Optimal Strategic Beliefs*

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Abstract

We provide a discipline for belief formation through a model of subjective beliefs, in which agents hold incorrect but strategic beliefs. More precisely, we consider beliefs as a strategic variable that agents can choose (consciously or not) in order to maximize their utility at the equilibrium. We find that strategic behaviour leads to belief subjectivity and heterogeneity. Optimism (resp. overconfidence) as well as pessimism (resp. doubt) both emerge as optimal beliefs. Furthermore, we obtain a positive correlation between pessimism (resp. doubt) and risk-tolerance.

Keywords: beliefs formation, strategic beliefs, optimal beliefs, distorted beliefs, pessimism

JEL codes: D81, D84, G12, D03

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1 Introduction

Classical economic theory assumes that decision makers have homogenous and rational expectations. However, the last 30 years have seen an accumulation of empirical tests that invalidate the theoretical conclusions of models based on rational expectations. The homogeneous prior beliefs assumption (Harsanyi doctrine) is weaker than the assumption of rational expectations that all agents' prior beliefs are equal to the objective probabilities. But like rational expectations, the common priors assumption is quite restrictive and does not allow agents to "agree to disagree" (Aumann, 1976). It suffices to observe the heterogeneity of analysts or professional forecasters forecasts or more generally of experts opinions to realize that this assumption is not realistic. Savage (1954) provides axiomatic foundations for a more general theory in which agents hold arbitrary prior beliefs, so agents can agree to disagree. But the alternative to rational expectations lacks discipline and if beliefs can be arbitrary, theory provides little structure or predictive power. Indeed, once the assumption of rational and homogenous expectations is relaxed, the following questions arise : How do agents form their beliefs? Do beliefs exhibit optimism? pessimism? overconfidence? doubt (or underconfidence)? How are these possible biases related to the agents preferences? How are agents' beliefs affected by strategic interaction? What is the impact of these beliefs or expectations on individual decisions and on equilibrium characteristics?

We provide a discipline for belief formation through a model of subjective beliefs, in which agents hold incorrect but strategic beliefs. More precisely, we consider beliefs as a strategic variable that agents can choose (consciously or not) to maximize their utility from trade. Our aim is to provide a rationale for belief heterogeneity that enlightens the reflexion about the questions above.

Each agent can adopt a belief to maximize his utility from trade, taking into account the effect his choice has on price and taking as given the strategy of the other agents. More precisely, we suppose that agents adopt strategic beliefs, so that the associated demand is optimal in the sense of a Nash equilibrium in demands, as presented in Kyle (1989). Then, an agent that forms beliefs strategically forms beliefs that are (the most) useful to him. The strategic explanation of beliefs should be contrasted with 'rational' approaches to beliefs where agents try to reflect the 'world as it is' in their beliefs. We also contrast the strategic explanation of beliefs to approaches in which forward-looking agents optimally distort beliefs and in which beliefs are of intrinsic value to agents, as with wishful thinking or fear of disappointment (Akerlof-Dickens, 1982, Brunnermeier-Parker, 2005, Gollier-Muerman, 2006). In these models, beliefs result from an individual optimization problem while our model of belief formation is strategic.

Our optimal strategic beliefs concept is similar to the pragmatic beliefs concept of Hvide (2002) in the sense that both concepts rely on a game theoretic approach. As underlined by Hvide (2002), "*an immediate criticism against the idea of pragmatic beliefs is that human agents are incapable of constructing beliefs that they do not really believe in*". The main justification provided for such pragmatic beliefs is dynamic: pragmatic beliefs in a population are formed either by the selection pressure towards agents that are born pragmatists, or by agents gradually learning that a certain way of forming beliefs is more rewarding than other ways. As Hvide (2002) underlines it, the notion of pragmatic beliefs refers then to the philosophical school known as Pragmatism. Russell (1945) interprets one of its main ideas as follows: agents should (or do) hold beliefs that have good consequences. Note that Hvide (2002) uses this concept in a framework that

is very different from ours. More precisely, he considers a principal-agent model in a job market framework where agents have (pragmatically built) beliefs about their ability.

Our model is embedded in a simple, standard equilibrium problem with one source of risk and beliefs are about the distribution of this risk. Its structure may be applied to several problems in which risk-averse agents have to choose the optimal exposure to a risk. This is the case, for example, when individuals interact on a financial market or when an insurance company has to negotiate an optimal retention rate with a reinsurance company or when entrepreneurs have to fix the optimal proportion of equity to retain for a given project.

Our findings are the following. A strategic behavior leads to belief subjectivity and heterogeneity. This means that in a standard portfolio/equilibrium problem in which beliefs are strategically determined, the objective belief is not optimal, and agents differ in their optimal strategic beliefs. Indeed, optimism (resp. overconfidence) as well as pessimism (resp. doubt) both emerge as optimal beliefs. Furthermore, we find a positive correlation between pessimism (resp. doubt) and risk-tolerance. The intuition is as follows. For a relatively risk-tolerant agent, his demand in the risky asset is positive, so that his expected utility from trade is decreasing in the price of the risky asset. A pessimistic belief is associated to a lower demand, hence to a lower price, and balances this benefit of pessimism against the costs of worse decision making. The converse reasoning applies to a very risk-averse agent, who, at the equilibrium, has a negative demand in the risky asset and benefits from optimism. Such a positive correlation has been observed in empirical studies in a purely behavioral setting (Ben Mansour et al., 2008).

In an exponential utility and normal distribution setting, the consensus belief (or the representative agent belief), which is given by the average of the individual beliefs weighted by the risk-tolerance, is pessimistic (resp. exhibits doubt). Intuitively, the more risk-tolerant agents make the market, and the consensus belief reflects the characteristics of the more risk-tolerant. Since we have just seen that the more risk-tolerant agents are pessimistic, it is consistent to obtain a pessimistic consensus belief. Moreover, the average (unweighted) belief is also pessimistic, which means that the pessimistic risk-tolerant agents are more pessimistic than the optimistic are optimistic and there is then a pessimistic bias in individual beliefs. Such a pessimistic bias has been observed in empirical studies in a purely behavioral setting (Ben Mansour et al., 2006), in a decision theory framework (Wakker, 2001) or in a market framework (Giordani-Söderlind, 2005).

One may argue that various empirical studies of professionals' economic forecasts as well as psychological surveys have opposite conclusions: optimism (see e.g. Fried and Givoly, 1982, O'Brien, 1988, Francis and Philbrick, 1993, Kang et al., 1994 and Dreman and Berry, 1995) and overconfidence (Rabin, 1998, Hirshleifer, 2001, Giordani and Söderlind, 2006). However, it has been repeatedly argued in the literature that professionals' forecasts may be biased by environmental factors (see e.g. Schipper, 1991, Mc Nichols and O'Brien, 1997, Darrough and Russell, 2002). Furthermore, these studies are generally based on self-assessment. For instance, in psychology, personal pessimism measures how individuals perceive their future while it is clear that individuals have an influence on this future. When this self assessment dimension disappears, it has been shown in various studies that the optimistic bias disappears too¹. Finally, Ben Mansour

¹For instance, Wengert and Rosen (2000) conclude to optimism for items like *I will have a happy life*, *I will keep my best friends* and to neutrality for items like *There will be a third world war*, *The unemployment rate shall fall*, *Life expectancy shall increase*.

et al (2006) have shown that the optimistic bias is transformed into a pessimistic bias when we focus on how individuals perceive the future through items that do have a clear direct impact on their well-being but on which they have no influence. This is coherent with the fact that people tend to attribute success to their own actions but failure to external factors (Zuckerman, 1979, Fiske and Taylor, 1991, Baumeister, 1998, Duval and Silvia, 2002, Van den Steen, 2004).

It is interesting from this perspective to note that pragmatic beliefs also lead to overconfidence in Hvide (2002) (while they lead to underconfidence in our framework). Hvide (2002) focuses on overconfidence in the sense of "hubris" since agents have beliefs about their own ability while we focus on overconfidence in the sense of excessive confidence in the statistical sense (as opposed to doubt) since agents have beliefs about external sources of risk².

Our results obtained in a strategic interaction framework differ from those obtained in an *optimal* beliefs/illusions setting, in which there is almost no belief heterogeneity and an optimistic bias³ (Brunnermeier-Parker, 2005, Brunnermeier et al., 2007, Gollier, 2005). This bias results from the specific mental process they consider. Indeed, in these models, subjective beliefs maximize the agents' expected well-being defined as the time average of expected felicity over all periods. Since agents that care about future utility flows have a higher current felicity if they are optimistic, the optimal beliefs balance this benefit of optimism against the costs of worse decision making.

In a financial markets framework, the pessimistic bias we obtain at the aggregate level is interesting in light of the risk premium puzzle (Mehra and Prescott, 1985). Indeed, the fact that a pessimistic bias and a positive correlation between risk-tolerance and pessimism lead to an increase of the market price of risk has been underlined by Abel (2002), Calvet et al. (2002), Detemple-Murthy (1994), Gollier (2007) and Jouini-Napp (2006, 2007); in their models, beliefs are exogenously given. In the insurance industry, our results lead to a situation where the more risk-averse agent (the insured) is optimistic and the less risk-averse agent (the insurer) is pessimistic. The average belief is pessimistic leading to a higher insurance premium, which might help to explain the purchase of vastly overpriced insurance in a range of situations (Cutler-Zeckhauser, 2004). In corporate finance, IPO's can be modeled as a decision for a risk-averse entrepreneur to sell shares of his firm to more risk-tolerant investors. The application of our results to such a setting leads to a pessimistic consensus belief. As a result, the firm is underpriced and the short run return is large, which is consistent with the empirical literature on IPO's (Ibbotson and Ritter, 1995). Obviously, we don't pretend that strategic interaction, such as in our simple model, is the unique explanation for these puzzles, however, it is interesting to remark that our model helps to explain these puzzles as well as belief heterogeneity without introducing any information asymmetry.

The paper is organized as follows. Section 2 introduces and discusses the model as well as the concept of Nash equilibrium in beliefs and presents the results for an economy with two agents and one risky asset. Explicit computations are provided in a setting with

²Hvide (2002) makes such a distinction and refers to these two possible definitions for overconfidence as $overconfidence_1$ and $overconfidence_2$ and then adopts $overconfidence_1$.

³Gollier-Muerman (2006) consider a model of optimal beliefs with ex ante savoring and ex post disappointment. Depending upon the intensity of anticipatory feelings and disappointment they might also obtain a systematic pessimistic bias.

exponential utility functions and normal distributions in which the strategic variable is the expected payoff of the risky asset. Qualitative results are provided in the setting with more general utility functions and distributions. We also compare our results with those obtained in an *optimal* beliefs setting. Section 3 considers extensions of the model of Section 2 in essentially two directions; a model in which the strategic variable is the variance of the payoff of the risky asset and a model with multiple sources of risk. Section 4 concludes.

2 A model of optimal strategic beliefs

We consider a standard equilibrium model, except that we allow strategic interaction to be incorporated into the analysis.

More precisely, we assume that the economy is composed of two agents, who differ in their level of risk-aversion. The agents live for one period and consumption takes place at the end of the period. There is a single risky asset in the economy, whose payoff at the end of the period is denoted by $\tilde{x}$. We let p denote the unit price of the risky asset, which means that both agents can sell their property rights on the risky asset against the delivery of the sure quantity p at the end of the period. We assume that the agents have the same endowment, which consists of a half unit of the risky asset. As in the standard portfolio problem, agents determine the optimal composition of their portfolio, in other words their optimal exposure to the risk. The difference with the standard model stems from the fact that agents take into account their impact on prices. Since these prices depend upon agents' beliefs, the agents (consciously or not) modify their beliefs to take advantage of this impact. For example, it may be optimal for an agent, who is risk-tolerant, hence willing to be quite highly exposed to the risk, or equivalently interested in buying a high quantity of the risky asset, to underestimate the asset in order to benefit from a lower price. The structure of this problem is quite general and may be applied to several equilibrium problems in which heterogeneous risk-averse agents have to choose their optimal exposure to a given risk. For instance, when an insurance company and a reinsurance company have to determine an optimal retention rate.

We are interested in the characteristics of this economy at the (Nash) equilibrium. More precisely, we are interested in the following questions. Does this model of strategic beliefs lead to subjectivity in optimal beliefs? Does it generate heterogeneous beliefs? Is there a link between risk-tolerance and optimal belief and what is the nature of this link? Is there a pessimistic/optimistic bias at the equilibrium at the individual as well as at the collective level?

For analytical tractability, we first consider exponential utility functions with normal distributions and provide explicit results. We then compare our results with those obtained under *optimal* expectations (as in Brunnermeier and Parker, 2005). Finally, we explore to which extent our results are robust to more general utility functions and distributions.

2.1 The exponential case

In this section, agents have CARA utility functions for consumption, more precisely, $u_1(c) = -\exp\left(-\frac{c}{\theta_1}\right)$ and $u_2(c) = -\exp\left(-\frac{c}{\theta_2}\right)$, where $\theta_i > 0$ denotes the degree of (absolute) risk-tolerance of agent i . Moreover, we assume that $\tilde{x}$ is normally distributed,

with mean μ and variance σ^2 .

In the competitive Walrasian equilibrium model, an equilibrium price is such that agents reach optimal demands and markets clear. More precisely, the optimal demand $\alpha_i(p)$ of the risky asset that agent i will retain, given price p , maximizes the expected utility from trade

$$E \left[u_i \left(\frac{1}{2}p + \alpha_i(\tilde{x} - p) \right) \right].$$

It is well known and easy to obtain that

$$E \left[-\exp \left(-\frac{\frac{1}{2}p + \alpha_i(\tilde{x} - p)}{\theta_i} \right) \right] = -\exp \left(-\frac{\frac{1}{2}p + \alpha_i(\mu - p)}{\theta_i} + \frac{1}{2} \frac{\alpha_i^2 \sigma^2}{\theta_i^2} \right),$$

so that $\alpha_i(p) = \theta_i \frac{\mu - p}{\sigma^2}$. The market clearing condition $\alpha_1(p) + \alpha_2(p) = 1$ imposes then that the equilibrium price of the risky asset is given by $p^* = \mu - \frac{\sigma^2}{\theta_1 + \theta_2}$. Moreover, we obtain $\alpha_i^* = \frac{\theta_i}{\theta_1 + \theta_2}$ so that the agent with a higher (resp. lower) risk-tolerance level has a positive (resp. negative) net demand in $\tilde{x}$ and the part of the risk borne by agent i is exactly given by his relative level of risk-tolerance.

As a simple extension of this model, we can consider the case in which agents have exogeneously given heterogeneous expectations about the payoff of the risky asset, i.e., agent 1 believes that $\tilde{x}$ is normal with mean μ_1 and variance σ^2 and agent 2 believes that $\tilde{x}$ is normal with mean μ_2 and variance σ^2 with⁴ $\mu_1 \neq \mu_2$. The optimal demand of agent i given price p is then given by $\alpha_i(p, \mu_i) = \theta_i \frac{\mu_i - p}{\sigma^2}$. The market clearing price is given by

$$p(\mu_1, \mu_2) = \frac{\theta_1}{\theta_1 + \theta_2} \mu_1 + \frac{\theta_2}{\theta_1 + \theta_2} \mu_2 - \frac{\sigma^2}{\theta_1 + \theta_2}, \quad (1)$$

which is the equilibrium price in an economy in which agents share the same expectations given by $\frac{\theta_1 \mu_1 + \theta_2 \mu_2}{\theta_1 + \theta_2}$. In other words, the equilibrium price in an economy with heterogeneous beliefs is the equilibrium price in an economy in which the belief of the representative agent (whose risk-tolerance $\bar{\theta}$ is given, as in the standard setting, by the sum of the individual risk-tolerances) is given by the average of the heterogeneous beliefs, weighted by the risk-tolerance. Moreover, replacing p by the expression of the equilibrium price, we obtain that the optimal demand α_i^* of agent i at the equilibrium is given by

$$\alpha_i^*(\mu_1, \mu_2) = \alpha_i(p(\mu_1, \mu_2), \mu_i) = \frac{\theta_i}{\bar{\theta}} \left[1 + \theta_j \frac{\mu_i - \mu_j}{\sigma^2} \right] \quad (2)$$

and the part of the risk beared by agent i depends upon both his level of risk-tolerance and his belief.

In the present paper, the individual subjective beliefs are determined endogenously and we analyze their properties, especially in terms of pessimism and correlation between pessimism and risk-tolerance.

Instead of assuming that the equilibrium is a competitive one, we consider agents who act as imperfect competitors and take into account explicitly the effect they have on prices. The agents will choose their beliefs optimally to maximize their utility by taking into account the effect their beliefs (on the mean of $\tilde{x}$) have on prices (through

⁴Walrasian equilibrium models with heterogeneous beliefs have been studied, among others, by Williams (1977), Abel (1989), Detemple-Murthy (1994), Calvet et al. (2002), Jouini-Napp (2006, 2007).

Equation 1) as well as on the trading quantity (through Equation 2). We assume that each agent takes as given the strategy of the other agent which leads us to the notion of Nash equilibrium in beliefs. This notion is the analogue of the notion of Nash equilibrium in demand schedules⁵ (introduced by Kyle, 1989) for demand schedules of a specific form.

More precisely, the choice of a belief $\hat{\mu}_i$ taking the belief μ_j of agent j as given is determined by

$$\hat{\mu}_i = \arg \max_{\mu_i} E \left[u_i \left\{ \frac{1}{2} p(\mu_i, \mu_j) + \{\tilde{x} - p(\mu_i, \mu_j)\} \alpha_i^*(\mu_i, \mu_j) \right\} \right].$$

It may seem puzzling that the agents maximize their utility under the objective probability while we are assuming on the other hand that they have subjective beliefs. In fact, agents *experience* their utility and *choose* their beliefs strategically. We could think of optimal strategic beliefs as the outcome of a process where the agents repeatedly⁶ face a similar situation, record the different received payoffs in terms of utility level and adopt the most rewarding belief. For instance, if it turns out that 'pessimistic' beliefs give a higher payoff than more neutral beliefs, then no wonder that the agent adopts those 'pessimistic' beliefs. In this sense, these beliefs may be considered as pragmatic. Of course, the agent himself does not have to judge his own beliefs as 'pessimistic'⁷. The agent takes into account what pays rather than what is true. It is not even clear why he should be interested in the concept of 'truth' at all.

Definition 1 *A Nash equilibrium in beliefs (on the mean) is defined as a pair of strategies on the mean $M = (\hat{\mu}_1, \hat{\mu}_2)$ such that for any other pair of strategies M' differing only in the i -th component, for $i = 1, 2$, the strategy M yields a utility level no less than M' :*

$$E \left[u_i \left(\frac{1}{2} p(M) + (\tilde{x} - p(M)) \alpha_i^*(M) \right) \right] \geq E \left[u_i \left(\frac{1}{2} p(M') + (\tilde{x} - p(M')) \alpha_i^*(M') \right) \right]$$

The construction of endogenous subjective beliefs that are solutions of a given utility maximization problem has been considered in recent literature by Brunnermeier-Parker (2005), Gollier (2005), Gollier-Muerman (2006), Brunnermeier et al. (2007). In our framework, the subjective beliefs are not only optimal but *strategic*. Indeed, they do not result from an individual utility maximization problem but from a Nash equilibrium, in which each agent takes into account the impact of his choices on the equilibrium price and allocations. As we shall analyze it in detail in Section 2.2, in a non-strategic setting where agents choose their belief in order to maximize a criterion related to their well-being, it is immediate that the *optimal* belief must be optimistic for all agents and that all agents

⁵As underlined by Kyle "(this) is perhaps the most obvious modification of the conventional competitive rational expectations concept. It preserves market clearing through a Walrasian mechanism and keeps the Nash flavour of a competitive equilibrium." In fact, the concept of Nash equilibrium in demand schedules is the analogon, from the consumers point of view, of the Cournot-Nash equilibrium in supply for producers.

⁶If the number of repetitions is large enough, the average utility level obtained by the agent converges to the expected utility under the objective probability.

⁷As underlined by Brunnermeier and Parker (2005), "*most human behavior is not based on conscious cognition, but is automatic, processed only in the limbic system and not the prefrontal cortex*".

select a riskier portfolio. In our setting, there is no such immediate intuition for a given systematic bias.

Proposition 2 *In a model with two agents, exponential utility functions and normal distributions, there exists a unique Nash equilibrium in beliefs (on the mean). It satisfies the following properties.*

1. The optimal beliefs $M = (\hat{\mu}_1, \hat{\mu}_2)$ are given by

$$\hat{\mu}_1 = \mu - \frac{\sigma^2}{4\theta_2(\theta_1 + \theta_2)}(\theta_1 - \theta_2), \quad \hat{\mu}_2 = \mu - \frac{\sigma^2}{4\theta_1(\theta_1 + \theta_2)}(\theta_2 - \theta_1). \quad (3)$$

The more risk-tolerant agent is pessimistic, in the sense that he behaves as if the mean of $\tilde{x}$ lied below its true value, and the less risk-tolerant agent is optimistic. Moreover, the more risk-tolerant agent is more pessimistic than the less risk-tolerant agent is optimistic and the unweighted average of the beliefs is pessimistic:

$$\frac{\hat{\mu}_1 + \hat{\mu}_2}{2} = \mu - \frac{1}{8} \frac{(\theta_1 - \theta_2)^2 \sigma^2}{\theta_1 \theta_2 \bar{\theta}}. \quad (4)$$

2. The representative agent is pessimistic, i.e. the average of the individual beliefs weighted by the risk-tolerance is pessimistic. More precisely,

$$\frac{\theta_1 \hat{\mu}_1 + \theta_2 \hat{\mu}_2}{\theta_1 + \theta_2} = \mu - \frac{1}{4} \frac{(\theta_1 - \theta_2)^2 \sigma^2}{\theta_1 \theta_2 (\theta_1 + \theta_2)}. \quad (5)$$

Note first that our construction of strategic beliefs leads to subjective and heterogeneous optimal beliefs. Indeed, optimal strategic beliefs differ from the objective belief, agents 1 and 2 differ in their optimal belief and, as expressed in Equations (3), belief heterogeneity takes its roots in the difference in risk-aversion levels. Besides, more than just being "heterogeneous", optimal beliefs are "antagonistic" in the sense that one of the agents is optimistic ($\hat{\mu}_i > \mu$) and the other one is pessimistic ($\hat{\mu}_i < \mu$).

The pessimism of the more risk-tolerant agent can be interpreted as follows. Suppose that agent 1 is more risk-tolerant. At the equilibrium, since agents initially only differ in their level of risk-aversion, the risky asset's demand for agent 1 is positive. His expected utility from trade is then decreasing in the price of the risky asset. The choice of a pessimistic belief is associated to a lower demand, hence to a lower price and a higher expected utility. The optimal belief balances this benefit of pessimism against the costs of worse decision making. The converse reasoning applies to agent 2, who, at the equilibrium, has a negative demand in the risky asset and benefits from optimism. Another way to interpret the pessimism of the more risk-tolerant agent is to analyze the situation in the neighborhood of the objective belief and the associated equilibrium, i.e. the Walrasian equilibrium. Indeed, a deviation from the objective belief has potentially two effects on the utility level: a quantity effect and a price effect. The quantity effect is equal to zero due to the optimal quantity choice condition in the Walrasian equilibrium and the price effect is positive for the less risk-tolerant agent (i.e. the agent that has a negative net demand) and is negative for the more risk-tolerant agent.

As a consequence of the positive correlation between pessimism (optimism) and risk-tolerance (risk-aversion), the more risk-tolerant will insure the less risk-tolerant less than in the standard setting, which induces less risk-sharing.

The average (unweighted) belief is pessimistic, which means that the risk-tolerant agent is more pessimistic than the risk-averse is optimistic. This result can be understood as follows. As we have seen, the optimal belief results from an arbitrage between the benefit of a low price induced by pessimism for the risk-tolerant (resp. the benefit of a high price induced by optimism for the risk-averse) against the costs of worse decision making. Let us explore this point further. At the Walrasian equilibrium, the marginal utility of the less risk-tolerant agent associated to a marginal increase of μ is positive and equal to the marginal utility of the more risk-tolerant agent associated to a marginal decrease of μ . By definition, the optimal beliefs correspond to zero marginal utilities. When the more risk-tolerant agent becomes more pessimistic and when the less risk-tolerant agent becomes more optimistic, their marginal utilities decrease at different rates. The difference between these two rates originates in the variance terms in the utility functions and more precisely in the terms $(\Delta\alpha_1)^2$ and $(\Delta\alpha_2)^2$. Due to the market clearing condition, these two terms are equal; however, by definition of the risk-tolerance coefficient, they are weighted respectively by $\frac{1}{\theta_1^2}$ and $\frac{1}{\theta_2^2}$ in the utility functions. This leads to a slower decrease of the marginal utility for the more risk-tolerant agent and then to a more pronounced divergence from the objective belief for that agent.

The consensus belief, which is given by the average of the individual beliefs weighted by the risk-tolerance, is then obviously pessimistic. Intuitively, the more risk-tolerant agents make the market, and the consensus belief reflects the characteristics of the more risk-tolerant. Since we have just seen that the more risk-tolerant is pessimistic, it is consistent to obtain a pessimistic consensus belief.

To sum up, our construction of endogenous beliefs leads to optimal beliefs that are different from the objective belief, heterogeneous, and antagonistic (one is optimistic and the other is pessimistic). There is a positive correlation between risk-tolerance (resp. risk-aversion) and pessimism (resp. optimism), which leads to less risk-sharing.

Note that, as underlined by Jouini and Napp (2006, 2007), a positive correlation between risk-tolerance (resp. risk-aversion) and pessimism (resp. optimism) and a pessimistic risk-tolerance weighted average belief lead to a higher equilibrium risk premium which is interesting in light of the risk premium puzzle (Mehra and Prescott, 1985)

Our results are robust to variations in the initial endowments as long as the more risk-tolerant agent has a positive net demand, i.e. as long as the more risk-tolerant agent insures the less risk-tolerant one, which is a natural situation. At first sight, a negative supply in the risky asset seems to lead to an optimistic bias. Indeed, in that case, the strategic behaviour induces an upward bias on the mean of the risky asset distribution but this corresponds to a pessimistic bias on the total wealth of the economy. The unique situation where all the effects we exhibited disappear corresponds to the case where there is no aggregate risk (i.e. when the total supply in risky assets is equal to zero). Indeed, in such a framework, there is no trade at the Walrasian equilibrium and there is then no price effect and no utility gain associated to a deviation from the objective belief.

2.2 Strategic vs "optimal" beliefs

Let us compare our results with those that are obtained in an *optimal* non-strategic framework. More precisely, adopting the same framework and notations as above, we consider the following concept of optimal beliefs, which corresponds to a simplified version of Brunnermeier-Parker (2005), Gollier (2005) and Brunnermeier et al. (2007).

Definition 3 *For a given price p , an optimal (non-strategic) belief $\bar{\mu}_i(p)$ for agent i is*

defined as the solution of

$$\arg \max_{\mu_i \in K} E_i \left[u_i \left(\frac{1}{2}p + \alpha_i(p, \mu_i) (\tilde{x} - p) \right) \right]$$

where E_i is the expectation operator associated⁸ to the belief μ_i and where K is a given set of admissible values for μ_i that contains the objective belief μ .

The belief $\bar{\mu}_i(p)$ is optimal in the sense that it maximizes over the set K the well-being of agent i .

The original definition of optimal beliefs introduced by Brunnermeier-Parker (2005) and further studied by Brunnermeier et al. (2007) and Gollier (2005) considers a weighted average of the objectively expected utility and of the subjectively expected utility. Our definition is more simple but it is easy to check that the results obtained below, under Definition 3, remain valid under the original definition. This will be further discussed at the end of this section.

We can now define an associated equilibrium concept as follows.

Definition 4 *An equilibrium price with optimal (non-strategic) beliefs is defined as a price $\bar{p}$ such that agents have optimal demands and optimal (non-strategic) beliefs and such that markets clear, i.e.*

$$\alpha_1(\bar{p}, \bar{\mu}_1(\bar{p})) + \alpha_2(\bar{p}, \bar{\mu}_2(\bar{p})) = 1.$$

Let us assume that $K = [a, b]$.

Proposition 5 *In the setting of the previous section (exponential utility and normal distributions), we have*

1. *For a given price p , the optimal (non-strategic) belief $\bar{\mu}_i(p)$ for agent i solves*

$$\max_{\mu \in \{a, b\}} (\mu - p)^2.$$

2. *If $\frac{\sigma^2}{\theta_1 + \theta_2} \geq \frac{b-a}{2}$, then the equilibrium is characterized by $\bar{p} = b - \frac{\sigma^2}{\theta_1 + \theta_2}$ and $\bar{\mu}_1(\bar{p}) = \bar{\mu}_2(\bar{p}) = b$. The agents share the same optimistic belief.*
3. *If $\frac{\theta_1 a + \theta_2 b}{\theta_1 + \theta_2} - \frac{\sigma^2}{\theta_1 + \theta_2} = \frac{a+b}{2}$, where $\theta_1 < \theta_2$, then the equilibrium is characterized by $\bar{p} = \frac{a+b}{2}$, $\bar{\mu}_1(\bar{p}) = a$ and $\bar{\mu}_2(\bar{p}) = b$. The more risk-tolerant agent is the more optimistic and the consensus belief $\frac{\theta_1 a + \theta_2 b}{\theta_1 + \theta_2}$ is more optimistic than the equally weighted belief $\frac{a+b}{2}$.*

We have then two possible situations but both of them induce an optimistic bias at the aggregate level. Furthermore, unless $\frac{\theta_1 a + \theta_2 b}{\theta_1 + \theta_2} - \frac{\sigma^2}{\theta_1 + \theta_2} = \frac{a+b}{2}$, there is no belief heterogeneity and both agents are optimistic. In fact, even when $\frac{\theta_1 a + \theta_2 b}{\theta_1 + \theta_2} - \frac{\sigma^2}{\theta_1 + \theta_2} = \frac{a+b}{2}$, the more risk averse agent is not completely pessimistic. Indeed, it is easy to check that this agent

⁸More precisely, E_i is the expectation operator associated to a probability P_i that represents agents i 's belief and under which $\tilde{x} \sim \mathcal{N}(\mu_i, \sigma^2)$.

is short in the risky asset and is then optimistic with respect to his own allocation, i.e. overestimate the return of his own portfolio.

Notice that for $\frac{\sigma^2}{\theta_1+\theta_2} < \frac{b-a}{2}$ and $\frac{\sigma^2}{\theta_1+\theta_2} \neq \frac{a+b}{2} - \frac{\theta_1 a + \theta_2 b}{\theta_1 + \theta_2}$ where agent 1 is the less risk-tolerant one, there is no equilibrium. A natural extension of the model would consist in allowing for mixed strategies or for a continuum of agents. We retrieve then an equilibrium in which a proportion α of the agents choose the belief a and a proportion $(1 - \alpha)$ choose the belief b . For instance, if we assume that the distribution of beliefs is independent of the distribution of risk-tolerances, the market clearing condition leads to

$$\alpha a + (1 - \alpha) b - \frac{\sigma^2}{\int \theta_i di} = \frac{a + b}{2}. \quad (6)$$

The proportion α is then perfectly determined if $\frac{\sigma^2}{\int \theta_i di} \leq \frac{b-a}{2}$. The solution α is always lower than $\frac{1}{2}$ which means that the consensus belief is always optimistic. This equilibrium in which each agent is indifferent between two possible beliefs and in which the market clearing condition imposes the proportions of agents choosing each belief resembles the equilibrium obtained in Brunnermeier-Parker (2005).

These results are analogous to those of Brunnermeier-Parker (2005) even if in their case there is no aggregate risk⁹.

We would obtain the same kind of results if we considered a weighted average of the objectively expected utility and the subjectively expected utility as in the original model of Brunnermeier-Parker (2005)

$$\max_{\mu_i \in K} \left\{ \beta E \left[u_i \left(\frac{1}{2} p + \alpha_i(p, \mu_i) (\tilde{x} - p) \right) \right] + (1 - \beta) E_i \left[u_i \left(\frac{1}{2} p + \alpha_i(p, \mu_i) (\tilde{x} - p) \right) \right] \right\}.$$

For β large enough, in other words when the weight on the objective expectation is beyond a given threshold, then the agents share the same belief and this belief is optimistic. Otherwise, there is not a unique optimal belief, agents have extreme beliefs (i.e. a or b), but the possible equilibria still lead to an optimistic average belief¹⁰. In all cases, the average optimal belief is optimistic. These results are similar to those obtained by Gollier (2005) in a general discrete distributions setting.

To conclude, in the optimal (non-strategic) setting, agents' beliefs are always optimistic (with respect to their own allocations). Furthermore, except for specific degenerate situations (see Condition 6), the agents share the same belief. The difference between *optimal* (non strategic) and strategic beliefs is now clear, since in the latter setting, there is belief heterogeneity, one agent is optimistic while the other is pessimistic.

⁹In this case, there is no absolute concept of optimism or pessimism and both agents are optimistic with respect to their own equilibrium allocation.

¹⁰More precisely, for $\beta < \frac{1}{2}$, the agents have extreme beliefs, a or b , as above and there might exist equilibria with heterogeneous optimal beliefs if the model parameters satisfy a condition like condition (6). For $\beta > \frac{1}{2}$, i.e. when there is more weight on the objective expectation, and if b is sufficiently large ($b > b^*$ for some b^*) the agents share the same belief and this belief is an interior point of $[a, b]$. For $\beta = \frac{1}{2}$ or for $b \leq b^*$ the agents share the same belief b .

2.3 More general utility functions and distributions

The purpose of this section is to analyze the robustness of the results of Section 2.1 to more general utility functions and distributions. We consider a family of beliefs $(P_{\tilde{x}}^\mu)_{\mu \in K}$, corresponding to the possible subjective distributions for $\tilde{x}$, where $K \subset \mathbb{R}_+$ is a given set of admissible beliefs (including the objective one). For $\mu \in K$, we let $f(\cdot, \mu)$ denote the density function of $P_{\tilde{x}}^\mu$ with respect to the Lebesgue measure on $\mathbb{R}_+$.

As in Section 2.1, our economy is composed of two agents, initially endowed with a half unit of the risky asset $\tilde{x}$, who can manipulate their beliefs and choose an optimal composition of their portfolio, taking into account the effect their trading has on price.

We make the following assumptions.

Assumption (A)

- The utility functions u_1 and u_2 are increasing, strictly concave and twice continuously differentiable on $\mathbb{R}_+$,
- Inada conditions: $u'_i(0) = +\infty$ and $\lim_{x \rightarrow \infty} u'_i(x) = 0$,
- The family $(P_{\tilde{x}}^\mu)_{\mu \in K}$ is increasing in the sense of the first-order stochastic dominance, i.e. for all $x > 0$ we have $\int_0^x f(s, \mu) ds \geq \int_0^x f(s, \mu') ds$ for $\mu' \geq \mu$ in K .
- The functions $s \mapsto su'_i(s)$, $i = 1, 2$, are increasing.

The first condition is standard. The second one guarantees interior solutions to the individual portfolio choice problem. The third condition ensures an order on the set of admissible beliefs. The setting of Section 2.1 satisfies this condition. More generally, any family of beliefs $(P_{\tilde{x}}^\mu)_{\mu \in \mathbb{R}_+}$ such that $f(s, \mu) = g(s - \mu)$ for a given distribution function g on $\mathbb{R}_+$ satisfies this monotonicity condition. Another example is provided by a family of log-normal distributions, $(\ln \mathcal{N}(\mu, \sigma^2))_{\mu \in \mathbb{R}}$. The fourth condition guarantees that a first-order stochastic dominance shift in the risky asset's payoffs increases the demand for the risky asset (see Gollier, 2001). The same portfolio property can be obtained without this condition if we replace the first-order stochastic dominance of the third condition by the monotone likelihood ratio order (Landsberger and Meilijson, 1990).

In the next, we also assume that all the considered expectations exist and are finite. We may assume without any loss of generality that the objective distribution corresponds to $\mu = 0$ and we will simply denote by E (instead of E^0) the associated expectation operator¹¹.

As in the previous section the agents choose their beliefs which means here that they choose a given μ and its associated subjective distribution P_X^μ for $\tilde{x}$. For a given belief $P_{\tilde{x}}^\mu$ and for a given price p , the demand function of agent i is given by

$$\alpha_i(p, \mu) = \arg \max_{\alpha_i} E^\mu \left[u_i \left(\frac{1}{2}p + \alpha_i(\tilde{x} - p) \right) \right].$$

For a pair of beliefs (μ_1, μ_2) , the equilibrium price $p(\mu_1, \mu_2)$ is determined by the market-clearing condition $\alpha_1(p(\mu_1, \mu_2), \mu_1) + \alpha_2(p(\mu_1, \mu_2), \mu_2) = 1$ and the associated optimal demand for agent i is defined by $\alpha_i^*(\mu_1, \mu_2) = \alpha_i(p(\mu_1, \mu_2), \mu_i)$. Finally, the optimal belief $\hat{\mu}_i$ of agent i is determined, as in Section 2.1, by the following optimization program

¹¹We let E^μ denote the expectation operator under the density $f(\cdot, \mu)$, i.e. $E^\mu[g(\tilde{x})] = \int g(s)f(s, \mu)dx$.

$$\arg \max_{\mu_i} E \left[u_i \left(\frac{1}{2} p(\mu_1, \mu_2) + \alpha_i^*(\mu_1, \mu_2) (\tilde{x} - p(\mu_1, \mu_2)) \right) \right]. \quad (7)$$

If this problem admits an interior solution $\hat{\mu}_i$ in K , this solution satisfies the following first order condition

$$E \left[\left(\frac{d\alpha_i^*}{d\mu_i} (\tilde{x} - p) + \left(\frac{1}{2} - \alpha_i^* \right) \frac{dp}{d\mu_i} \right) u_i' \left(\frac{1}{2} p + \alpha_i^* (\tilde{x} - p) \right) \right] = 0. \quad (8)$$

Proposition 6 *Under Assumption (A), the functions $\alpha_i(p, \mu)$, $p(\mu_1, \mu_2)$ and $\alpha_i^*(\mu_1, \mu_2)$ are well defined and satisfy $\frac{\partial \alpha_i}{\partial p}(p, \mu) \leq 0$, $\frac{\partial \alpha_i}{\partial \mu}(p, \mu) \geq 0$, $\frac{\partial p}{\partial \mu_i} \geq 0$, $\frac{\partial \alpha_i^*}{\partial \mu_i} \geq 0$, $\frac{\partial \alpha_i^*}{\partial \mu_j} \leq 0$, $i = 1, 2$, $j \neq i$.*

If the optimization program (7) admits an interior solution, then one of the agents (agent i) is pessimistic and the other one (agent j) is optimistic and we have $\alpha_j^(\hat{\mu}_1, \hat{\mu}_2) \leq \frac{1}{2} \leq \alpha_i^*(\hat{\mu}_1, \hat{\mu}_2)$.*

If one of the agents (say agent 1) is more risk-averse than the other one in the sense of Arrow-Pratt, then $\alpha_1^(\hat{\mu}_1, \hat{\mu}_2) \leq \frac{1}{2}$, hence there is a positive correlation between pessimism and risk-tolerance.*

We obtain first that the optimal demand of the agents (as a function of the price and the belief) increases with the belief and decreases with the price, which are natural properties. As a consequence, the equilibrium price increases with the beliefs, which is also natural; if the asset is more “desirable”, its equilibrium price increases. An increase in the belief of agent i has then two effects on his demand α_i^* at the equilibrium, a direct positive effect and an indirect negative effect due to the price increase. The global effect is positive. The effect of an increase of the belief of agent i on the equilibrium demand α_j^* of the other agent is negative because there is only one effect, namely the price effect.

We obtain that the heterogeneity of optimal beliefs is robust to the choice of more general utility functions and distributions. Moreover, as in Section 2.1, one agent is optimistic and the other agent is pessimistic. The pessimistic agent is the one for which the net demand is positive. This result can be explained as before. For the agent who expresses a positive net demand for the risky asset, the choice of a pessimistic belief is associated to a lower price and a higher expected utility; the optimal belief balances this benefit of pessimism against the costs of worse decision making. The converse reasoning applies to the other agent, who, at the equilibrium, has a negative net demand in the risky asset and benefits from optimism.

The positive correlation between pessimism and risk-tolerance is also robust to this more general setting. When one of the agents is more risk-tolerant, his net demand is necessarily positive. Otherwise, he would have a negative demand which would lead to an optimistic belief while the other agent would be pessimistic, more risk-averse with a positive net demand. This is obviously impossible. The positive correlation follows.

3 Extensions of the model

In this section, we analyze the robustness of our results (heterogeneity of optimal beliefs and positive correlation between pessimism and risk-tolerance) to other specifications of the model. In previous sections we analyzed the impact of strategic behavior on beliefs when the set of possible beliefs is ordered for first-order stochastic dominance shifts. Such

shifts correspond to changes on the mean for normal distributions and more generally can be interpreted in terms of optimism and pessimism. We can also be interested by the impact of strategic behavior on beliefs when the set of possible beliefs is ordered for mean preserving spreads. Such spreads correspond to changes on the variance for normal distributions and more generally can be interpreted in terms of overconfidence and doubt/underconfidence. For tractability reasons, we will analyze this impact in an exponential utility and normal distributions framework (as in Section 2.1). We will also analyze, in an analogous framework, if our results are robust to the introduction of multiple sources of risk.

3.1 Nash equilibrium in beliefs on the variance

In this section we consider a Nash equilibrium in beliefs where the agents differ by their estimation of the variance of the risky asset. The model is the same as in Section 2.1 except that the strategic variable is now the variance of $\tilde{x}$. The payoff of the risky asset $\tilde{x}$ is still normally distributed with mean μ and variance σ^2 .

In the competitive Walrasian equilibrium model, both agents agree on the true value of the variance of the payoff of the risky asset; as we have seen in Section 2, the optimal demand of agent i , given the equilibrium price p , is given by $\alpha_i(p) = \theta_i \frac{\mu - p}{\sigma_i^2}$ and the market clearing condition $\alpha_1(p) + \alpha_2(p) = 1$ imposes that the equilibrium price of the risky asset is given by $p = \mu - \frac{\sigma^2}{\theta_1 + \theta_2}$.

If agents have exogeneously given heterogeneous expectations about the risky asset (i.e., agent 1 believes that $\tilde{x}$ has variance σ_1^2 and agent 2 believes that $\tilde{x}$ has variance σ_2^2 with $\sigma_1 \neq \sigma_2$), then the optimal demand of agent i is given by $\alpha_i(p, \sigma_i) = \theta_i \frac{\mu - p}{\sigma_i^2}$ and the market clearing price is given by $p(\sigma_1, \sigma_2) = \mu - \left(\frac{\theta_1}{\sigma_1^2} + \frac{\theta_2}{\sigma_2^2} \right)^{-1}$. The bias with respect to the objective belief can here be interpreted as a form of doubt ($\sigma_i^2 > \sigma^2$) or overconfidence ($\sigma_i^2 < \sigma^2$) instead of the pessimism/optimism biases¹² of Section 2. Note that the obtained equilibrium price corresponds to the equilibrium price in an economy in which agents share the same belief, namely the harmonic average of the initial beliefs, weighted by the risk-tolerance. In other words, it is the equilibrium price in an economy in which the belief of the representative agent (whose risk-tolerance is given by $\bar{\theta}$, as in the standard setting) is given by the average of the initial beliefs, weighted by the risk-tolerance¹³.

We assume that the strategic variable for the agents consists in the variance of $\tilde{x}$. The choice of a belief $\hat{\sigma}_i$ is then determined by

$$\hat{\sigma}_i = \arg \max_{\sigma_i} E \left[u_i \left\{ \frac{1}{2} p(\sigma_i, \hat{\sigma}_j) + \{ \tilde{x} - p(\sigma_i, \hat{\sigma}_j) \} \alpha_i^*(\sigma_i, \hat{\sigma}_j) \right\} \right].$$

where $\alpha_i^*(\sigma_i, \hat{\sigma}_j) = \alpha_i(p(\sigma_i, \hat{\sigma}_j), \sigma_i)$ and where $\hat{\sigma}_j$, for $j \neq i$, is considered as given.

Definition 7 *A Nash equilibrium in beliefs on the variance is defined as a pair of variance strategies $M = (\hat{\sigma}_1, \hat{\sigma}_2)$ such that for any other pair of strategies M' differing only in the*

¹²See Abel (2002) for concepts of pessimism and doubt related to first and second order stochastic dominance.

¹³Walrasian equilibrium models in which agents have heterogeneous beliefs on the variance of the asset under consideration have been studied by, among others, Abel (1989, 2002) and Jouini-Napp (2006).

i -th component, for $i = 1, 2$, the strategy M yields a utility level no less than M' :

$$E \left[u_i \left\{ \frac{1}{2} p(M) + \{\tilde{x} - p(M)\} \alpha_i^*(M) \right\} \right] \geq E \left[u_i \left\{ \frac{1}{2} p(M') + \{\tilde{x} - p(M')\} \alpha_i^*(M') \right\} \right].$$

Proposition 8 *There exists a unique Nash equilibrium in beliefs on the variance. It satisfies the following properties.*

1. The optimal beliefs $M = (\hat{\sigma}_1, \hat{\sigma}_2)$ are given by

$$\hat{\sigma}_1^2 = \sigma^2 \left(1 + \frac{\theta_1 - \theta_2}{4\theta_2} \right), \quad \hat{\sigma}_2^2 = \sigma^2 \left(1 + \frac{\theta_2 - \theta_1}{4\theta_1} \right). \quad (9)$$

The more risk-tolerant agent exhibits doubt, in the sense that he overestimates the variance of $\tilde{x}$, and the less risk-tolerant agent is overconfident. Moreover, the more risk-tolerant agent exhibits more doubt than the less risk-tolerant agent exhibits overconfidence, and the unweighted (harmonic) average of the beliefs exhibits doubt:

$$2 \left(\frac{1}{\hat{\sigma}_1^2} + \frac{1}{\hat{\sigma}_2^2} \right)^{-1} = \sigma^2 \left(1 + \frac{(\theta_1 - \theta_2)^2}{2(\theta_1^2 + \theta_2^2 + 6\theta_1\theta_2)} \right). \quad (10)$$

2. The representative agent exhibits doubt, i.e. the harmonic average of the individual beliefs weighted by the risk-tolerance exhibits doubt. More precisely,

$$(\theta_1 + \theta_2) \left(\frac{\theta_1}{\hat{\sigma}_1^2} + \frac{\theta_2}{\hat{\sigma}_2^2} \right)^{-1} = \sigma^2 \left(1 + \frac{3(\theta_1 - \theta_2)^2}{16\theta_1\theta_2} \right).$$

The Nash equilibrium in beliefs on the variance has then the same properties as the Nash equilibrium in beliefs on the mean except that pessimism is replaced by doubt.

3.2 The model with two sources of risk

The model is essentially the same as in Section 2 except that we now suppose that there are two sources of risk in the economy, whose associated payoffs at the end of the period are respectively denoted by $\tilde{x}$ and $\tilde{y}$. We let p (resp. q) denote the price of $\tilde{x}$ (resp. $\tilde{y}$) and we assume that $\tilde{x}$ and $\tilde{y}$ are normally distributed, more precisely $\tilde{x} \sim \mathcal{N}(\mu, \sigma^2)$ and $\tilde{y} \sim \mathcal{N}(\nu, \varpi^2)$. We let ρ denote the correlation between $\tilde{x}$ and $\tilde{y}$, i.e., $\rho \equiv \frac{\text{cov}(\tilde{x}, \tilde{y})}{\sigma\varpi}$. Each agent is initially endowed with one half unit of each risky asset.

We assume that agents' strategic set is the set of pairs (μ_i, ν_i) and as previously we look for a Nash equilibrium in beliefs (on the means). The definition and the notations are straightforward generalizations of those introduced in Section 2.

Proposition 9 *There exists a unique Nash equilibrium in beliefs. The optimal beliefs $M = ((\hat{\mu}_i, \hat{\nu}_i); i = 1, 2)$ are given by*

$$\hat{\mu}_i = \mu - \frac{(\theta_i - \theta_j)(\sigma^2 + \sigma\varpi\rho)}{4\theta_j\bar{\theta}}, \quad \hat{\nu}_i = \nu - \frac{(\theta_i - \theta_j)(\varpi^2 + \sigma\varpi\rho)}{4\theta_j\bar{\theta}}.$$

As far as the whole market portfolio $\tilde{x} + \tilde{y}$ is concerned, the beliefs ξ_i^M on the average market return are given by

$$\xi_i^M = \xi - \frac{(\theta_i - \theta_j) \sigma_M^2}{4\theta_j \bar{\theta}}$$

where $\xi = \mu + \nu$ and $\sigma_M^2 = \varpi^2 + 2\rho\sigma\varpi + \sigma^2$ correspond respectively to the objective market portfolio return and variance. These formulas are exactly the same as in the one-asset framework which means that the more risk-tolerant (risk-averse) agent is pessimistic (resp. optimistic) at the aggregate level and the consensus belief is pessimistic at the aggregate level. The formulas for individual assets that are provided in the proposition are similar to those obtained in the one asset framework. However, for each asset, the variance term in the one-asset formula is replaced by the covariance of the considered asset payoffs with the market portfolio payoffs. Recall that in the Walrasian setting (CAPM setting), the equilibrium price for a given asset depends on the covariance of the payoffs of this asset with the payoffs of the market portfolio and not on the total variance of the asset payoffs. Since beliefs' choice is governed by beliefs' impact on prices, it is natural to obtain optimal beliefs that depend on the covariance with the market portfolio and not on the total variance. The aggregate level properties (pessimism, correlation between pessimism and risk-tolerance,...) are then retrieved at the individual assets level as far as these assets are positively correlated with the market portfolio.

It is interesting to note that these effects are more pronounced for the riskier asset. Intuitively, the strategic behavior leads to more beliefs dispersion for the riskier asset and hence to a more pronounced impact on the market for the riskier asset .

4 Conclusion

The introduction of strategic interaction provides a rationale for belief heterogeneity; it leads to optimal beliefs that are subjective, heterogeneous and antagonistic. The selection of optimal strategic beliefs is governed by very precise rules. These beliefs must be related to the individual level of risk-aversion: the beliefs of more risk-averse agents exhibit optimism and/or overconfidence and the beliefs of more risk-tolerant agents exhibit pessimism and/or doubt. As a consequence, there is a positive correlation between pessimism/doubt and risk-tolerance. In a setting with exponential utility and normal distributions, the average belief exhibits pessimism and/or doubt as well as the consensus belief. This is compatible with the observation that subjects in experimental and empirical studies exhibit a dose of pessimism (Wakker, 2001, Ben Mansour et al., 2006, Giordani-Söderlind, 2005). This induced pessimism/doubt of investors might be helpful to solve the equity premium puzzle. It is also helpful to explain the purchase of vastly overpriced insurance contracts or the large short run returns of IPOs.

This work suggests further investigation in several directions. First, in this paper we have let aside information asymmetry and heterogeneity in order to focus on the impact of strategic interactions on individual beliefs and from there on equilibrium prices and allocations. It would be useful to consider a more general model including both a strategic use of private information and a strategic choice of beliefs. Second, it would be also useful to analyze how our results can be transposed to a dynamic setting. Third, we have only considered totally ordered families of possible subjective distributions for the risky asset payoffs. In particular, all beliefs deformations can be interpreted in terms of pessimism/optimism or in terms of doubt/overconfidence. It would be interesting to

consider more general possible deformations of the objective distribution in particular in terms of higher order moments (as skewness and kurtosis). Finally, it would be interesting to analyze a model with a large number of players. Since beliefs are built strategically, we may argue that the impact of the strategic behavior will vanish when the number of players becomes large. However, “the real issue is not so much how many (agents) there are, but to what extent (agents) cluster in their beliefs” [Shefrin, 2005, p216]. Situations with a small number of clusters would correspond to situations with a small number of leaders (highly concentrated decision power, presence of gurus...). This should then lead to extend the model in order to take into account leadership, beliefs contagion and herding behavior.

Appendix

Proof of Proposition 2

1. For chosen beliefs $(\mu_i)_{i=1,2}$, we have seen that the optimal demand for agent i is given by $\alpha_i(p, \mu_i) = \theta_i \frac{\mu_i - p}{\sigma^2}$, and the market clearing price is then given by $p(\mu_1, \mu_2) = \left(\frac{\theta_1}{\theta_1 + \theta_2}\right) \mu_1 + \left(\frac{\theta_2}{\theta_1 + \theta_2}\right) \mu_2 - \frac{\sigma^2}{\theta_1 + \theta_2}$. The resulting expected utility from trade for agent i , given the belief μ_j of agent j , $j \neq i$, can be written

$$\begin{aligned} U_i(\mu_i) &= E \left[-\exp \left(-\frac{\frac{1}{2}p(\mu_1, \mu_2) + \alpha_i(\mu_1, \mu_2)(\tilde{x} - p(\mu_1, \mu_2))}{\theta_i} \right) \right] \\ &= -\exp \left[-\left(\frac{\frac{1}{2}p(\mu_1, \mu_2) + \alpha_i(\mu_1, \mu_2)(\mu - p(\mu_1, \mu_2))}{\theta_i} + \frac{1}{2} \left(\frac{\alpha_i(\mu_1, \mu_2)}{\theta_i} \right)^2 \sigma^2 \right) \right] \end{aligned}$$

where $\alpha_i(\mu_1, \mu_2) = \theta_i \frac{\mu_i - p(\mu_1, \mu_2)}{\sigma^2}$.

Maximizing this quantity amounts to maximizing

$$A_i(\mu_i) = \frac{1}{2}p(\mu_1, \mu_2) + \alpha_i(\mu_1, \mu_2)(\mu - p(\mu_1, \mu_2)) - \frac{1}{2} \frac{1}{\theta_i} (\alpha_i(\mu_1, \mu_2))^2 \sigma^2.$$

This program is concave and the maximum is reached for μ_i such that $\frac{dA_i}{d\mu_i}(\mu_i) = 0$. This leads to

$$\mu_i = \frac{2\mu\theta_j(\theta_1 + \theta_2) + 2\theta_i\theta_j\mu_j + \sigma^2(\theta_j - \theta_i)}{4\theta_j\theta_i + 2\theta_i^2}. \quad (11)$$

We solve then for $(\hat{\mu}_1, \hat{\mu}_2)$ and obtain Equations (3).

2. Straightforward using Equations (3). ■

Proof of Proposition 5

The utility level of agent i is given by $E_i[u_i(\frac{1}{2}p + \alpha_i(p, \mu_i)(\tilde{x} - p))]$ with $\alpha_i(p, \mu_i) = \theta_i \frac{\mu_i - p}{\sigma^2}$. Then, for a given p , the agent maximizes $\theta_i \frac{(\mu_i - p)^2}{\sigma^2}$.

When $p > \frac{a+b}{2}$, all the agents have the same belief a and the equilibrium price, if it exists, must satisfy $\bar{p} = a - \frac{\sigma^2}{\theta_1 + \theta_2}$ which is not compatible with the condition $p > \frac{a+b}{2}$. When $p < \frac{a+b}{2}$, all the agents have the same belief b and the equilibrium price, if it exists, must satisfy $\bar{p} = b - \frac{\sigma^2}{\theta_1 + \theta_2}$ which is compatible with the condition $p < \frac{a+b}{2}$ only if $\frac{\sigma^2}{\theta_1 + \theta_2} > \frac{b-a}{2}$. When $p = \frac{a+b}{2}$, both agents may choose the same belief b leading to an equilibrium only if $\frac{\sigma^2}{\theta_1 + \theta_2} = \frac{b-a}{2}$. They may also choose different beliefs. If agent 1 (resp. 2) chooses a (resp. b), the market clearing condition leads to

$$\frac{\theta_1 a + \theta_2 b}{\theta_1 + \theta_2} - \frac{\sigma^2}{\theta_1 + \theta_2} = \frac{a + b}{2}. \quad (12)$$

If $\mu = \frac{a+b}{2}$ and $\theta_1 < \theta_2$, then $\frac{\theta_1 a + \theta_2 b}{\theta_1 + \theta_2} > \mu$. ■

Proof of Proposition 6

It is well known that due to Inada conditions, the demand function is characterized by the following first order condition

$$E^\mu \left[(\tilde{x} - p) u'_i(\alpha_i(p, \mu)(\tilde{x} - p) + \frac{1}{2}p) \right] = 0$$

and the partial derivatives of $\alpha_i(p, \mu)$ with respect to p and μ are given by

$$\begin{aligned} \frac{\partial \alpha_i}{\partial p}(p, \mu) &= - \frac{E^\mu \left[\left(\frac{1}{2} - \alpha_i(p, \mu) \right) (\tilde{x} - p) u''_i(c(p, \mu)) - u'_i(c(p, \mu)) \right]}{E^\mu \left[(\tilde{x} - p)^2 u'_i(c(p, \mu)) \right]} \\ \frac{\partial \alpha_i}{\partial \mu}(p, \mu) &= - \frac{\frac{\partial}{\partial \mu} E^\mu \left[(\tilde{x} - p) u'_i(c(p, \mu)) \right] \Big|_{(\mu, p, \alpha_i(p, \mu))}}{E^\mu \left[(\tilde{x} - p)^2 u'_i(c(p, \mu)) \right]}. \end{aligned}$$

with $c(p, \mu) = \alpha_i(p, \mu)(\tilde{x} - p) + \frac{1}{2}p$. Letting $\tilde{c}$ denote $c(p, \mu)$, remark that

$$E^\mu \left[\left(\frac{1}{2} - \alpha \right) (\tilde{x} - p) u''_i(\tilde{c}) - u'_i(\tilde{c}) \right] = E^\mu \left[-u'_i(\tilde{c}) - \tilde{c} u''_i(\tilde{c}) + \frac{1}{2} \tilde{x} u''_i(\tilde{c}) \right].$$

Hence, $\frac{\partial \alpha_i}{\partial p}(p, \mu)$ is negative. Furthermore, $(\tilde{x} - p) u'_i(\tilde{c}) = \frac{1}{\alpha} \tilde{c} u'(\tilde{c}) - \frac{1}{2} \frac{p}{\alpha} u'(\tilde{c})$ and is then increasing. By the first-stochastic dominance property, we have $\frac{\partial \alpha_i}{\partial \mu}(p, \mu) \geq 0$.

We have then

$$\frac{\partial p}{\partial \mu_i} = - \frac{\frac{\partial \alpha_i}{\partial \mu_i}(p, \mu_i)}{\frac{\partial \alpha_1}{\partial p}(p, \mu_1) + \frac{\partial \alpha_2}{\partial p}(p, \mu_2)}$$

hence $\frac{\partial p}{\partial \mu_i} \geq 0$, $i = 1, 2$.

For $i \neq j$, we have $\frac{\partial \alpha_i^*(\mu_1, \mu_2)}{\partial \mu_i} = \frac{\partial \alpha_i(p, \mu_i)}{\partial \mu_i} + \frac{\partial \alpha_i(p, \mu_i)}{\partial p} \frac{\partial p}{\partial \mu_i} = \frac{\frac{\partial \alpha_i}{\partial \mu_i}(p, \mu_i) \frac{\partial \alpha_j}{\partial p}(p, \mu_j)}{\frac{\partial \alpha_1}{\partial p}(p, \mu_1) + \frac{\partial \alpha_2}{\partial p}(p, \mu_2)} \geq 0$.

If Problem (7) admits an interior solution, the first-order condition for agent i gives

$$E \left[\left(\frac{\partial \alpha_i^*}{\partial \mu_i} (\tilde{x} - p) + \left(\frac{1}{2} - \alpha_i^* \right) \frac{\partial p}{\partial \mu_i} \right) u'_i(\alpha_i^*(\tilde{x} - p) + \frac{1}{2}p) \right] = 0.$$

If $\frac{1}{2} - \alpha_i^*(\hat{\mu}_1, \hat{\mu}_2) \leq 0$ then $\left(\frac{1}{2} - \alpha_i^*(\hat{\mu}_1, \hat{\mu}_2) \right) \frac{\partial p}{\partial \mu_i} \leq 0$, hence $E[(\tilde{x} - p(\hat{\mu}_1, \hat{\mu}_2)) u'_i] \geq 0$. As previously, by the first-order stochastic dominance property we obtain $\hat{\mu}_i \leq 0$. Analogously $\frac{1}{2} - \alpha_i^*(\hat{\mu}_1, \hat{\mu}_2) \geq 0$ leads to $\hat{\mu}_i \geq 0$. We have then proved that the agent for which $\alpha_i^*(\hat{\mu}_1, \hat{\mu}_2) \geq \frac{1}{2}$ (resp. $\alpha_i^*(\hat{\mu}_1, \hat{\mu}_2) \leq \frac{1}{2}$) is pessimistic (resp. optimistic).

If one of the utility functions (let us say u_1) is more risk-averse than the other one in the sense of Arrow-Pratt, let us prove that $\alpha_1^*(\hat{\mu}_1, \hat{\mu}_2) \leq \frac{1}{2}$. If this is not the case, we have $\alpha_2^*(\hat{\mu}_1, \hat{\mu}_2) \leq \frac{1}{2}$ and agent 2 is optimistic while agent 1 is pessimistic. We have then

$$\frac{1}{2} < \alpha_1(p(\hat{\mu}_1, \hat{\mu}_2), \hat{\mu}_1) \leq \alpha_2(p(\hat{\mu}_1, \hat{\mu}_2), \hat{\mu}_1)$$

because agent 1 is more risk-averse. Furthermore we have $\alpha_2(p(\hat{\mu}_1, \hat{\mu}_2), \hat{\mu}_1) \leq \alpha_2(p(\hat{\mu}_1, \hat{\mu}_2), \hat{\mu}_2)$ because $\hat{\mu}_2$ is larger than $\hat{\mu}_1$. We would have then $\alpha_2^*(\hat{\mu}_1, \hat{\mu}_2) > \frac{1}{2}$ which contradicts our assumption. ■

Proof of Proposition 8

1. Agent i maximizes

$$A_i(\sigma_1, \sigma_2) = \frac{1}{2}p + \alpha_i(\mu - p) - \frac{1}{2} \frac{1}{\theta_i} (\alpha_i)^2 \sigma^2$$

with respect to σ_i where p and α_i both depend on σ_i and are given by $p = \mu - \left(\frac{\theta_1}{\sigma_1^2} + \frac{\theta_2}{\sigma_2^2}\right)^{-1}$ and $\alpha_i = \theta_i \frac{\mu - p}{\sigma_i^2}$.

This problem is concave in $\left(\frac{\theta_1}{\sigma_1^2} + \frac{\theta_2}{\sigma_2^2}\right)^{-1}$. Setting $\frac{dA_i}{d\sigma_i}(\hat{\sigma}_1, \hat{\sigma}_2) = 0$ leads to

$$\hat{\sigma}_i^2 = \frac{2}{3}\sigma^2 + \frac{1}{3} \left(\frac{\theta_i}{\theta_j} \hat{\sigma}_j^2 \right). \quad (13)$$

The optimal beliefs are then given by Equations (9). Equation (10) follows.

Since

$$\left| \left(\frac{\theta_1 - \theta_2}{4\theta_2} \right) \left(\frac{4\theta_1}{\theta_2 - \theta_1} \right) \right| = \frac{\theta_1}{\theta_2}$$

the more risk-tolerant agent exhibits more doubt than the less risk-tolerant agent exhibits overfidence.

2. Straightforward using Equations (9). ■

Proof of Proposition 9

1. As in the previous proofs, direct computations lead to the following equilibrium prices and quantities in a Walrasian setting when agents are endowed with beliefs (μ_i, ν_i) :

$$\begin{cases} p = \sum_{i=1}^2 \frac{\theta_i}{\bar{\theta}} \mu_i - \frac{\sigma^2 + \sigma\varpi\rho}{\bar{\theta}}, & q = \sum_{i=1}^2 \frac{\theta_i}{\bar{\theta}} \nu_i - \frac{\varpi^2 + \sigma\varpi\rho}{\bar{\theta}} \\ \alpha_1 = \theta_1 \frac{\mu_1 - p}{\sigma^2(1-\rho^2)} - \theta_1 \frac{(\nu_1 - q)\rho}{\sigma\varpi(1-\rho^2)} = \frac{\theta_1}{\bar{\theta}} \left[1 + \frac{\theta_2(\mu_1 - \mu_2)}{\sigma^2(1-\rho^2)} - \frac{\theta_2(\nu_1 - \nu_2)}{\sigma\varpi(1-\rho^2)} \right] \\ \beta_1 = \theta_1 \frac{\nu_1 - q}{\varpi^2(1-\rho^2)} - \theta_1 \frac{(\mu_1 - p)\rho}{\sigma\varpi(1-\rho^2)} = \frac{\theta_1}{\bar{\theta}} \left[1 + \frac{\theta_2(\nu_1 - \nu_2)}{\varpi^2(1-\rho^2)} - \frac{\theta_2(\mu_1 - \mu_2)}{\sigma\varpi(1-\rho^2)} \right] \end{cases}. \quad (14)$$

In the setting of the proposition, agent i maximizes

$$A_i(\mu_1, \nu_1, \mu_2, \nu_2) = \frac{1}{2}p + \alpha_i(\mu - p) + \frac{1}{2}q + \beta_i(\nu - q) - \frac{1}{2} \frac{1}{\theta_i} (\alpha_i^2 \sigma^2 + \beta_i^2 \varpi^2 + 2\alpha_i \beta_i \sigma\varpi\rho)$$

with respect to (μ_i, ν_i) taking $(\mu_j, \nu_j) = (\hat{\mu}_j, \hat{\nu}_j)$, $j \neq i$, as given. The maximization programs under consideration are concave. Setting $\frac{dA_i}{d\mu_i} = \frac{dA_i}{d\nu_i} = 0$ leads to

$$\hat{\mu}_i = \mu - \frac{(\theta_i - \theta_j)(\sigma^2 + \sigma\varpi\rho)}{4\theta_j\bar{\theta}}, \quad \hat{\nu}_i = \nu - \frac{(\theta_i - \theta_j)(\varpi^2 + \sigma\varpi\rho)}{4\theta_j\bar{\theta}}$$

which is the unique solution of the Nash equilibrium in beliefs. ■

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