A structural model with jump-diffusion processes

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Abstract

In this paper, we extend the framework of Leland's 94 by examining corporate debt, equity and firm values with jump-diffusion processes. We choose two kinds of jumps such as the uniform and double exponential jumps to modelise the distribution of the log jump sizes. By this choice, we are able to derive closed-form results in both models for equity, debt and firm values. Our results have the same forms as those of Leland's 94. However, in both our models, the spreads are modified significantly in comparison with those of Leland due to jumps' assumption.

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Abstract

In this paper, we extend the framework of Leland's 94 by examining corporate debt, equity and firm values with jump-diffusion processes. We choose two kinds of jumps such as the uniform and double exponential jumps to modelise the distribution of the log jump sizes. By this choice, we are able to derive closed-form results in both models for equity, debt and firm values. Our results have the same forms as those of Leland's 94. However, in both our models, the spreads are modified significantly in comparison with those of Leland due to jumps' assumption.

1 Introduction

Credit risk has raised a increasing interests for both academic and professional researchers for the past ten years. Although there have been many papers written on the subjects of credit risk and related credit derivatives, there are still many questions left unresolved and unsatisfied. There are mainly three principal credit risk pricing approaches: the structural approach, the intensity approach and the rating based approach. The pricing of credit derivatives is mostly explicitly priced inside these three different approaches. The structural approach is marked by a series of papers by Leland [19] and Leland and Toft [20] which consider the question of the optimal capital structure of the firm and its endogenous default barrier. Although the ideas are very appealing, interesting and intuitive, the results of their models are unsatisfactory (for example the spread does not fit empirical data) due to the assumption that the underlying process follows a geometric Brownian motion with constant parameters as in the Black Scholes framework. Since then, many other solutions proposed to modify this model such as changing the underlying process, using the stochastic volatility or interest rate, or even introducing transaction costs. Among these propositions, the presence of jumps in the asset prices are further supported by empirical papers of Bates [1] and others.

In the tendency of using jump-diffusion processes for replacing the Black Scholes framework, we can refer to Merton [22], Zhou [29] for using

a log normal jump-diffusion. Hillberink and Rogers [15] first introduce a spectrally negative Lévy process into the framework of Leland's model. However, the restriction of only negative jumps yields the results of the model less applaudable. Boyarchenski and Levendorskii [4] propose a structural model whose assets follow a regular Lévy process of exponential type. They propose the calculations mostly by the Wiener Hoft factorization. Although their model proposes two approximate formulations in extreme cases (very close to or very far from the bankruptcy level), these formulations are still too complicated to apply in general because the explicit calculation of the Wiener Hoft factorization is difficult. The papers of Eberlein and Ozkan [28],[27] use the Lévy process for credit risk mostly in the intensity approach. (We do not cite a series of papers using the Lévy process or jump-diffusion process for pricing options).

Our paper is on the line of the Leland approach [19] using jump diffusions processes with two different kinds of jumps. One is the simple uniform density and the other is a special case of the Levy processes called the double exponential jump diffusion which is first proposed by Kou [16]. We choose these two kind of jumps which are totally different for two main reasons. The first one is to verify the importance of the different amplitude of jumps in the valuations of the firm's value, its debt and equity. The second reason is these two kinds of jumps can give us analytical solutions. With this reason, we can also choose the log normal jump-diffusion process as in Merton [22] or the bivariate jumps used by Gukhal [13]. Other reasons can be the fact that the uniform jump is the most simple jump process, however the derivative formula is not evident. While the double exponential jump diffusion for the assets process is a special case of the Lévy processes which has the two interesting properties of the exponential distribution¹. The first property is that the double exponential distribution has the leptokurtic feature² of the jump size that provides the peak and tails of the return distribution found in reality. The second property is that the double exponential distribution has the memoryless property which makes it easier in the calculation of expected average and variance terms and in

¹See Kou (2002) for more detail explications of the advantages of the double exponential jump diffusion model and the comparison with other models as the constant elasticity model (CEV), the normal jump diffusion model, model based on t-distribution, stochastic volatility model, affine jump diffusion model, model based on Levy process.

²The leptokurtic feature means that the return distribution of assets may have a higher peak and two asymmetrically heavier tails than those of the normal distributions.

the computation of the distribution of the first passage times. Thanks to this memoryless property, the problem of overshoots, a problem occurs when a jump diffusion process crosses a barrier level b , as sometimes it hits the barrier level exactly S_{τ_b} and sometimes it incurs an "overshoot" $S_{\tau_b} - b$ over the barrier, can be resolved.

We find that in general, the debt, equity and firm values in our models have the same forms as those in Leland's 94 which confirm that his model is robust. However, the spreads in both our models are higher than those in Leland's due to the fact of jumps. Our contribution can be seen in two sides: mathematical and finance. In the mathematical side, we derive analytic formulas for the value of American perpetual option, ainsi que the formula for the debt, equity and firm values in two jump diffusion frameworks, the uniform and double exponential jumps. In the financial side, we test influences of jumps in the firm's related values of a structural model with endogenous default barrier.

The paper is organized as follows. In section 2, we present the debt equity structure of the firm and also its general formulas of valuations. We present the two models of jump-diffusion for the assets' process in section 3. Formulas for each process and also the results which are compared with the original Leland's 94 model are presented in section 4. We end with conclusion in section 5. Technical results are reported in appendices. Appendix A derives the formula of a perpetual American put option in the double exponential jump diffusion framework. Appendix B derives the equivalent formula for the uniform jump diffusion framework.

2 The debt-equity structure of the firm

We suppose that the value of firm's assets follows a jump diffusion under the **risk neutral probability** Q^3 at time t as equation:

$$\frac{dV_t}{V_{t-}} = (r - \delta - \lambda\xi)dt + \sigma dW_t + d\left(\sum_{i=1}^{N_t}(S_i - 1)\right) \quad (1)$$

where r is the riskless interest rate and δ is the payout ratio. This ratio δ is a proportional rate at which profit is distributed to investors (both shareholders and bondholders). The other parameters will be precise later in section 3.

Debt structure

Suppose that the firm is partly financed by debt. This debt promises a perpetual coupon payment C ($C \geq 0$ non negative) until bankruptcy. If bankruptcy occurs (it means that the value of the firm's assets V is less than V_B , the bankruptcy trigger level), a fraction of value of the firm's assets α will be lost due to bankruptcy costs or to the reorganization and the absolute priority rule is respected.

Time of default

Default happens at the first time τ that the value of the firm's assets falls down to some level V_B (default barrier that will be calculated optimally later).

$$\tau = \inf \{t > 0, V_t \leq V_B\} \quad (2)$$

The value at default V_τ may be different from the value of default barrier because of jumps.

The value of the debt is the expected value of the actual value of coupon payment C if there is no default and the value at default $(1 - \alpha)V_\tau$ if there is default. We have, under the risk neutral probability Q the formula of the debt value as follows:

$$D = D(V, V_B) = E \left[\left(\int_0^\tau C e^{-ru} du \right) + (1 - \alpha) V_\tau e^{-r\tau} 1_{\tau < \infty} \right]$$

$$\iff \frac{C}{r} (1 - E(e^{-r\tau} 1_{\tau < \infty})) + E[(1 - \alpha) V_\tau e^{-r\tau} 1_{\tau < \infty}]$$

³Note that this risk neutral measure Q is not unique and it depends on the utility function of representative agent.

$$D = \frac{C}{r} - E \left[e^{-r\tau} 1_{\tau < \infty} \left(\frac{C}{r} - (1 - \alpha) V_\tau \right) \right] \quad (3)$$

The value of the firm is the expected value of the firm's assets (V) plus the expected value of the tax benefit (TB) (the tax rate is κ) minus the value of expected bankruptcy cost value (BC). Thus we can write the value of the firm as follows:

$$\begin{aligned} v = v(V, V_B) &= E \left[V + \frac{\kappa C}{r} (1 - e^{-r\tau} 1_{\tau < \infty}) - \alpha V_\tau e^{-r\tau} 1_{\tau < \infty} \right] \\ &\iff V + \frac{\kappa C}{r} (1 - E(e^{-r\tau} 1_{\tau < \infty})) - E[\alpha V_\tau e^{-r\tau} 1_{\tau < \infty}] \\ v &= V + \frac{\kappa C}{r} - E \left[e^{-r\tau} 1_{\tau < \infty} \left(\frac{\kappa C}{r} + \alpha V_\tau \right) \right] \end{aligned} \quad (4)$$

The value of firm's equity which is equal to the firm's value minus the debt value, can be written down easily:

$$E = E(V, V_B) = v(V, V_B) - D(V, V_B)$$

To rearrange the term, we have the final formula of the firm's equity value:

$$E = V - \frac{(1 - \kappa) C}{r} + E \left[e^{-r\tau} 1_{\tau < \infty} \left(\frac{(1 - \kappa) C}{r} - V_\tau \right) \right] \quad (5)$$

And the shareholders objective is to maximize the value of equity E by choice of V_τ or the moment of default τ .

Introducing $K = (1 - \kappa) C/r$, the problem of maximization of the firm's equity value E becomes:

$$\max_{V_\tau} E[e^{-r\tau} 1_{\tau < \infty} (K - V_\tau)] = \sup_{\tau} E[e^{-r\tau} 1_{\tau < \infty} (K - V_\tau)] \quad (6)$$

This formula looks quite familiar to us and we realize that is the formulation of a **perpetual American put with the strike K**.

Recall the value of a perpetual American put is

$$\sup_{\tau} E[e^{-r\tau} (K - V_\tau)^+]$$

The only two differences between the two formulas are: $(K - V_\tau)^+$ and $1_{\tau < \infty}$

- It is well known that the optimal stopping time τ^* verifies that $(K - V_\tau) \geq 0$ i.e., $(K - V_\tau) = (K - V_\tau)^+$

Indeed, suppose that τ^* is optimal and $(K - V_\tau) < 0$ with positive probability on $\tau^* < \infty$, we can always find

$$\tau^{**} = \tau^* 1_{(K-V_\tau) > 0} 1_{\tau < \infty} + \infty 1_{(K-V_\tau) < 0} 1_{\tau < \infty} + \infty 1_{\tau = \infty}$$

then τ^{**} is also a stopping time. We have:

$$\begin{aligned} & E \left[e^{-r\tau^{**}} (K - V_{\tau^{**}}) 1_{\tau^{**} < \infty} \right] \\ &= E \left[e^{-r\tau^*} (K - V_{\tau^*}) 1_{\tau^* < \infty} 1_{(K-V_\tau) > 0} \right] \\ &> \sup_\tau E \left[e^{-r\tau} (K - V_\tau) 1_{\tau < \infty} \right] \end{aligned}$$

so if τ^* is optimal, it verifies that $(K - V_\tau)^+$

- We now demonstrate that

$$E \left[e^{-r\tau} 1_{\tau < \infty} \right] = E \left[e^{-r\tau} \right]$$

This equality is trivial, in fact at $\tau = \infty$, $e^{-r\tau} = 0$

Our question is now simple to look for the value of the perpetual American put!

Under the jump diffusion model, the rational expectation equilibrium price of an American put option under the risk neutral probability Q is given:

$$PA(V) = \sup_\tau E \left(e^{-r\tau} (K - V_\tau)^+ \right) = \sup_\tau E \left(e^{-r\tau} [K - V_0 e^{X_\tau}]^+ \right)$$

The definition of X_τ will be presented in the section 3.

3 The Two Models of Jump Diffusion Processes

3.1 The General Jump Diffusion Model

Under the jump diffusion model, the dynamics of the firm's assets has two parts, a continuous part driven by a geometric Brownian motion, and a jump part. The value of the firm's assets following a jump diffusion model is written as follows, under the **objective probability measure** P :

$$\frac{dV_t}{V_{t-}} = (\mu - \delta)dt + \sigma dW_t + d \left(\sum_{i=1}^{N_t} (S_i - 1) \right) \quad (7)$$

where $(W_t, t \geq 0)$ is a one dimensional standard Brownian motion, $(\mu - \delta)$ is the drift, δ is the payout ratio, σ is the volatility. $(N_t, t \geq 0)$ is a Poisson process with rate λ and $\{S_i, i \geq 0\}$ a sequence of independent identically distributed (*i.i.d*) non negative random variables.

In the model, the three sources of randomness, $(N_t, t \geq 0)$, $(W_t, t \geq 0)$ and $(S_i, i \geq 1)$ are assumed to be independent. All parameters as the risk free interest rate r , the drift μ , the payout ratio δ and the volatility σ are assumed to be constants. These assumptions, however can also be relaxed in general case, for the expense of heavier calculations.

To solve the stochastic differential equation (7), we consider the interval between two jumps. In each interval $[T_n, T_{n+1}]$, the processus V is just as a geometric Brownian motion

$$V_t = V_{T_n} \left(\mu - \delta - \frac{\sigma^2}{2} \right) (t - T_n) + \sigma(W_t - W_{T_n})$$

and at T_n the jumps of V is $V_{T_n} = S_{T_n} V_{T_n-}$

So we have the formula of V_t as follows:

$$\begin{aligned} V_t &= V_0 \exp \left\{ \left(\mu - \delta - \frac{\sigma^2}{2} \right) t + \sigma W_t \right\} \prod_{i=1}^{N_t} S_i \\ &\iff V_0 \exp \left\{ \left(\mu - \delta - \frac{\sigma^2}{2} \right) t + \sigma W_t + \sum_{i=1}^{N_t} Y_i \right\} \end{aligned}$$

where V_0 is the value at time 0, $Y_i = \ln(S_i)$.

We assume that the jump risk is diversified. That means the risk premium of jumps equals to zero. And there exists a risk neutral probability measure Q , in which the drift of the value of the firm's assets is as follows:

$$RP : \mu - \delta + \lambda\xi = r - \delta \implies RP = \mu + \lambda\xi - r$$

with $\lambda\xi$ is the expected mean of jump part and

$$\xi = E[S_i] - 1 = E[e^{Y_i}] - 1.$$

To change the Brownian from under the objective probability measure P

$$P : \frac{dV_t}{V_{t-}} = (\mu - \delta - RP)dt + \sigma d\left(W_t + \frac{RP}{\sigma}\right) + d\left(\sum_{i=1}^{N_t}(S_i - 1)\right)$$

into under the risk neutral probability measure Q , we have a news dynamic of V :

$$Q : \frac{dV_t}{V_{t-}} = (r - \delta - \lambda\xi)dt + \sigma d\hat{W}_t + d\left(\sum_{i=1}^{N_t}(S_i - 1)\right) \quad (8)$$

and using the same argument as under the probability P to solve the equation (8), we have:

$$V_t = V_0 \exp \left\{ \left(r - \delta - \frac{\sigma^2}{2} - \lambda\xi \right) t + \sigma W_t + \sum_{i=1}^{N_t} Y_i \right\} = V_0 e^{X_t} \quad (9)$$

where V_0 is the value at time 0 and

$$X_t = \left(r - \delta - \frac{\sigma^2}{2} - \lambda\xi \right) t + \sigma W_t + \sum_{i=1}^{N_t} Y_i$$

One important element in doing pricing with the jump-diffusion process is to find out the moment generating function.

As the processus V has the Levy's character, means the infinitely divisible character of Lévy processes. From the Lemma on the Levy-Khinchine presentation in Bertoin [2], there exists a function $G(\cdot)$ such that $e^{\beta X_t - tG(\beta)}$ is a martingale, so we have:

$$E[e^{\beta X_t}] = \exp\{G(\beta)t\} \quad (10)$$

where

$$G(\beta) = \beta \left(r - \delta - \frac{\sigma^2}{2} - \lambda \xi \right) + \frac{\sigma^2}{2} \beta^2 + \lambda(v - 1), v = E(e^{\beta Y_1}) \quad (11)$$

The demonstration for this formula (11) is as follows:

From the equation (10), take the expectation with the filtration $(F_s, s \leq t)$, we have:

$$E(e^{\beta X_t - tG(\beta)} | F_s) = e^{\beta X_s - sG(\beta)}$$

Arrange the term X and G , we have

$$E(e^{\beta(X_t - X_s)} | F_s) = e^{(t-s)G(\beta)} \quad (12)$$

We calculate now the left side of the equation (12)

$$E(e^{\beta(X_t - X_s)} | F_s) \stackrel{acc.ind}{=} E(e^{\beta(X_t - X_s)}) = E(e^{\beta X_{t-s}})$$

Replace $e^{\beta X_{t-s}}$ by its value as equation (9) and the jump's part $E(N_{t-s}) = n$ earn:

$$\begin{aligned} E(e^{\beta X_{t-s}}) &= E \left[\exp \beta \left(\left(r - \delta - \frac{\sigma^2}{2} - \lambda \xi \right) (t-s) + \sigma W_{t-s} + \sum_{i=1}^{N_{t-s}} Y_i \right) \right] \\ &\iff E \left[\exp \beta \left(\left(r - \delta - \frac{\sigma^2}{2} - \lambda \xi \right) (t-s) + \sigma W_{t-s} \right) \right] \end{aligned}$$

$$E \left[\exp \sum_{i=1}^{N_{t-s}} Y_i \right] \iff \sum_1^\infty E \left[\exp \beta \left(\sum_{i=1}^n Y_i \right) \right] e^{-\lambda(t-s)} \frac{(\lambda(t-s))^n}{n!}$$

Arrange the terms, we have:

$$E(e^{\beta X_{t-s}}) = e^{(t-s)[\beta(r-\delta-\frac{\sigma^2}{2}-\lambda\xi)+\frac{\sigma^2}{2}\beta^2]} e^{-\lambda(t-s)} \sum_1^\infty [E(e^{\beta Y_1})]^n \frac{\lambda^n (t-s)^n}{n!}$$

Note $v = E(e^{\beta Y_1})$, we can reduce the left side of the equation (12) into

$$E(e^{\beta X_{t-s}}) = e^{(t-s)\left[\beta\left(r-\delta-\frac{\sigma^2}{2}-\lambda\xi\right)+\frac{\sigma^2}{2}\beta^2\right]} e^{-\lambda(t-s)} e^{-\lambda v(t-s)}$$

Recall the right side of the equation (12) is $e^{(t-s)G(\beta)}$

We deduce the value of $G(\beta)$ as follows:

$$G(\beta) = \beta\left(r - \delta - \frac{\sigma^2}{2} - \lambda\xi\right) + \frac{\sigma^2}{2}\beta^2 + \lambda(v - 1) \quad (13)$$

Until now we have not yet specified the distribution of jump sizes S_i or its log jump size Y_i . We focus next on the two special kinds of jumps. The log jump sizes Y_i in first case follows a double exponential jump and the second one follows an uniform jump.

