DIVERGENCE, WAGE-GAP AND GEOGRAPHY

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ABSTRACT. We develop a geographic growth model where nominal wages are allowed to diverge between the two considered countries. Removing the standard assumption entailing that both countries always own a traditional sector, we argue that, as trade gets freer, the traditional sector of one country might cease to exist so that wages increase: it gives rise to an additional dispersion force independent of trade costs. Hence, the core-periphery outcome might never be reached, which contradicts previous literature's results. We also question a hallmark of the literature since we argue that full agglomeration of firms might actually lead to slower growth for both countries.

JEL Classification: F15; O41; R11.

Keywords: Wage Differential; New Economic Geography;
Endogenous Growth; Knowledge Spillovers.

RÉSUMÉ. Cet article présente un modèle géographique de croissance à deux pays dans lequel les salaires nominaux des pays peuvent diverger. Ecartant l'hypothèse classique selon laquelle chaque pays a toujours un secteur traditionnel, cette étude montre qu'avec la libéralisation du commerce, un pays peut voir disparaître son secteur traditionnel, ce qui induit une hausse des salaires: ceci crée une force de dispersion supplémentaire qui n'est pas liée aux coûts du commerce. De ce fait, l'équilibre centre-périphérie peut n'être jamais atteint ce qui contredit les résultats présentés jusqu'ici dans la littérature. Nous remettons aussi en question un des points importants de la littérature en démontrant que l'agglomération complète des firmes peut entraîner un ralentissement de la croissance pour les deux pays.

Classification JEL: F15, O41, R11.

Mots-clefs: Différentiels de salaires; nouvelle économie géographique;
croissance endogène; *spillovers* de connaissances.

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■ INTRODUCTION

A striking feature of the developed world is that economic activity is regrouped in clusters which experience high economic growth to the detriment of peripheral regions. An all-important question is whether these clusters tend to grow over time, or, on the contrary, whether there are forces that weaken this process. These questions are crucial in the European Union because it already experienced large differences amongst its 15 members (hereafter referred to as "the EU15").² Moreover, on May 1, 2004, ten new countries from Central and Eastern Europe have joined the EU. These countries are much poorer than the EU15: indeed, average GDP per head in the ten new members is only 46% of the EU15 average. Our point is that these members are very different from the EU15 on many aspects. One such crucial difference concerns nominal wages. Indeed, a clear incentive for Western firms to move their production eastwards is cheaper labor costs in Central Europe. Nowadays, the threat of delocalizing to low-wage countries is so omnipresent, that one alleged reason (among many, admittedly) for which French voters rejected the European Constitution was precisely the fear that it would have paved the way for more delocalizations towards the likes of Slovakia or Slovenia. It must be borne in mind however that, assessing the impact of delocalizations on the GDP per head of the sending and the receiving country is already a difficult task but *what about the long-term impact on the growth rate of those countries?* This topic has not received as much attention as it should have.

Hence, accounting for the potential discrepancies in labor costs alluded above, this paper addresses, from a theoretical perspective, the critical question of what impact a greater integration will have on: (i) the location of production, (ii) the wage differential and (iii), the growth rates of Western and Eastern European countries.

We build our work upon the New Economic Geography literature,³ and, in particular, on two papers: Puga (1999) and Baldwin *et al.* (2001). Using a framework encompassing both

2. Puga (2002) points out that one fourth of EU citizens live in regions which would be eligible to receive some assistance under "Objective 1" of the Structural Fund, that is, have a Gross Domestic Product (GDP henceforth) per head below 75% of the EU average. Those same structural funds (which aim at reducing the discrepancies between the EU members) have been allocated almost 200 billion euros for the period 2000-2006, which amounts to more than 30% of total EU spending: the question of income convergence is thus one with far-reaching political implications.

3. Another related strand of literature is the one linking foreign direct investment and growth: papers like Glass and Saggi (2002) and Dinopoulos and Segerstrom (2005) investigate how globalization affects the development of multinational firms. For instance, in the latter (i.e. a Schumpeterian growth model with 2 countries), Northern firms can locate their manufacturing facilities in the low-wage South after incurring some "adaptive R&D costs" (their terminology). Then, these authors study the effects of globalization, which is assimilated to either, (i) an increase in the population size of the South, or (ii), an increase in the efficiency of adaptive R&D (thus encouraging the formation of multinational firms). Nevertheless, the geographical dimension is absent from their paper, so that they can't consider a decrease of trade costs as a possible facet of globalization.

Krugman (1991) and Krugman and Venables (1995), Puga (1999)⁴ shows that reducing trade costs between identical countries initially results in an agglomeration of firms in a single country. Nevertheless, since workers are not internationally mobile, this concentration of firms eventually leads to a divergence of wages, which, for low enough trade costs, can foster de-agglomeration. This paper doesn't take growth into account however.

In a 2-countries setting blending economic geography and endogenous growth, Baldwin *et al.* (2001) show that firms tend to concentrate in a single country when trade costs are low, and that this agglomeration is growth-enhancing for the economy as a whole (growth plays the role of an additional centripetal force). Nevertheless, their model rules out the very plausible scenario where wages diverge between countries, which is slightly puzzling since their aim is to study a divergence process: indeed, when wages are equalized between countries, one gets the impression that the two countries have already converged towards each other. Although this might be acceptable in the static setting of standard New Economic Geography models, in a growth setting such as in their paper, this is unappealing. As it is, they also abstract from important general equilibrium effects since wages remain constant.

In this paper, we account for the possibility that wages can diverge by removing the so-called "*Non-Full Specialization*" (NFS for short) assumption, which is a condition over the parameters that holds at all times in most (if not all) recent NEG models, ensuring that both countries always possess the constant returns sector: hence, coupled with the assumption that the homogeneous good is traded without cost, wages are equalized between countries. The key parameter in the NFS condition is the share of consumer expenditure on industrial goods: when it is high, wages can indeed diverge at some point. In that case, the model encompasses two regimes⁵: (i) the first one is the only one acknowledged by Baldwin *et al.* (2001), i.e. the one with equal-wage trajectories; (ii) the second one occurs when agglomeration in one country is well advanced, entailing that its industrial sector becomes so large that it has drawn all labor away from the traditional sector, which drives the wage rate up. We argue that this rise is a dispersion force militating against agglomeration, thus counterbalancing the lower cost of research. Critically, this force does not depend on the level of trade costs. Hence, it slows agglomeration, yielding a realistic pattern of agglomeration, namely, as long

4. Other papers investigate settings where wages are not equalized between countries. Robert-Nicoud (2006) is the closest to Puga (1999) since he assumes a hidden factor (i.e. land) in the traditional sector so that there are decreasing returns to labor in this sector; wages are thus not necessarily identical between countries. Some other examples are Davis (1998) and Crozet and Trionfetti (2006) where selling the traditional good incurs a transport cost; however, their focus is on the existence and magnitude of the Home Market Effect. Picard and Zeng (2005) also assume positive transport costs in the traditional sector, and they even obtain analytical results thanks to the use of a quasi-linear utility function. However, this setting is unsuitable for the analysis of growth since a balanced growth path cannot be found with a quasi-linear utility function. Finally, two papers are worth mentioning even though they do not explicitly model a wage-gap: Ricci (1999) and Forslid and Wooton (2003) investigate the interplay between standard economic geography forces and Ricardian comparative advantage.

5. In this paper, regime does not have its standard meaning. For instance, it does mean that something is being imposed by the intervention of some external authority. Indeed, in science, regime might also mean a *particular state of affairs* where a particular physical phenomenon or boundary condition is significant.

as the share of industrial spending is high enough, both countries always own some firms. Thus, our results deviate from both Baldwin *et al.* (2001) and Puga (1999) since, contrary to the former, the Core-Periphery outcome is not necessarily reached when trade costs are low, and, contrary to the latter, the symmetric equilibrium does not become stable for very low trade costs. In the New Economic Geography and Growth (NEGG for short) literature,⁶ this is a new result since, to our knowledge, all models show that when trade tends to become costless, firms fully agglomerate in one country. In any case, this result is definitely less stark than those of previous studies where, for low trade costs, either firms are fully agglomerated in one country, or totally dispersed. Moreover, this is important for policy-making since it shows that, when both growth and wage differentials are taken into account, a greater integration will not unavoidably lead to a core-periphery structure, and as such, to more inequality within the EU. Finally, we nuance the existing literature in another respect: previous studies have shown that agglomeration is always beneficial to both countries in terms of steady-state real income growth rate, whereas we argue that the concentration of firms in a single country *might* actually lead to slower growth for both countries.

The rest of the paper unfolds as follows. It lays out the basic skeleton upon which we build. In the third section, we study the instantaneous equilibrium, i.e. the equilibrium at a specific point on the long-run growth path. Then, in the fourth section, we turn to the analysis of the long-run equilibrium: we first explain the behavior of the model in the first regime, i.e. when wages are equal, before turning to the analysis of the second regime. In the fifth section we present our simulations of the full-blown model encompassing the two regimes. Finally, the sixth section presents our concluding comments as well as some insights for future research.

■ THE SETTING

The two countries, North and South (henceforth, a * superscript indicates a Southern variable), are identical in all ways (i.e. same technology, preferences and endowments). Both countries are endowed with two internationally immobile factors that are inelastically supplied, i.e. capital (K) and labor (L). There are three sectors: the traditional sector (T) provides a single homogeneous good that can be freely traded. It is produced under constant returns to scale and perfect competition with a unitary labor requirement. The manufacturing sector (M) produces a differentiated good under Dixit-Stiglitz monopolistic competition, characterized by increasing returns to scale and costly trade (i.e. Iceberg trade costs). The cost function of a typical manufacturing variety is composed of a variable cost involving solely labor, and a fixed cost of a single unit of capital. Finally, in the R&D sector (I), firms solely use labor to design patents (i.e. new units of capital), building on previous discoveries, i.e. there are

6. For a survey of this particular strand of the NEG literature, see for instance Baldwin and Martin (2004). To our knowledge, no new NEGG models has been proposed since these surveys were written.

learning spillovers in the I-sector. Thus, we write for the North (the derivations for the South are isomorphic):

$$\dot{K} = \frac{L_I}{a_I}; \quad a_I \equiv \frac{1}{K^w A}; \quad A \equiv s_K + \lambda(1 - s_K); \quad 0 < \lambda < 1 \quad (1)$$

where L_I is the amount of labor devoted to research, a_I is the productivity parameter, K^w is the total number of units of capital worldwide, $s_K \equiv K/K^w$ is the share of northern capital, and λ is a measure of how localized spillovers are. Indeed, in the spirit of Grossman and Helpman (1991), researchers in the I-sector benefit not only from past discoveries of their own I-sector, but also from the discoveries made in the other country's I-sector.⁷

Since capital is also immobile between countries, firms must be set up in the country where their knowledge capital has been invented. Hence, with (1), we can define the rate of growth of the number of northern varieties as follows:

$$g \equiv \frac{\dot{K}}{K} = \frac{L_I A}{s_K} = \frac{L_I (K + \lambda K^*)}{K} \quad (2)$$

We can already note that for L_I to be constant in steady state requires that both the rate of growth of varieties and the share of capital be constant.

On the demand side, the representative consumer's utility function takes the standard Cobb-Douglas form with a CES sub-utility function for the differentiated good:

$$U = \int_0^\infty e^{-\rho t} \ln C_t dt; \quad C \equiv C_T^{1-\alpha} C_M^\alpha; \quad C_M \equiv \left[\int_{i=0}^{n^w} c_i^{1-\alpha} di \right]^{\frac{\sigma}{\sigma-1}}; \quad 0 < \alpha < 1 < \sigma \quad (3)$$

with C_T and C_M the consumptions of, respectively, the homogeneous traditional good and the composite industrial good, n^w , the total number (mass) of varieties worldwide (i.e. $n^w \equiv n + n^*$), σ , the elasticity of substitution between varieties, and α the share of expenditure that falls on the industrial good. Finally, ρ is the rate of time preference.

The demand for any variety j is given by:⁸

$$c_j = \frac{\alpha s_j E}{p_j} \quad \text{where} \quad s_j = \frac{p_j^{1-\sigma}}{\int_{i=0}^{n+n^*} p_i^{1-\sigma} di} \quad (4)$$

where p_j is the price of a variety j , E is Northern nominal income and s_j can be interpreted as the market share of a firm selling variety j .

7. When $\lambda = 1$, spillovers are *global* in the sense that researchers in one country equally benefit from knowledge accumulated in both countries' I-sector, while for $\lambda = 0$, spillovers are purely *local*.

8. Because of the usual symmetry of varieties in the Dixit-Stiglitz setting, in what follows we omit the subscript related to varieties.

Finally, standard optimal control calculations yield the Euler equation, that is, the time path of nominal expenditure: $\dot{E}/E = r - \rho$ where r is the interest rate, and a dot over a variable represents, as usual, the time derivative of that variable (i.e. $\dot{E} \equiv dE/dt$).

On to the supply-side. First, note that *as long as the two countries trade the T-good*, $p_T = p_T^*$. This is so, if and only if a condition over the parameters, the so-called "Non-Full Specialization" condition holds at all times. In this paper, we shall focus on what happens when it does not hold. We shall discuss this condition further in section 1. Let us now assume that full specialization in the North⁹ is a possible outcome. Hence, the equilibrium conditions of the T-sector are:

$$\begin{cases} p_T = \min(w, w^*) \\ S_T = 0 \quad \text{whenever } w > w^* \end{cases}$$

where $s_T \equiv Y_T/Y_T^W$ is the northern share of T-good output in total production. These conditions obviously yield two different regimes: (i) one where wages are equalized between countries and $s_T \neq 0$, (ii) and the other where northern wages are higher than southern ones and $s_T = 0$. These two regimes will be respectively called "*equal-wage trajectories*" and "*unequal-wage trajectories*" and form the main basis of our study in sections 3 and 4. Additionally, we henceforth take the T-good produced in the South as the *numeraire*.

Solving northern firms' optimization program yields the familiar result of so-called "mill pricing" (i.e. firms charge the same pre-trade price for local sales and exports, foreigners bearing all trade costs): $p_M = \sigma w a_M / (\sigma - 1)$ and $p_M^{EX} = \tau p_M$ where p_M^{EX} is the export price, and as such depends on $\tau > 1$, i.e. the trade costs,

and a_M is the productivity parameter. An adequate choice of units allows us to set $a_M = 1 - 1/\sigma$ which entails that $p_M = w$.

Free entry in the M-sector ensures that there are no pure profits. Hence, the operating profit of a northern firm is¹⁰: $\pi = \sigma^{-1} (p_M x_N^N + p_M^{EX} x_S^N)$. Using (4) and $p_M = w$ yields after a few simplifications:

$$\pi = B b \frac{E^W}{K^W}; \quad \pi^* = B^* b \frac{E^W}{K^W} \quad (5)$$

where $B \equiv \frac{s_E}{\Delta} + \phi \frac{s_E^*}{\Delta^*}$, $B^* \equiv \phi \frac{s_E}{\Delta} + \frac{s_E^*}{\Delta^*}$, $\Delta \equiv s_K + \phi s_K^* \omega$, $\Delta^* \equiv \phi s_K + s_K^* \omega$, $b \equiv \frac{\alpha}{\sigma} \omega \equiv \left(\frac{W^*}{W} \right)^{1-\sigma}$,

$s_E \equiv E/E^W$; $s_K^* \equiv 1 - s_K$ and E^W is world income.

Finally, $\phi \equiv \tau^{1-\sigma}$ is a measure of the degree of trade freeness and belongs to $[0, 1]$: whenever

9. We will focus on the case of full agglomeration in the North.

10. When necessary for clarity's sake, we will use both a subscript and a superscript. In that case, the superscript indicates the country of origin, while the subscript represents the country of destination.

ϕ is close to 0, trade costs are prohibitive, while when ϕ reaches 1, trade is costless.

The I-sector is perfectly competitive. Hence, free entry into R & D implies that the value generated by the I-sector is equal to its costs: in a lapse of time dt , L_I units of labor create a supplement of value v/a_I at a cost of w which entails that $\pi_{R\&D} = (v/a_I - w)L_I dt$.

Thus, the value v of a patent is v/a_I whenever $L_I > 0$, and $v < wa_I$ otherwise. The value of a firm equals the infinite stream of profits that will accrue to her. Time differentiating yields the standard no-arbitrage condition: $r = \pi/v + \dot{v}/v$.

As to the market clearings, the equilibrium condition for the northern labor market is simply $L = L_M + L_I + L_T$, which reduces to:¹¹

$$L = \frac{gs_K}{w} + \frac{E^w}{w} \left[bB(\sigma - 1)s_K + (1 - \alpha)s_T \right] \quad (6)$$

In a steady-state, the intersectoral allocation of labor is constant; equation (6) tells us that this requires constant wages and constant shares in each region. Finally, we can define the northern current account as follows:

$$\underbrace{Y_T - C_T}_{\text{Net supply of T-good}} = \underbrace{\tau K^* x_S^N}_{\text{Imports of M-good}} - \underbrace{\tau w K x_N^S}_{\text{Exports of M-good}} \quad (7)$$

This equation is pivotal for our study since it will enable us to find the frontier between the different regimes: indeed, the second regime is characterized by the absence of northern T-good production, that is, $Y_T = 0$: hence, the frontier is found by setting $Y_T = 0$ and $w = 1$ in (7). This is a subject we shall develop *infra*.

■ LONG-RUN EQUILIBRIUM

In the long run, the capital stocks of the two countries change as a result of capital accumulation.

The dynamics of the model

The dynamics of our economy is defined by four equations, that is, the northern Euler equation, the northern equation of accumulation of capital (i.e. equation (2)), and their symmetric for the South. We can decrease the dimension of the dynamical system by exploiting the properties of symmetry of the model: indeed, we shall be using s_K as a state variable instead of both K and K^* . Intuitively, this is readily understandable since what we are really interested in is the *distribution* of capital between the two countries. Formally, this is only possible because *both capital stocks appear in a linear way in the accumulation function of the I-sector* (i.e. equation (1)). Finally, using (1), (2), (5), the no-arbitrage condition and

11. Where L_T is the amount of labor devoted to the production of the traditional good.

12. From now on, we remove the time subscript since we focus on the steady-state.

$v = wa_l$, the dynamics of the economy reduce to:¹²

$$\begin{cases} \frac{\dot{E}}{E} = bE^w \left[\frac{B}{w}(A - s_K) - \lambda \frac{A}{A^*} B^*(1 - s_K) \right] + \left(\frac{E}{w} + \frac{A^*}{A} \lambda E^* \right) - L \left(1 + \lambda \frac{A^*}{A} \right) + \frac{\dot{w}}{w} - \rho \\ \frac{\dot{E}^*}{E^*} = bE^w \left[B^*(A^* - (1 - s_K)) - \lambda \frac{A^*}{A} B s_K \right] + \left(E^* + \frac{A}{A^*} \lambda E \right) - L \left(1 + \lambda \frac{A}{A^*} \right) - \rho \\ \dot{s}_K = (1 - s_K) A \left(L + \frac{bs_K B(E + E^*) - E}{w} \right) - s_K A^* (L + b(1 - s_K) B^*(E + E^*) - E^*) \end{cases} \quad (8)$$

where $w = (E, E^*, s_K)$ is the static definition of the wage rate as a function of the other endogenous: it is implicitly defined in equation (7) setting $y_T = 0$. Once again, this issue shall be developed *infra*.

Since we cannot analytically characterize the transitional dynamics of a system of dimension 3, we shall henceforth limit ourselves to the study of comparative statics between steady states. These steady-states are such that, on the one hand, the distribution of capital between the North and the South is constant (i.e. $\dot{s}_K = 0$) and on the other hand that nominal spending in both regions is constant¹³ (i.e. $\dot{E} = \dot{E}^* = 0$). Thus, inspecting the law of motion of s_K (i.e. $\dot{s}_K = s_K(1 - s_K)(g - \dot{g})$) allows us to conclude that there are only two possible long-run outcomes: (i) the interior equilibria, where both regions innovate at the same rate, such that $0 < s_K < 1$ and $g = \dot{g}$ which entails that $v = wa_l$ in both regions; (ii) only one region innovates, i.e. the so-called corner equilibria (i.e. $s_K = 0$ or 1) where only one region accumulates capital which implies, for instance, both $v = wa_l$ and $v^* < w^* a_l^*$: this is a core-periphery outcome where all firms are agglomerated in the North.¹⁴

Hence, an interior equilibrium is defined as a situation where the capital stocks of both regions adjust to the point where the value of an extra unit of capital (i.e. v) equals its cost (i.e. wa_l).

Likewise, for a CP outcome to arise, only one region must find it profitable to breed new blueprints. This model behaves like standard NEG models which prompts us to investigate the existence and the stability of both the symmetric equilibrium and the core-periphery equilibrium. This is what we do in the next subsections, but assuming first that the NFS condition holds so that both countries will produce the homogeneous good. Then, we shall turn to the analysis of the full-blown model where we remove this assumption. The interest of presenting a simpler version of the model is twofold: (i) first, as we shall see in section 4, introducing the possibility that wages can diverge adds another force of dispersion in the model. Hence, studying equal-wage trajectories only, allows us to focus on the impact of

13. This is easily seen using the northern T-good market clearing (i.e. $L_T = (1 - \alpha)E$) for instance: indeed, since L_T is constant in a steady-state, E has to be constant also.

14. When we allude to "the movement of firms", or "agglomeration of firms", we have M-firms in mind.

knowledge spillovers on industrial structures; (ii) then, since the simpler version is analytically solvable (while the full-blown version is not), it yields some very valuable insights about the dynamics of agglomeration.

Before turning to the analysis of the existence and the stability of the different types of equilibria, we derive a few useful preliminary results in the next subsection.

Value of firms, nominal spending and Tobin's q

In a steady-state, $\dot{E}/E = \dot{E}^*/E^* = 0$. Hence, the Euler equations imply that $\bar{r} = \bar{r}^* = \rho$.¹⁵ Using (5) and $\dot{\bar{s}}_E = \dot{\bar{s}}_K = 0$, we get $\dot{\pi}/\pi = -\bar{g}$. