ is better than best}))$  then
     $best \leftarrow S_i$ ;
 $n_c \leftarrow n_{init}$ ;
Sort_The_Weeds( $n_c$ );
if  $(n_c > n_{max})$  then  $n_c \leftarrow n_{max}$ ;
while Termination condition not satisfied do
   $n_{colony} \leftarrow n_c$ ;
  for  $i \leftarrow 1$  to  $n_{colony}$  do
     $seeds \leftarrow \text{Number\_of\_Seeds}(S_i)$ ;
    for  $j \leftarrow 1$  to  $seeds$  do
       $n_c \leftarrow n_c + 1$ ;
       $S_{n_c} \leftarrow \text{New\_Neighbor}(S_i)$ ;
      if  $S_{n_c}$  is better than best then  $best \leftarrow S_{n_c}$ ;
  Sort_The_Weeds( $n_c$ );
  if  $(n_c > n_{max})$  then  $n_c \leftarrow n_{max}$ ;
return best;

```

5. Integer linear programming approach to MWDDS

This section is devoted to our integer linear programming (ILP) approach for MWDDS. To formulate our ILP approach, we will make use of two sets $N_G^+(v_i)$ and $N_G^-(v_i)$ defined as follows: For all $v_i \in V$, $N_G^+(v_i)$ denotes the set of all the vertices $v_j \in V$ such that (i, j) is an arc of A , and, $N_G^-(v_i)$ is the set of all the vertices $v_j \in V$ such that $(j, i) \in A$. Clearly in this notational convention, $|N_G^+(v_i)|$ and $|N_G^-(v_i)|$ denote the outdegree and indegree of a vertex v_i respectively. In addition, $n = |V|$

is the order of the digraph G . In the sequel, we refer to our integer linear programming approach as ILP. We have used IBM CPLEX to solve the ILP.

5.1. ILP model of MWDDS

Let x_i be the binary decision variable associated with vertex $v_i \in V$: variable x_i is set to one if and only if v_i belongs to the minimum weight directed dominating set. The objective function of ILP is to minimize the total weight of the directed dominating set, and it has n inequalities, that specify that each vertex $v_i \in V$ should be part of the minimum weight directed dominating set, or should have at least one parent that is part of this set. The remaining n constraints are integrality constraints.

$$\begin{aligned}
 &\text{Minimize } \sum_{v_i \in V} w_i x_i \\
 &x_i + \sum_{v_j \in N_G^-(v_i)} x_j \geq 1 \quad \forall v_i \in V \\
 &x_i \in \{0, 1\} \quad \forall v_i \in V
 \end{aligned}$$

This formulation is strengthened as follows. For each vertex $v_i \in V$, let $z(v_i) = 1 + |N_G^+(v_i)| > 0$ be the maximum number of vertices that can be covered by vertex v_i . Since any solution to MWDDS should cover all the vertices, the following constraint holds: $\sum_{v_i \in V} z(v_i) x_i \geq n$. Let Z be the set of all the pairwise different values of $z(v_i)$ (the cardinality of Z is at most n). The previous constraint can be attempted for strengthening as follows:

$$\sum_{v_i \in V} \left\lceil \frac{z(v_i)}{q} \right\rceil x_i \geq \left\lceil \frac{n}{q} \right\rceil \quad \forall q \in Z$$

Since it is not guaranteed that one of these constraint dominates the original one, they are all enforced. If some of them are redundant, they can be removed at presolve stage by the ILP solver (CPLEX in our case).

These constraints can be shown useful by considering the circuit of order 3, with $w(i) = 1$ for all $v_i \in V$. The continuous relaxation of ILP yields 1.5 as the objective value, and a fractional solution $x_i = 0.5$ for all $v_i \in V$. Adding the (unique) constraint above, which is simply $x_1 + x_2 + x_3 \geq 2$ leads to an optimal solution to MWDDS.

5.2. Graph reduction pre-process

The following properties can be used as a pre-processing step to remove edges from the digraph G , and discard some vertices from the search of an optimal solution to MWDDS.

Property 0: If there exists $v_i \in V$ such that $N_G^-(v_i)$ is empty, then vertex v_i must be part of any dominating set. As a consequence, the coverage requirement of all the vertices in $N_G^+(v_i)$ is satisfied, and all the arcs to $N_G^+(v_i)$ can be removed.

Property 1 (Strong vertices): If v_i and v_j are such that

- $w_j \leq w_i$
- $N_G^+[v_i] \subseteq N_G^+(v_j)$

Then v_j is stronger than v_i , and any solution of MWDDS containing v_i can be turned into another one of lesser cost by deselecting v_i , and by selecting v_j instead. As a consequence, G can be modified by setting w_i to $+\infty$, and by removing all the arcs of $N_G^+(v_i)$ since v_i shall not dominate any vertex.

Property 2: If a vertex $v_i \in V$ is such that $w_i \geq \sum_{v_j \in N_G^+(v_i)} w_j$, and $N_G^+(v_i) \cap N_G^-(v_i) \neq \emptyset$ then v_i does not appear in any optimal solution to MWDDS. Indeed, if it was present, it could be replaced by selecting all the vertices of $N_G^+(v_i)$ leading to a solution of lesser cost. In addition, since $N_G^+(v_i) \cap N_G^-(v_i) \neq \emptyset$, vertex v_i is dominated. As a consequence, G can be modified by setting w_i to $+\infty$, and by removing all the arcs of $N_G^+(v_i)$ since v_i shall not dominate any vertex.

Algorithm 8: Pseudo code of MH Algorithm for MWDDS**Input:** Sets of parameters for MH & IWO and an MWDDS instance $G = (V, A, w)$ **Output:** Best solution found $best \leftarrow ILP(G, T_1, OPT);$ **if** ($OPT == 1$) **then return** $best$; $S \leftarrow IWO(\text{Set of parameters for IWO}, G);$ **if** (S is better than $best$) **then** $best \leftarrow S$; $R \leftarrow 90$; $T_2 \leftarrow 2T_1$;**while** Termination condition not satisfied **do** $S \leftarrow \text{Explore_Neighborhood}(G, best, R, T_2, T_A);$ **if** (S is better than $best$) **then** $best \leftarrow S$; $R \leftarrow 90$; $T_2 \leftarrow 2T_1$;**else if** ($T_A < T_2$) **then****if** ($R == 80$) **then return** $best$; $R \leftarrow 80$; $T_2 \leftarrow 4T_1$;**else****if** ($R == 80$) **then return** $best$;**if** ($R == 90$) **then** $R \leftarrow 95$; $T_2 \leftarrow T_1$;**else** $R \leftarrow 80$; $T_2 \leftarrow 4T_1$;**return** $best$;**6. Matheuristic approach to MWDDS**

We have also developed a matheuristic approach by combining our IWO approach with the ILP problem formulation described in the previous section. The reason for choosing IWO and not the ABC approach is the better performance of the former, in general, in comparison to the latter. We will refer to our matheuristic approach as *MH*.

Our matheuristic consists of solving the ILP formulation of previous section for T_1 amount of time. If ILP finds an optimal solution within this time then matheuristic stops. Otherwise IWO is executed and the best solution between the two solutions returned by ILP and IWO is chosen for further processing. We have used CPLEX to explore the neighborhood of the current solution, where the size of the solution space is controlled by the input parameter R as follows: We enforce that at least $R\%$ of vertices from the existing solution must be part of the new solution, and let CPLEX find the best possible solution for T_2 seconds. This neighborhood exploration through CPLEX can be perceived as a local search in the context of our matheuristic. We have used this local search in an iterative framework, where value of R varies over the set $\{80, 90, 95\}$ and the value of T_2 varies over the set $\{4T_1, 2T_1, T_1\}$. We start with $R = 90$ and $T_2 = 2T_1$ in the first iteration. With $R = 90$ and $T_2 = 2T_1$ in an iteration following are the possibilities:

- if the local search improves the solution then we retain this setting for the next iteration.
- If the local search terminates before T_2 time without improvement indicating no improved solution exists with this value of R , we use $R = 80$ and $T_2 = 4T_1$ in the next iteration.
- If the local search terminates at or after T_2 time without improvement thereby indicating value of R may be too low for a meaningful exploration, we use $R = 95$ and $T_2 = T_1$ in the next iteration.

With $R = 95$ and $T_2 = T_1$ in an iteration, following are the possibilities:

- if the local search improves the solution then we move to $R = 90$ and $T_2 = 2T_1$.
- If the local search fails to improve the solution, we use $R = 80$ and $T_2 = 4T_1$ in the next iteration.

With $R = 80$ and $T_2 = 4T_1$ in an iteration, following are the possibilities:

- if the local search improves the solution then we move to $R = 90$ and $T_2 = 2T_1$.
- If the local search fails to improve the solution, matheuristic terminates.

Algorithm 8 provides the pseudo-code of MH, where $ILP(G, T_1, OPT)$ is a function that solves the ILP formulation of previous section through CPLEX on MWDDS instance G for T_1 amount of time and returns the solution found. It also sets the OPT parameter to 1 if an optimal solution is found within this time, otherwise OPT is set to zero. $\text{Explore_Neighborhood}(G, best, R, T_2, T_A)$ is another function that explores the neighborhood of current solution $best$ of instance G through CPLEX for T_2 amount of time by retaining $R\%$ vertices of $best$ in any solution that gets generated. It also returns the actual time taken in parameter T_A . $IWO(\text{Set of parameters for IWO}, G)$ is yet another function implementing Algorithm 7.

7. Computational results

We have evaluated the performance of our approaches on three datasets, which we have derived from three standard datasets (Type-I, Type-II, and Unit disk graph (UDG)) available in the literature for MWDS on undirected graphs. The first two datasets, viz. Type-I and Type-II, are from Jovanovic et al. (2010). Both of these datasets have same graphs, but their vertex weights differ. For Type-I instances, vertices' weights are distributed uniformly at random in the closed interval of $[20, 70]$, whereas for Type-II instances, vertices' weights are distributed in the interval $[1, \text{degree}(v)^2]$, where $\text{degree}(v)$ is the degree of vertex v . In these datasets, the number of vertices varies from 50 to 1000, and the number of edges varies from 50 to 20000. There exist ten different instances for each combination of the number of vertices and the number of edges. Each of Type-I & Type-II datasets has 530 instances. Third dataset UDG is from Potluri and Singh (2013), where vertices' weights are distributed randomly in the interval $[1, 100]$. In this dataset, vertices are assumed to be distributed in an area of 1000×1000 uniformly at random, and any two vertices are considered connected by an edge if the distance between them is not exceeding a pre-specified range R_p . The number of vertices in this dataset is $\{50, 100, 250, 500, 750, 1000\}$. Two values are considered for range (R_p), viz. 150 and 200. There are ten instances for each combination of R and the number of nodes. So there are 120 instances in the UDG dataset. Hence, a total of 1180 instances are there in these three datasets for MWDS.

We have derived the datasets for MWDDS from the corresponding datasets of MWDS by following the same process as described in Habibulla et al. (2015). In general, for every edge $\{u, v\}$ in an undirected graph, there are two arcs or directed edges (u, v) and (v, u) in the corresponding directed graph. Therefore, in order to transform an instance of MWDS to an instance of MWDDS, for each edge in the MWDS instance, we have selected randomly one of the two corresponding arcs, and included it into the set of arcs for the instance of MWDDS. No change is made in weights of vertices when an MWDS instance is transformed into an MWDDS instance. These datasets for MWDDS will also be referred to as Type I, Type II and UDG.

Presented approaches are implemented in C and executed on Ubuntu based 3.6 GHz Core-i7-7700 system with 32 GB of RAM. IBM CPLEX 12.7 library is used to solve the ILP model. None of our approaches

Table 2

Comparison of H1, H2, H3, H4, H5, ABC, IWO, ILP, and MH on UDG dataset.

$ V $	Range	H1	H2	H3	H4	H5	ABC	ABC _{ET}	IWO	IWO _{ET}	ILP	ILP _{ET}	MH	MH _{ET}
50	150	820.0	818.1	818.1	828.0	828.0	805.2	1.9	805.2	1.9	805.2	0.0	805.2	0.0
50	200	588.1	588.1	588.1	605.3	605.3	573.5	2.2	573.5	2.6	573.5	0.0	573.5	0.0
100	150	1089.8	1091.9	1091.9	1111.3	1111.3	1065.8	6.9	1065.8	7.2	1065.8	0.0	1065.8	0.0
100	200	642.8	641.0	641.0	644.7	644.7	614.6	8.3	613.8	8.4	613.7	0.0	613.7	0.0
250	150	1025.2	1025.2	1024.1	1025.2	1024.1	985.4	32.4	985.1	31.1	984.1	0.0	984.1	0.0
250	200	517.2	517.2	517.2	517.2	517.2	492.8	42.4	493.4	38.2	490.4	0.0	490.4	0.0
500	150	856.9	856.9	855.1	856.9	855.1	814.8	115.0	814.9	101.9	813.2	0.1	813.2	0.1
500	200	375.3	375.3	374.8	375.3	374.8	356.8	134.0	356.3	126.1	354.2	0.1	354.2	0.1
800	150	659.0	659.0	659.0	659.0	659.0	625.5	271.7	624.3	242.9	619.9	0.3	619.9	0.3
800	200	302.5	302.5	302.5	302.5	302.5	288.2	342.5	286.9	307.4	286.3	0.4	286.3	0.4
1000	150	665.2	665.2	665.7	665.2	665.7	612.4	384.1	611.5	347.7	608.7	2.4	608.7	2.4
1000	200	290.2	290.2	291.0	290.2	291.0	276.8	504.9	276.7	478.4	275.9	1.0	275.9	1.0
Avg:		652.7	652.6	652.4	656.8	656.6	626.0	153.9	625.7	141.2	624.3	0.4	624.3	0.4

use any multi-core features. Our approaches are executed once on each instance of the dataset. All the datasets have ten instances in a group for each combination of the same number of nodes ($|V|$) and the same number of directed edges ($|A|$) or the same range (R). We have reported for each approach the average solution quality and the average execution time on each group of ten instances like the usual practice of reporting results in case of MWDS.

Parameter values for our ABC and IWO approaches are chosen empirically after a large number of trials. First, we will describe the ABC parameter values. The number of initial employee bees E_n is fixed to 75, the number of onlookers O_n is set to 125, $limit_{non_imp}$ is set to 600. When an employed bee solution has not improved continuously over a $limit_{non_imp}$ number of trials, then that becomes a scout. To select an onlooker, we have used the binary tournament selection mechanism. In this, the better fitness solution has been selected with the probability P_{ol} set to 0.8. In destroy-repair procedure, R is set to 0.6. In heuristic selection mechanism, after every I_{update} iterations the heuristic probabilities are recomputed. This parameter is set to 8. While the generation of the initial population, in the mixed random-greedy procedure, a dominating node has been selected randomly with P_r set to 0.3. Our ABC algorithm terminates when best solution has not improved over 2500 iterations.

As far as IWO parameter values are concerned, the number of initial weed n_{init} is set to 300. The minimum x_{min} and maximum x_{max} number of seeds that a weed can produce are set to 1 and 3 respectively. The maximum number of weeds allowed in the population n_{max} is set to 350. Same values of R and P_r as in ABC are used in IWO also, i.e., $R = 0.6$ and $P_r = 0.3$. I_{update} is set to 5. IWO algorithm terminates when best solution has not improved over 750 iterations.

ILP and MH approaches were allowed to execute for a maximum of 3600 s. In MH, T_1 is set to 1000 s and T_i is set to 60 s. Note that if MH finds a solution in less than 1000 s then it is the solution returned by ILP, and, hence, guaranteed to be optimal.

Tables 2, 3 and 4 present the detailed computational results of our four approaches, viz. ABC, IWO, ILP and MH, and, compare these results with those of five heuristic approaches, viz. H1, H2, H3, H4 and H5 (Nakkala and Singh, 2020). These five heuristic approaches are the only approaches available in the literature for MWDDS. These tables report the average solution quality on each group of 10 instances obtained by various approaches under the columns named after respective approaches. These tables also report the average execution times in seconds on each group of 10 instances for ABC, IWO, ILP and MH under the columns named after respective approaches with subscript ET , i.e., columns entitled ABC_{ET}, IWO_{ET}, ILP_{ET}, MH_{ET}. Execution times of H1, H2, H3, H4 and H5 being negligible will not be reported. Table 2 present the detailed results on UDG instances, Table 3 on Type-I instances and Table 4 on Type-II instances. Table 5 present the summary of comparisons among our 4 proposed approaches in terms of number of groups of instances in each dataset and overall on which an approach mentioned in the left performs better than (B) or same as (E) or worse than the approach mentioned at the top. From these tables, following observations can be made:

- As expected, all the four proposed approaches performed better than the five heuristics available in the literature on all the groups of instances in all the three datasets.
- IWO performed better than ABC in general, and the performance gap between them widens as the instance size increases.
- Barring 10 groups of instances in Type I dataset, where ILP could not finish in allotted time of 3600 s on some instances, it performed as good as or better than IWO and ABC both.
- MH performed as good as or better than IWO and ABC both except for 2 groups of instances in Type I dataset where ABC performed better. It is to be noted that MH will always perform as good as IWO due to its very design where IWO is one component.
- All UDG and Type II instances were solved to optimality by ILP and MH both with maximum execution time of around 100 s. Moreover, ILP and MH are several times faster than IWO and ABC on these two datasets.
- MH performed as good as or better than ILP on all groups of instances except for one group of instances in Type I dataset. MH can perform worse than ILP only on those instances where ILP can find a better solution after T_1 amount of time in comparison to the solution that it got within T_1 amount time.
- MH never required the maximum allotted time of 3600 s. It has terminated before that time in all the cases.
- Among the 4 methods that we have proposed, MH has performed the best. This is not surprising as for different dominating set variants, matheuristic methods are currently the best performing methods, e.g. Albuquerque and Vidal (2018), Pinacho-Davidson et al. (2019), and Nakkala et al. (2021). These matheuristic approaches not only provide solutions of better quality, but are also faster than several metaheuristic approaches for the same problem in most cases.

To determine the impact of parameter settings on the quality of solutions returned by ABC and IWO, for each of their parameters, we have evaluated the performance of these two approaches on three parameter values, viz. the original value, one lower value and one higher value. In all these experiments, all parameters other than the one whose performance is evaluated are set to their original values as reported previously. This has been done one-by-one for all parameters. For this performance evaluation, we have chosen three instance groups, viz. a Type-I instance group with 800 vertices and 5000 arcs, a Type-II instance group with 800 vertices and 10000 arcs, and a Type II instance group with 1000 vertices and 10000 arcs. Performance is reported as sum of average solution quality on each of these three groups. Table 6 reports the results of these experiments. From this table, it can be clearly seen that each of our chosen parameter values provide better results for both ABC and IWO in comparison to tested values on the lower and the higher sides.

Table 3

Comparison of H1, H2, H3, H4, H5, ABC, IWO, ILP, and MH on Type-I dataset.

$ V $	$ A $	H1	H2	H3	H4	H5	ABC	ABC_{ET}	IWO	IWO_{ET}	ILP	ILP_{ET}	MH	MH_{ET}
50	50	750.3	747.5	747.5	744.7	744.7	737.6	1.3	737.6	1.2	737.6	0.0	737.6	0.0
50	100	590.1	577.9	577.9	574.6	574.6	564.8	2.1	564.8	2.6	564.8	0.0	564.8	0.0
50	250	322.1	322.1	323.2	318.8	318.8	300.2	3.4	300.2	3.7	300.2	0.0	300.2	0.0
50	500	191.6	191.6	195.7	191.6	195.7	176.0	3.5	176.0	4.1	176.0	0.0	176.0	0.0
50	750	139.5	139.5	139.5	139.5	139.5	124.1	4.0	124.1	4.3	124.1	0.0	124.1	0.0
50	1000	99.8	99.8	99.8	99.8	99.8	93.2	3.7	93.2	4.4	93.2	0.0	93.2	0.0
100	100	1492.7	1481.1	1478.5	1485.8	1485.8	1461.6	3.0	1461.6	3.6	1461.6	0.0	1461.6	0.0
100	250	1027.2	1025.5	1025.5	1041.4	1041.4	978.3	5.9	978.3	6.7	978.3	0.0	978.3	0.0
100	500	674.8	668.4	668.4	667.0	667.0	618.3	8.5	617.6	9.2	617.6	0.1	617.6	0.1
100	750	483.8	486.4	486.4	488.0	488.0	448.2	10.1	448.2	10.9	448.2	0.1	448.2	0.1
100	1000	398.0	398.0	398.0	398.0	398.0	360.7	11.5	360.7	12.3	360.7	0.2	360.7	0.2
100	2000	217.1	217.1	217.1	217.1	217.1	199.4	12.8	199.4	12.5	199.4	0.3	199.4	0.3
150	150	2275.8	2260.5	2260.7	2262.7	2262.7	2237.4	5.2	2237.4	6.0	2237.4	0.0	2237.4	0.0
150	250	1909.8	1880.4	1876.5	1871.8	1874.3	1821.6	8.2	1821.6	9.0	1821.6	0.0	1821.6	0.0
150	500	1287.8	1278.9	1278.9	1273.6	1273.6	1222.4	12.2	1222.2	12.6	1222.1	0.1	1222.1	0.1
150	750	1023.7	1015.7	1015.7	998.8	998.8	927.5	14.3	927.5	15.1	927.5	0.2	927.5	0.2
150	1000	829.3	829.3	838.3	826.2	835.2	751.5	17.2	751.5	17.8	751.5	1.2	751.5	1.2
150	2000	476.1	476.1	476.1	476.1	476.1	425.5	24.0	425.4	23.9	425.4	4.7	425.4	4.6
150	3000	345.3	345.3	345.3	345.3	345.3	306.0	27.5	306.0	26.8	306.0	10.7	306.0	10.6
200	250	2833.8	2808.9	2811.8	2802.3	2805.2	2748.6	11.2	2748.0	10.8	2748.0	0.0	2748.0	0.0
200	500	2065.7	2062.8	2062.8	2049.4	2049.3	1958.8	16.1	1958.3	16.6	1957.8	0.0	1957.8	0.0
200	750	1646.9	1629.0	1625.3	1624.4	1624.8	1530.2	20.6	1528.0	21.0	1527.8	0.2	1527.8	0.2
200	1000	1336.9	1340.5	1342.7	1341.1	1341.1	1226.1	20.8	1225.9	21.7	1225.9	0.5	1225.9	0.5
200	2000	808.0	808.0	803.1	808.0	803.1	723.8	32.9	724.6	31.3	723.8	29.2	723.8	28.9
200	3000	587.4	587.4	587.4	587.4	587.4	518.6	42.4	518.6	39.7	518.6	72.0	518.6	71.7
250	250	3709.2	3678.5	3679.6	3664.7	3663.3	3641.7	11.4	3641.9	10.7	3641.5	0.0	3641.5	0.0
250	500	2943.3	2910.5	2909.1	2906.4	2906.0	2797.1	20.7	2797.0	20.3	2796.9	0.0	2796.9	0.0
250	750	2369.7	2365.2	2365.2	2382.1	2382.1	2234.4	25.7	2233.6	25.3	2231.4	0.1	2231.4	0.1
250	1000	1956.7	1965.5	1967.5	1942.4	1939.6	1816.2	28.2	1815.8	31.3	1815.1	0.4	1815.1	0.4
250	2000	1197.4	1197.4	1197.1	1197.4	1197.1	1073.9	41.7	1074.5	37.1	1073.9	85.9	1073.9	85.1
250	3000	884.1	884.1	884.1	884.1	884.1	784.7	56.3	785.8	47.5	784.7	773.2	784.7	516.8
250	5000	577.4	577.4	578.1	577.4	578.1	514.6	66.0	514.5	60.7	514.3	1670.5	514.5	795.1
300	300	4572.9	4530.6	4530.6	4534.1	4534.1	4487.6	17.7	4487.0	14.3	4486.5	0.0	4486.5	0.0
300	500	3850.4	3811.0	3807.7	3810.4	3813.1	3700.5	21.4	3698.7	23.3	3696.9	0.0	3696.9	0.0
300	750	3205.9	3161.3	3168.7	3152.0	3154.0	3010.6	34.2	3010.4	29.2	3007.8	0.0	3007.8	0.0
300	1000	2667.9	2639.7	2636.1	2635.4	2635.4	2491.0	31.0	2490.7	34.8	2488.1	0.4	2488.1	0.4
300	2000	1658.5	1658.7	1651.5	1652.8	1648.8	1500.4	47.6	1499.9	45.0	1499.3	118.6	1499.3	116.3
300	3000	1199.4	1199.4	1195.5	1199.4	1195.5	1095.9	67.1	1097.6	56.8	1097.6	2332.1	1096.5	1018.9
300	5000	799.0	799.0	799.0	799.0	799.0	719.6	83.5	719.6	79.1	722.4	3393.9	719.6	1079.9
500	500	7546.1	7485.4	7480.4	7482.4	7482.4	7406.3	37.8	7406.0	29.0	7399.2	0.0	7399.2	0.0
500	1000	5833.9	5789.2	5786.7	5796.7	5796.7	5597.3	55.8	5594.8	56.7	5590.0	0.0	5590.0	0.0
500	2000	4001.9	3985.7	3986.3	3973.6	3973.6	3691.8	89.9	3689.8	90.6	3686.0	20.6	3686.0	20.0
500	5000	2012.6	2012.6	2012.6	2012.6	2012.6	1811.7	119.9	1811.0	104.7	1825.4	3600.3	1811.0	1146.1
500	10000	1200.2	1200.2	1200.2	1200.2	1200.2	1044.8	252.6	1048.7	183.3	1075.4	3600.3	1048.1	1206.5
800	1000	11238.7	11088.2	11087.1	11085.4	11087.3	10947.5	75.4	10946.0	77.2	10933.5	0.1	10933.5	0.1
800	2000	8442.3	8382.7	8378.6	8350.2	8350.2	7974.2	170.0	7968.1	161.5	7956.8	0.7	7956.8	0.7
800	5000	4664.6	4639.0	4634.8	4639.1	4636.8	4204.1	236.4	4193.6	222.9	4210.7	3600.2	4189.1	1574.5
800	10000	2697.9	2697.9	2691.5	2697.9	2691.5	2434.2	487.8	2432.0	240.9	2483.7	3600.1	2431.7	1531.1
1000	1000	14956.5	14847.4	14839.8	14863.6	14863.6	14725.8	84.0	14722.3	87.3	14708.9	0.1	14708.9	0.1
1000	5000	6768.5	6814.3	6816.6	6790.3	6792.0	6238.5	437.0	6229.9	344.0	6228.1	3600.2	6213.4	1801.5
1000	10000	4050.6	4050.6	4057.7	4047.0	4054.1	3647.7	462.4	3648.3	322.0	3728.3	3600.1	3647.3	1719.0
1000	15000	2945.7	2945.7	2943.7	2945.7	2943.7	2651.1	624.8	2650.3	456.1	2734.9	3600.1	2650.3	1849.6
1000	20000	2331.0	2331.0	2331.0	2331.0	2331.0	2102.5	869.6	2093.7	608.4	2215.9	3600.0	2093.7	1973.2
Avg:		2530.2	2515.6	2515.1	2513.0	2513.1	2392.6	92.1	2391.7	72.3	2397.9	704.2	2390.0	312.4

Table 4

Comparison of H1, H2, H3, H4, H5, ABC, IWO, ILP, and MH on Type-II dataset.

$ V $	$ A $	H1	H2	H3	H4	H5	ABC	ABC_{ET}	IWO	IWO_{ET}	ILP	ILP_{ET}	MH	MH_{ET}
50	50	100.6	101.0	100.8	103.4	103.5	100.0	1.5	100.0	1.3	100.0	0.0	100.0	0.0
50	100	165.9	166.7	165.8	185.6	182.2	163.4	2.3	163.4	2.8	163.4	0.0	163.4	0.0
50	250	379.5	379.5	379.5	382.6	382.6	370.0	4.0	370.0	4.4	370.0	0.0	370.0	0.0
50	500	521.4	521.4	521.4	521.4	521.4	495.9	4.5	495.9	4.7	495.9	0.0	495.9	0.0
50	750	674.3	674.3	674.3	674.3	674.3	633.0	4.5	634.8	4.5	633.0	0.0	633.0	0.0
50	1000	618.6	618.6	618.6	618.6	618.6	598.3	4.7	598.3	3.2	598.3	0.0	598.3	0.0
100	100	198.9	198.6	198.4	202.3	202.3	196.6	3.2	196.6	3.8	196.6	0.0	196.6	0.0
100	250	390.7	386.9	386.9	414.5	414.5	376.4	7.0	376.3	7.6	376.3	0.0	376.3	0.0
100	500	712.3	710.8	711.3	723.5	724.0	686.1	10.8	686.2	10.6	686.0	0.0	686.0	0.0
100	750	932.9	932.9	932.9	930.7	930.7	889.3	11.8	888.6	13.1	888.6	0.0	888.6	0.0
100	1000	1233.9	1233.9	1233.9	1233.9	1233.9	1148.5	15.9	1148.5	15.1	1148.5	0.0	1148.5	0.0
100	2000	1743.8	1743.8	1743.8	1743.8	1743.8	1583.0	17.0	1583.0	16.4	1583.0	0.0	1583.0	0.0
150	150	307.9	304.9	305.2	308.2	308.2	302.8	5.4	302.8	6.1	302.8	0.0	302.8	0.0

(continued on next page)

Table 4 (continued).

$ V $	$ A $	H1	H2	H3	H4	H5	ABC	ABC _{ET}	IWO	IWO _{ET}	ILP	ILP _{ET}	MH	MH _{ET}
150	250	419.9	417.7	417.4	439.8	439.4	411.4	9.3	411.4	9.9	411.4	0.0	411.4	0.0
150	500	743.4	744.0	744.0	766.0	765.2	724.1	13.2	723.8	14.5	723.8	0.0	723.8	0.0
150	750	1064.2	1062.3	1060.8	1064.9	1063.4	1018.5	18.2	1017.4	18.1	1016.1	0.0	1016.1	0.0
150	1000	1358.0	1358.0	1358.0	1358.0	1358.0	1298.2	23.8	1297.5	21.3	1297.5	0.0	1297.5	0.0
150	2000	2066.4	2066.4	2066.4	2066.4	2066.4	1989.5	31.5	1986.7	31.4	1986.1	0.1	1986.1	0.1
150	3000	2550.3	2550.3	2550.3	2550.3	2550.3	2432.8	38.3	2432.8	34.7	2428.8	0.1	2428.8	0.1
200	250	472.7	470.3	470.9	486.8	486.8	466.6	11.3	466.6	11.5	466.6	0.0	466.6	0.0
200	500	795.2	792.5	792.5	846.3	847.4	772.6	20.4	771.8	19.0	771.1	0.0	771.1	0.0
200	750	1128.4	1127.5	1127.5	1156.3	1156.3	1076.4	22.1	1076.0	23.2	1075.1	0.0	1075.1	0.0
200	1000	1404.6	1407.3	1407.3	1402.6	1402.6	1350.5	27.3	1349.4	25.7	1349.0	0.0	1349.0	0.0
200	2000	2336.1	2336.1	2336.1	2336.1	2336.1	2215.1	45.0	2212.7	39.1	2211.6	0.1	2211.6	0.1
200	3000	2999.0	2999.0	2999.0	2999.0	2999.0	2851.9	53.3	2851.9	50.7	2851.4	0.1	2851.4	0.1
250	250	498.7	496.0	497.4	502.5	502.4	492.7	11.2	492.7	10.9	492.7	0.0	492.7	0.0
250	500	850.7	849.6	849.2	895.3	893.9	832.3	24.9	832.1	22.1	832.1	0.0	832.1	0.0
250	750	1138.5	1135.7	1135.3	1181.7	1182.6	1107.7	28.2	1107.3	27.2	1106.6	0.0	1106.6	0.0
250	1000	1511.1	1510.1	1510.1	1551.8	1551.3	1445.5	34.5	1443.0	32.6	1442.5	0.0	1442.5	0.0
250	2000	2505.3	2505.3	2505.3	2505.3	2505.3	2397.7	57.0	2395.4	45.1	2389.4	0.1	2389.4	0.1
250	3000	3568.3	3568.3	3568.3	3568.3	3568.3	3267.4	61.1	3264.7	59.4	3264.5	0.1	3264.5	0.1
250	5000	4633.5	4633.5	4633.5	4633.5	4633.5	4379.3	91.8	4373.4	81.8	4373.4	0.2	4373.4	0.2
300	300	629.1	625.7	626.2	632.8	632.8	621.6	15.6	621.6	14.1	621.6	0.0	621.6	0.0
300	500	891.8	888.0	887.4	941.2	941.7	871.4	25.0	871.0	25.8	870.3	0.0	870.3	0.0
300	750	1256.0	1250.7	1249.8	1296.0	1296.5	1216.0	32.3	1215.8	32.8	1215.4	0.0	1215.4	0.0
300	1000	1630.4	1625.8	1627.9	1665.4	1663.5	1556.6	36.4	1555.5	36.8	1552.7	0.0	1552.7	0.0
300	2000	2657.2	2657.3	2657.3	2672.2	2672.2	2531.3	56.3	2527.1	51.2	2523.2	0.1	2523.2	0.1
300	3000	3717.0	3717.0	3719.1	3717.0	3719.1	3536.5	75.1	3534.2	74.0	3520.9	0.2	3520.9	0.1
300	5000	5115.8	5115.8	5115.8	5115.8	5115.8	4797.3	104.6	4788.1	97.3	4776.9	0.2	4776.9	0.2
500	500	1022.5	1018.2	1017.0	1034.0	1033.8	1011.2	36.1	1011.3	31.6	1011.0	0.0	1011.0	0.0
500	1000	1711.4	1699.2	1699.9	1800.7	1802.5	1675.0	62.2	1674.5	59.8	1672.3	0.0	1672.3	0.0
500	2000	3038.9	3036.9	3036.9	3111.0	3114.9	2915.3	104.3	2915.4	94.0	2904.0	0.0	2904.0	0.0
500	5000	6116.7	6116.7	6118.6	6116.7	6118.6	5792.5	148.4	5789.3	127.9	5769.9	0.4	5769.9	0.4
500	10000	9761.6	9761.6	9761.6	9761.6	9761.6	9084.1	251.8	9063.2	243.7	9043.2	0.7	9043.2	0.7
800	1000	1891.8	1880.4	1879.7	1945.6	1945.2	1864.0	91.6	1863.1	86.2	1860.5	0.1	1860.5	0.1
800	2000	3342.2	3339.7	3340.4	3493.5	3494.7	3247.1	165.1	3243.7	158.6	3233.0	0.1	3233.0	0.1
800	5000	6905.2	6909.9	6909.9	6917.3	6917.3	6605.8	273.5	6604.9	311.9	6561.7	0.4	6561.7	0.4
800	10000	11541.3	11541.3	11541.3	11541.3	11541.3	10802.6	407.3	10778.1	376.8	10741.5	1.5	10741.5	1.5
1000	1000	2023.7	2012.2	2012.4	2041.1	2040.4	2001.5	90.7	2001.3	91.8	2000.6	0.1	2000.6	0.1
1000	5000	7365.4	7354.1	7354.6	7449.7	7450.3	7030.7	375.6	7027.8	408.6	6975.0	0.5	6975.0	0.5
1000	10000	12489.7	12489.7	12485.1	12475.4	12470.8	11745.2	488.6	11702.7	602.8	11642.5	7.1	11642.5	6.8
1000	15000	16447.3	16447.3	16447.3	16447.3	16447.3	15414.2	618.3	15374.2	492.1	15267.1	104.6	15267.1	99.0
1000	20000	19694.3	19694.3	19694.3	19694.3	19694.3	18278.0	835.8	18240.6	693.6	18172.5	95.5	18172.5	90.9
Avg:		3005.2	3003.5	3003.5	3023.1	3023.1	2842.9	94.6	2838.8	90.3	2829.6	4.1	2829.6	3.9

Table 5

Performance comparison summary of 4 proposed approaches.

Dataset	Method	ABC			IWO			ILP			MH		
		B	E	W	B	E	W	B	E	W	B	E	W
UDG	ABC	–	–	–	2	3	7	0	3	9	0	3	9
	IWO	7	3	2	–	–	–	0	3	9	0	3	9
	ILP	9	3	0	9	3	0	–	–	–	0	12	0
	MH	9	3	0	9	3	0	0	12	0	–	–	–
Type-I	ABC	–	–	–	7	18	28	9	20	24	2	21	30
	IWO	28	18	7	–	–	–	8	22	23	0	26	27
	ILP	24	20	9	23	22	8	–	–	–	1	42	10
	MH	30	21	2	27	26	0	10	42	1	–	–	–
Type-II	ABC	–	–	–	4	15	34	0	14	39	0	14	39
	IWO	34	15	4	–	–	–	0	19	34	0	19	34
	ILP	39	14	0	34	19	0	–	–	–	0	53	0
	MH	39	14	0	34	19	0	0	53	0	–	–	–
Overall	ABC	–	–	–	13	36	69	9	37	72	2	38	78
	IWO	69	36	13	–	–	–	8	44	66	0	48	70
	ILP	72	37	9	66	44	8	–	–	–	1	107	10
	MH	78	38	2	70	48	0	10	107	1	–	–	–

8. Conclusions

Minimum weight dominating set problem (MWDS) and minimum directed dominating set problem (MDDS) are well-known variants of minimum dominating set problem and many approaches are available in the literature to solve them. However, the minimum weight directed dominating set problem (MWDDS) that has several real-world applications and can be considered as a generalization of MWDS and MDDS both has not received much attention. Only five problem

specific heuristics were available in the literature (Nakkala and Singh, 2020), and this problem has neither been studied in the literature from metaheuristic perspective nor from integer linear programming perspective nor from matheuristic perspectives. This paper proposed two swarm intelligence based metaheuristics, viz. ABC and IWO, an ILP approach and a matheuristic approach MH and experimentally evaluated their performance on three datasets derived from respective datasets of MWDS. All the four approaches performed better than five problem specific heuristics. IWO performed better than ABC. MH and

Table 6
Influence of parameter settings on solution quality returned by ABC and IWO.

ABC				IWO			
Parameter	Tested values	Corresponding results	Best value	Parameter	Tested values	Corresponding results	Best value
E_n	{50, 75, 100}	{26763.2, 26751.9, 26770.6}	75	n_{init}	{250, 300, 350}	{26723.6, 26674.4, 26746.1}	300
O_n	{100, 125, 150}	{26784.7, 26751.9, 26786.3}	125	x_{min}	{0, 1, 2}	{26746.7, 26674.4, 26711.1}	1
$limit_{non_imp}$	{500, 600, 700}	{26776.0, 26751.9, 26780.3}	600	x_{max}	{2, 3, 4}	{26693.6, 26674.4, 26718.7}	3
R	{0.5, 0.6, 0.7}	{26791.3, 26751.9, 26787.2}	0.6	R	{0.5, 0.6, 0.7}	{26754.7, 26674.4, 26747.4}	0.6
I_{update}	{7, 8, 9}	{26774.2, 26751.9, 26769.9}	8	I_{update}	{4, 5, 6}	{26708.8, 26674.4, 26717.5}	5
P_{ol}	{0.7, 0.8, 0.9}	{26780.7, 26751.9, 26772.3}	0.8	n_{max}	{300,350,400}	{26757.0, 26674.4, 26716.3}	350
P_r	{0.2, 0.3, 0.4}	{26775.2, 26751.9, 26769.6}	0.3	P_r	{0.2, 0.3, 0.4}	{26745.3, 26674.4, 26736.1}	0.3

ILP performed better than both ABC and IWO. MH performed better than ILP, and, hence, it is the overall best approach to solve MWDDS.

As a future work, we intend to work on few more metaheuristic and matheuristic approaches for MWDDS. The adaptive mechanism used here for selecting a heuristic can be used for other problems also where more than one problem-specific heuristics are available. Success of MH again stresses the effectiveness of matheuristics in solving the dominating set variants (Albuquerque and Vidal, 2018; Pinacho-Davidson et al., 2019; Nakkala et al., 2021). Hence, new matheuristic approaches need to be explored for solving different dominating set variants and other related problems like vertex cover and independent set variants.

CRedit authorship contribution statement

Mallikarjun Rao Nakkala: Conceptualization, Methodology, Software, Formal analysis, Investigation, Writing – original draft, Visualization. **Alok Singh:** Supervision, Conceptualization, Methodology, Formal analysis, Investigation, Validation, Writing - review & editing, Resources. **André Rossi:** Conceptualization, Methodology, Software, Validation, Formal analysis, Writing – original draft, Writing – review & editing.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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