3.2 The Double Exponential Jump Diffusion Model

Suppose that the log jump sizes Y_i follows an asymmetric double exponential distribution with the density:

$$f_E(y) = p\eta_1 e^{-\eta_1 y} 1_{\{y \geq 0\}} + q\eta_2 e^{\eta_2 y} 1_{\{y < 0\}}, \quad \eta_1 > 1, \eta_2 > 0$$

where $p, q \geq 0$, $p + q = 1$, represent the probabilities of upward and downward jumps, and

$$\log(S) = Y \stackrel{d}{=} \begin{cases} \xi^+, & \text{with probability } p \\ -\xi^-, & \text{with probability } q \end{cases}$$

where ξ^+ and ξ^- are exponential random variables with means $1/\eta_1$ and $1/\eta_2$, respectively, and the notation $\stackrel{d}{=}$ means equal in distribution.

The condition $\eta_1 > 1$ is used to ensure that the assets' value has finite expectation, means $E(S) < \infty$ and $E(V_t) < \infty$. This condition means that the average upward jump cannot exceed 100%.

We have

$$E(Y) = \frac{p}{\eta_1} - \frac{q}{\eta_2} \quad \text{and} \quad \text{Var}(Y) = pq\left(\frac{1}{\eta_1} + \frac{1}{\eta_2}\right)^2 + \left(\frac{p}{\eta_1^2} + \frac{q}{\eta_2^2}\right)$$

We now calculate the values of v and ξ

$$v = E(e^{\beta Y}) = \int_{-\infty}^{\infty} e^{\beta u} f_Y(u) du$$

$$\Longleftrightarrow \int_{-\infty}^0 e^{\beta u} q \eta_2 e^{\eta_2 u} du + \int_0^{\infty} e^{\beta u} p \eta_1 e^{-\eta_1 u} du$$

$$v = \frac{p \eta_1}{\eta_1 - \beta} + \frac{q \eta_2}{\eta_2 + \beta}$$

$$\xi = E(e^Y) - 1 = \frac{p \eta_1}{\eta_1 - 1} + \frac{q \eta_2}{\eta_2 + 1} - 1$$

To insert the values of v and ξ into the equation (13), we have the equation of moment generating function in the case of double exponential jump-diffusion as follows:

$$G_E(\beta) = \frac{\sigma^2}{2} \beta^2 + \left(r - \delta - \frac{\sigma^2}{2} - \lambda \xi \right) \beta + \lambda \left(\frac{p \eta_1}{\eta_1 - \beta} + \frac{q \eta_2}{\eta_2 + \beta} - 1 \right) \quad (14)$$

We can show by graphs and by verification of sign that the equation $G_E(\beta) = \alpha, \alpha > 0$ has exactly four roots: $\beta_{1,\alpha}, \beta_{2,\alpha}, -\beta_{3,\alpha}, -\beta_{4,\alpha}$ where $0 < \beta_{1,\alpha} < \eta_1 < \beta_{2,\alpha} < \infty, 0 < \beta_{3,\alpha} < \eta_2 < \beta_{4,\alpha} < \infty$

By verification of sign

$$\begin{array}{cccccccc} \beta & -\infty & -\eta_{2-} & -\eta_{2+} & 0 & \eta_{1-} & \eta_{1+} & +\infty \\ G_E(\beta) & +\infty & \searrow -\infty & +\infty & \searrow 0 & \nearrow +\infty & -\infty & \nearrow +\infty \end{array}$$

So $G_E(\beta) = \alpha, \alpha > 0$ has at least 4 roots. But as the equation is a polynomial equation with degree four, it has at most four real roots. It deduces that each interval $(-\infty, -\eta_{2-}), (-\eta_{2+}, 0), (0, \eta_{1-})$ and $(\eta_{1+}, +\infty)$ has only one root.

By graphic

Assume $\alpha = r = 0.07, \delta = 0.01, \sigma = 0.2, \eta_1 = 1/0.02, \eta_2 = 1/0.03, \lambda = 3$, we plot in the **Figure 1**, the curves of $G_E(\beta)$ and α .

3.3 The Uniform Jump Diffusion Model

The log jump sizes Y_i follows an uniform distribution with the density:

$$f_U(y) = c1_{[a,b]} \text{ with } c = \frac{1}{b-a}$$

We have

$$E(Y) = \frac{b+a}{2} \quad \text{and} \quad Var(Y) = \frac{(b-a)^2}{12}$$

We now calculate the values of v and ξ

$$v = E(e^{\beta Y}) = \int_a^b e^{\beta u} \frac{1}{b-a} du = \frac{(e^{\beta b} - e^{\beta a})}{b-a} \frac{1}{\beta}$$

$$\xi = E(e^Y) - 1 = \frac{e^b - e^a}{b-a} - 1$$

To insert the values of v and ξ in the equation (11), the moment generating function $G(\beta)$ in the uniform jump diffusion becomes:

$$G_U(\beta) = \frac{\sigma^2}{2}\beta^2 + \left(r - \delta - \frac{\sigma^2}{2} - \lambda\xi\right)\beta + \lambda\left(\frac{(e^{\beta b} - e^{\beta a})}{(b-a)\beta} - 1\right) \quad (15)$$

For reasonable variables of a, b we can show that $G_U(\beta) = \alpha, \alpha > 0$ has two roots: $\beta_{1,\alpha}, -\beta_{2,\alpha}$

By graphic

Assume $\alpha = r = 0.07, \delta = 0.01, \sigma = 0.2, a = -0.5, b = 0.7, \lambda = 3$, we plot in the Figure 2, the curves of $G_U(\beta), \alpha$.

4 Formulations of the option, equity, debt and firm's values

4.1 The Double Exponential Jump Diffusion Model

Under the double exponential jump diffusion model, the rational expectation equilibrium price of a perpetual American put option is given:

$$PA(V) = \sup_{V_\tau} E^Q \left(e^{-r\tau} (K - V(\tau))^+ \right) = \sup_{\tau} E \left(e^{-r\tau} [K - V_0 e^{X(\tau)}]^+ \right)$$

Then the optimal stopping time is given by

$$\tau = \inf \{t \geq 0, V_t \leq V_B\}$$

where V_B is the optimal default barrier and V_t is as in equation (8).

And then the value of the perpetual American put option is given by $PA(V) = u(V)$, where the value function V is given by

$$u(V) = \begin{cases} K - V & ; \text{if } V < V_B \\ AV^{-\beta_{3,r}} + BV^{-\beta_{4,r}} & ; \text{if } V \geq V_B \end{cases}$$

with the optimal exercise boundary V_B

$$V_B = K \frac{\eta_2 + 1}{\eta_2} \frac{\beta_{3,r}}{1 + \beta_{3,r}} \frac{\beta_{4,r}}{1 + \beta_{4,r}}$$

the coefficients A, B :

$$A = V_B^{\beta_{3,r}} \frac{1 + \beta_{4,r}}{\beta_{4,r} - \beta_{3,r}} \left[\frac{\beta_{4,r}}{1 + \beta_{4,r}} K - V_B \right] > 0$$

$$B = V_B^{\beta_{4,r}} \frac{1 + \beta_{3,r}}{\beta_{4,r} - \beta_{3,r}} \left[V_B - \frac{\beta_{3,r}}{1 + \beta_{3,r}} K \right] > 0$$

and the $1/\eta_2$ is the mean of downward jumps and $-\beta_{3,r}, -\beta_{4,r}$ are roots of the equation (14) : $G_E(\beta) = r$.

The demonstration of this formula is given in the appendix A

To insert the formulation of perpetual American put option into the formulas of equity, debt and firm's values, equations (5, 3, 4) in the case $V \geq V_B$ we have:

The value of the firm's equity is directly equal to:

$$E(V) = V - \frac{(1 - \varkappa)C}{r} + [AV^{-\beta_{3,r}} + BV^{-\beta_{4,\alpha}}]$$

The value of the debt can be written easily as:

$$D(V) = \frac{C}{r} - (1 - \alpha) [A'V^{-\beta_{3,r}} + B'V^{-\beta_{4,\alpha}}]$$

where the values A' , B' and K' are as follows:

$$A' = V_B^{\beta_{3,r}} \frac{1 + \beta_{4,r}}{\beta_{4,r} - \beta_{3,r}} \left[\frac{\beta_{4,r}}{1 + \beta_{4,r}} K' - V_B \right] > 0$$

$$B' = V_B^{\beta_{4,r}} \frac{1 + \beta_{3,r}}{\beta_{4,r} - \beta_{3,r}} \left[V_B - \frac{\beta_{3,r}}{1 + \beta_{3,r}} K' \right] > 0$$

$$K' = \frac{K}{(1 - \alpha)(1 - \varkappa)}$$

The value of the firm is then:

$$v(V) = E(V) + D(V) = V + \frac{\varkappa C}{r} + [A''V^{-\beta_{3,r}} + B'V^{-\beta_{4,\alpha}}]$$

where the values A'' , B'' are as follows:

$$A'' = -V_B^{\beta_{3,r}} \frac{1 + \beta_{4,r}}{\beta_{4,r} - \beta_{3,r}} \left[\frac{\varkappa}{1 - \varkappa} \frac{\beta_{4,r}}{1 + \beta_{4,r}} K - \alpha V_B \right]$$

$$B'' = V_B^{\beta_{4,r}} \frac{1 + \beta_{3,r}}{\beta_{4,r} - \beta_{3,r}} \left[\alpha V_B + \frac{\varkappa}{1 - \varkappa} \frac{\beta_{3,r}}{1 + \beta_{3,r}} K \right]$$

4.2 The Uniform Jump Diffusion Model

Under the uniform jump diffusion model, the rational expectation equilibrium price of a perpetual American put option is given by the formula:

$$PA(V) = \sup_{\tau} E^Q(e^{-r\tau}(K - V(\tau))^+) = \sup_{\tau} E(e^{-r\tau} [K - V_0 e^{X(\tau)}]^+)$$

Then the optimal stopping time is given by

$$\tau = \inf \{t \geq 0, V_t \leq V_B\}$$

where V_B is the optimal default barrier and V_t is as in equation (9).

And then the value of the perpetual American put option is given by $PA(V) = u(V) = u(e^x)$, where the value function V is given by

$$u(e^x) = \begin{cases} K - e^x; & \text{if } e^x < e^{x_B} \\ Ae^{-x\beta_2}; & \text{if } e^x \geq e^{x_B} \end{cases}$$

with the optimal exercise boundary e^{x_B}

$$V_B = e^{x_B} = K \frac{\beta_{2,r}}{1 + \beta_{2,r}}$$

the coefficients A :

$$A = \frac{e^{x_B\beta_2} K}{1 + \beta_{2,r}} > 0$$

and $-\beta_{2,r}$ is root of the equation(15) : $G_U(\beta) = r$.

The demonstration of this formula is given in the appendix *B*.

To insert the formulation of perpetual American put option into the formulas of equity, debt and firm's values, equations (5,3,4) in the case $e^x \geq e^{x_B}$ we have:

The value of the firm's equity is directly equal to:

$$E(V) = V - \frac{(1 - \kappa)C}{r} + Ae^{-x\beta_{2,r}}$$

The value of the debt can be written easily as:

$$D(V) = \frac{C}{r} - (1 - \alpha)A'e^{-x\beta_{2,r}}$$

where the values A' and K' are as follows:

$$A' = \frac{e^{x_B\beta_{2,r}} K'}{1 + \beta_{2,r}} > 0$$

$$K' = \frac{K}{(1 - \alpha)(1 - \kappa)}$$

The value of the firm is then:

$$v(V) = E(V) + D(V) = V + \frac{\kappa C}{r} - A''e^{-x\beta_{2,r}}$$

where the values A'' is written as:

$$A'' = \frac{\kappa}{1 - \kappa} A$$

4.3 The results

In order to compare with the model of Leland's 94, we take his inputs into our model. They are $\sigma = 20\%, 15\%$ or 25% , $r = 6\%$, $\varkappa = 0.35$, $\alpha = 0.5$

Other parameters of the double exponential jump:

$$p = 0.3, q = 0.7, \eta_1 = 1/0.02, \eta_2 = 1/0.03, \lambda = 3$$

Other parameters of the uniform jump:

$$a = -0.5, b = 0.7, \lambda = 3$$

GRAPHICS

Graphics of the moment generating function

Figure 1 : The curve of $G_E(\beta)$ and α

Figure 2 : The curve of $G_U(\beta)$ and α

Leland’s 94 principal graphics

Figure 3

Figure 4

Figure 5

Figure 6

Graphics in the double exponential jump diffusion model

Figure 7

Figure 8

Figure 9

Figure 10

Figure 11

Figure 12

The spreads in our models are much higher than those of Leland's 94

Graphics in the uniform jump diffusion model

Figure 13

I think have a problem of scale with these graphs

Figure 14

Or these results due to the choice of variables [a,b] for the uniform distribution !

Figur 15

Figure 16

Figure 17

Figure 18

Appendix A: Demonstration of the formula of perpetual American put option in the double exponential jump diffusion model

The demonstration below strongly resembles of the Appendix 1 in Kou and Wang [18]. However, to clarify, we present it here in further detail.

Assumption:

Suppose there exist some $x_B : e^{x_B} < K$ and a non negative function $u(x) \in C^1$ such that:

1. $u(x)$ is C^2 on $\mathbb{R} \setminus \{x_B\}$, $u(x_{B-})$ and $u(x_{B+})$ existing and $u(x)$ is convex
2. $(Lu)(x) - ru(x) = 0$ for $\forall x \geq x_B$
3. $(Lu)(x) - ru(x) < 0$ for $\forall x < x_B$
4. $u(x) > (K - e^x)^+$ for $\forall x \geq x_B$
5. $u(x) = (K - e^x)^+$ for $\forall x < x_B$

and suppose there exists a random variable Z with $E[Z] < \infty$, such that:

$$e^{-r(t \wedge \tau \wedge \tau^*)} u(X(t \wedge \tau \wedge \tau^*) + x) \leq Z, \forall t > 0, x, \forall \tau$$

Results:

Then the optimal stopping time is given by

$$\tau^* = \inf \{t \geq 0; V_t \leq e^{x_B}\}$$

And then the value of the perpetual American put option is given by $PA(V) = u(V)$ where the value function V is given by

$$u(x) = \begin{cases} K - e^x & \text{if } x < x_B \\ Ae^{-x\beta_{3,r}} + Be^{-x\beta_{4,\alpha}} & \text{if } x \geq x_B \end{cases}$$

The infinitesimal generator of the double jump diffusion process is given by

$$Lu(x) = \frac{1}{2}\sigma^2 u''(x) + \mu u'(x) + \lambda \int_{-\infty}^{+\infty} [u(x+y) - u(x)] f_Y(y) dy$$

where $u(x)$ is a twice continuously differential function
and $\mu = \left(r - \delta - \frac{\sigma^2}{2} - \lambda\xi\right)$.

To compute the quantity $(Lu)(x)$, we need to first calculate the term $\int_{-\infty}^{+\infty} u(x+y) f_Y(y) dy$

Since the value of u depends on the position with respect to x_B , we shall consider two cases.

We first consider the case $\mathbf{x} \geq \mathbf{x}_B$

In that case,

$$\begin{aligned} \int_{-\infty}^{+\infty} u(x+y) f_Y(y) dy &= \int_{-\infty}^{x_B-x} (K - e^{x+y}) q \eta_2 e^{y \eta_2} dy \\ &\quad + \int_{x_B-x}^0 (A e^{-(x+y) \beta_3} + B e^{-(x+y) \beta_4}) q \eta_2 e^{y \eta_2} dy \\ &\quad + \int_0^{+\infty} (A e^{-(x+y) \beta_3} + B e^{-(x+y) \beta_4}) p \eta_1 e^{-y \eta_1} dy \\ &= I_1 + I_2 + I_3 \end{aligned}$$

The quantities I_i are obtained as follows:

$$\begin{aligned} I_1 &= \frac{1}{\eta_2} q \eta_2 K e^{y \eta_2} \Big|_{-\infty}^{x_B-x} - \frac{1}{1+\eta_2} e^x q \eta_2 e^{y(1+\eta_2)} \Big|_{-\infty}^{x_B-x} \\ &= q e^{(x_B-x)} \left[K - \frac{\eta_2}{1+\eta_2} e^{x_B \eta_2} \right] \\ I_2 &= \frac{1}{(\eta_2 - \beta_3)} e^{-\beta_3 x} q \eta_2 A e^{-(\eta_2 - \beta_3) y} \Big|_{x_B-x}^0 + \frac{1}{(\eta_2 - \beta_4)} e^{-\beta_4 x} q \eta_2 B e^{-(\eta_2 - \beta_4) y} \Big|_{x_B-x}^0 \\ &= \frac{\eta_2}{(\eta_2 - \beta_3)} q A \left[e^{-\beta_3 x} - e^{-\beta_3 x_B} e^{\eta_2(x_B-x)} \right] + \frac{\eta_2}{(\eta_2 - \beta_4)} q B \left[e^{-\beta_4 x} - e^{-\beta_4 x_B} e^{\eta_2(x_B-x)} \right] \\ I_3 &= \frac{\eta_1}{-(\eta_1 + \beta_3)} e^{-\beta_3 x} p A e^{-(\eta_1 + \beta_3) y} \Big|_0^\infty + \frac{\eta_1}{-(\eta_1 + \beta_4)} e^{-\beta_4 x} p B e^{-(\eta_1 + \beta_4) y} \Big|_0^\infty \\ &= \frac{\eta_1}{(\eta_1 + \beta_3)} p A e^{-\beta_3 x} + \frac{\eta_1}{(\eta_1 + \beta_4)} p B e^{-\beta_4 x} \end{aligned}$$

We can now to compute $(Lu)(x) - ru(x)$:

$$\begin{aligned}
(Lu)(x) - ru(x) &= \frac{1}{2}\sigma^2 u''(x) + \mu u'(x) + \lambda \int_{-\infty}^{+\infty} [u(x+y) - u(x)] f_Y(y) dy - ru(x) \\
&= \frac{1}{2}\sigma^2 (A\beta_3^2 e^{-x\beta_3} + B\beta_4^2 e^{-x\beta_4}) - \mu (A\beta_3 e^{-x\beta_3} + B\beta_4 e^{-x\beta_4}) \\
&\quad - r (Ae^{-x\beta_3} + Be^{-x\beta_4}) - \lambda (Ae^{-x\beta_3} + Be^{-x\beta_4}) \\
&\quad + \lambda \left[\begin{aligned} &qe^{(x_B-x)} \left[K - \frac{\eta_2}{1+\eta_2} e^{x_B\eta_2} \right] \\ &+ \frac{\eta_2}{(\eta_2-\beta_3)} qA \left[e^{-\beta_3 x} - e^{-\beta_3 x_B} e^{\eta_2(x_B-x)} \right] \\ &+ \frac{\eta_2}{(\eta_2-\beta_4)} qB \left[e^{-\beta_4 x} - e^{-\beta_4 x_B} e^{\eta_2(x_B-x)} \right] \\ &+ \frac{\eta_1}{(\eta_1+\beta_3)} pAe^{-\beta_3 x} + \frac{\eta_1}{(\eta_1+\beta_4)} pBe^{-\beta_4 x} \end{aligned} \right]
\end{aligned}$$

recall the function $G(x)$ is:

$$G(x) = \frac{1}{2}\sigma^2 x^2 + \mu x + \lambda \left(\frac{p\eta_1}{\eta_1 - x} + \frac{q\eta_2}{\eta_2 + x} - 1 \right)$$

we can pose

$$f(x) = G(-x) - r = \frac{1}{2}\sigma^2 x^2 - \mu x + \lambda \left(\frac{p\eta_1}{\eta_1 + x} + \frac{q\eta_2}{\eta_2 - x} - 1 \right)$$

then we have $(Lu)(x) - ru(x)$

$$\begin{aligned}
(Lu)(x) - ru(x) &= Ae^{-x\beta_3} f(\beta_3) + Be^{-x\beta_4} f(\beta_4) \\
&\quad + \lambda qe^{(x_B-x)\eta_2} \left[K - \frac{\eta_2}{1+\eta_2} e^{x_B} - \frac{\eta_2}{\eta_2-\beta_3} Ae^{-\beta_3 x_B} - \frac{\eta_2}{\eta_2-\beta_4} Be^{-\beta_4 x_B} \right]
\end{aligned}$$

As $-\beta_3, -\beta_4$ are roots of $G(\beta) = r$ so $f(\beta_3) = 0$ and $f(\beta_4) = 0$

We can deduce that $(Lu)(x) - ru(x) = 0$ implies

$$\left[K - \frac{\eta_2}{1+\eta_2} e^{x_B} - \frac{\eta_2}{\eta_2-\beta_3} Ae^{-\beta_3 x_B} - \frac{\eta_2}{\eta_2-\beta_4} Be^{-\beta_4 x_B} \right] = 0$$

We have three unknowns e^{x_B}, A, B so we need three equations in order to solve these unknowns.

When we recall the function $u(x)$ at x_B , we have the first equation:

$$K - e^{x_B} = Ae^{-x_B\beta_3} + Be^{-x_B\beta_4}$$

The smooth fit condition⁴ at x_B gives us the second equation:

⁴The validation of smooth fit condition can be found in Boyarchenko and Levendorskii (2002)

$$e^{x_B} = A\beta_3 e^{-x_B\beta_3} + B\beta_4 e^{-x_B\beta_4}$$

And the third equation, we have from $(Lu)(x) - ru(x) = 0$:

$$\left[K - \frac{\eta_2}{1 + \eta_2} e^{x_B} - \frac{\eta_2}{\eta_2 - \beta_3} A e^{-\beta_3 x_B} - \frac{\eta_2}{\eta_2 - \beta_4} B e^{-\beta_4 x_B} \right] = 0$$

From the first and second equation, we have the formulas of A and B :

$$A = -\frac{\beta_4 K - e^{x_B} \beta_4 - e^{x_B}}{(\beta_3 - \beta_4) e^{-\beta_3 x_B}}$$

$$B = \frac{-e^{x_B} + \beta_3 K - \beta_3 e^{x_B}}{(\beta_3 - \beta_4) e^{-\beta_4 x_B}}$$

Inserting the values of A and B into the third equation generates these formulas:

$$A = e^{x_B \beta_3} \frac{1 + \beta_4}{\beta_4 - \beta_3} \left(\frac{\beta_4}{1 + \beta_4} K - e^{x_B} \right) \quad (16)$$

$$B = e^{\beta_4 x_B} \frac{1 + \beta_3}{\beta_4 - \beta_3} \left(e^{x_B} - \frac{\beta_3}{1 + \beta_3} K \right)$$

$$e^{x_B} = K \frac{\eta_2 + 1}{\eta_2} \frac{\beta_3}{1 + \beta_3} \frac{\beta_4}{1 + \beta_4}$$

So we have proven the condition $n^\circ 2$.

The conditions $n^\circ 1, 4, 5$ follow easily as we have $u(x_{B+}) = u(x_{B-})$ and $0 \leq u(x) \leq K$

The only remaining condition that we have to prove is the condition $n^\circ 3$. That is in the case $x < x_B$.

In that case:

$$\begin{aligned} \int_{-\infty}^{+\infty} u(x+y) f_Y(y) dy &= \int_{-\infty}^0 (K - e^{x+y}) q \eta_2 e^{y \eta_2} dy \\ &+ \int_0^{x_B-x} (K - e^{x+y}) p \eta_1 e^{-y \eta_1} dy \\ &+ \int_{x_B-x}^{+\infty} (A e^{-(x+y)\beta_3} + B e^{-(x+y)\beta_4}) p \eta_1 e^{-y \eta_1} dy \\ &= II_1 + II_2 + II_3 \end{aligned}$$

The quantities II_i are obtained as follows:

$$\begin{aligned}
II_1 &= \frac{\eta_2}{\eta_2} q K e^{y\eta_2} \Big|_{-\infty}^0 - \frac{\eta_2}{1+\eta_2} e^x q e^{y(1+\eta_2)} \Big|_{-\infty}^0 \\
&= Kq - \frac{\eta_2}{1+\eta_2} e^x q \\
II_2 &= -\frac{\eta_1}{\eta_1} p K e^{-y\eta_1} \Big|_0^{x_B-x} - \frac{\eta_1}{(\eta_1-1)} p e^x e^{-y(1+\eta_1)} \Big|_0^{x_B-x} \\
&= pK - pK e^{-\eta_1(x_B-x)} - \frac{\eta_1}{(\eta_1-1)} p e^x + \frac{\eta_1}{(\eta_1-1)} p e^{x_B} e^{-\eta_1(x_B-x)} \\
II_3 &= \frac{\eta_1}{-(\eta_1+\beta_3)} e^{-\beta_3 x} p A e^{-(\eta_1+\beta_3)y} \Big|_{x_B-x}^{\infty} + \frac{\eta_1}{-(\eta_1+\beta_4)} e^{-\beta_4 y} p B e^{-(\eta_1+\beta_4)y} \Big|_{x_B-x}^{\infty} \\
&= \frac{\eta_1}{(\eta_1+\beta_3)} e^{-\beta_3 x_B} p A e^{-\eta_1(x_B-x)} + \frac{\eta_1}{(\eta_1+\beta_4)} e^{-\beta_4 x_B} p B e^{-\eta_1(x_B-x)}
\end{aligned}$$

To arrange the terms of the quantities II_i , we have:

$$\begin{aligned}
II_1 + II_2 + II_3 &= K - e^x \left[\frac{p\eta_1}{\eta_1-1} + \frac{q\eta_2}{1+\eta_2} \right] \\
&\quad - p e^{-\eta_1(x_B-x)} \left[K - \frac{1}{\eta_1-1} e^{x_B} - \frac{\eta_1}{\eta_1+\beta_3} A e^{-\beta_3 x_B} - \frac{\eta_1}{\eta_1+\beta_4} B e^{-\beta_4 x_B} \right]
\end{aligned}$$

We can now compute $(Lu)(x) - ru(x)$ in this case of $x < x_B$:

$$\begin{aligned}
(Lu)(x) - ru(x) &= \frac{1}{2} \sigma^2 u''(x) + \mu u'(x) + \lambda \int_{-\infty}^{+\infty} [u(x+y) - u(x)] f_Y(y) dy - ru(x) \\
&= \frac{1}{2} \sigma^2 e^x - \mu e^x - r(K - e^x) - \lambda(K - e^x) \\
&\quad + \lambda \left[-p e^{-\eta_1(x_B-x)} \left[K - \frac{1}{\eta_1-1} e^{x_B} - \frac{\eta_1}{\eta_1+\beta_3} A e^{-\beta_3 x_B} - \frac{\eta_1}{\eta_1+\beta_4} B e^{-\beta_4 x_B} \right] \right]
\end{aligned}$$

We have $\xi = \frac{p\eta_1}{\eta_1-1} + \frac{q\eta_2}{\eta_2+1}$ and $\mu = (r - \delta - \frac{1}{2}\sigma^2 - \lambda\xi)$. Then $(Lu)(x) - ru(x)$ is equal

$$\Longleftrightarrow -rK + \delta e^x - \lambda p e^{-\eta_1(x_B-x)} \left[K - \frac{1}{\eta_1-1} e^{x_B} - \frac{\eta_1}{\eta_1+\beta_3} A e^{-\beta_3 x_B} - \frac{\eta_1}{\eta_1+\beta_4} B e^{-\beta_4 x_B} \right]$$

To insert the values of A, B as in (16) into the third part

$$\begin{aligned}
& K - \frac{1}{\eta_1 - 1} e^{x_B} - \frac{\eta_1}{\eta_1 + \beta_3} A e^{-\beta_3 x_B} - \frac{\eta_1}{\eta_1 + \beta_4} B e^{-\beta_4 x_B} \\
= & K - \frac{1}{\eta_1 - 1} e^{x_B} - \frac{\eta_1}{\eta_1 + \beta_3} \frac{1 + \beta_4}{\beta_4 - \beta_3} \left(\frac{\beta_4}{1 + \beta_4} K - e^{x_B} \right) \\
& - \frac{\eta_1}{\eta_1 + \beta_4} \frac{1 + \beta_3}{\beta_4 - \beta_3} \left(e^{x_B} - \frac{\beta_3}{1 + \beta_3} K \right)
\end{aligned}$$

We can also rearrange the equation above with terms e^{x_B} and K

$$\begin{aligned}
= & e^{x_B} \left(-\frac{1}{\eta_1 - 1} + \frac{\eta_1}{\eta_1 + \beta_3} \frac{1 + \beta_4}{\beta_4 - \beta_3} + \frac{\eta_1}{\eta_1 + \beta_4} \frac{1 + \beta_3}{\beta_4 - \beta_3} \right) \\
& + K \left(1 - \frac{\eta_1}{\eta_1 + \beta_3} \frac{\beta_4}{\beta_4 - \beta_3} + \frac{\eta_1}{\eta_1 + \beta_4} \frac{\beta_3}{\beta_4 - \beta_3} \right)
\end{aligned}$$

Replacing e^{x_B} with its value as in (16) generates:

$$\begin{aligned}
= & K \frac{\eta_2 + 1}{\eta_2} \frac{\beta_3}{1 + \beta_3} \frac{\beta_4}{1 + \beta_4} \left(\frac{-\beta_3 \beta_4 \eta_1}{(\eta_1 - 1)(\eta_1 + \beta_3)(\eta_1 + \beta_4)} \right) \\
& + K \frac{\beta_3}{\eta_1 + \beta_3} \frac{\beta_4}{\eta_1 + \beta_4} \\
= & -K \frac{\beta_3}{\eta_1 + \beta_3} \frac{\beta_4}{\eta_1 + \beta_4} \frac{\eta_2 + \eta_1}{\eta_2(\eta_1 - 1)}
\end{aligned}$$

Then, we can generate the reduced form of $(Lu)(x) - ru(x)$ which is:

$$(Lu)(x) - ru(x) = -rK + \delta e^x + \lambda p e^{-\eta_1(x_B - x)} K \frac{\beta_3}{\eta_1 + \beta_3} \frac{\beta_4}{\eta_1 + \beta_4} \frac{\eta_2 + \eta_1}{\eta_2(\eta_1 - 1)} \leq 0$$

So the condition No3 is proven.

Appendix B: Demonstration of the formula of perpetual American put option in the uniform jump diffusion case.

Work in advance...to complete

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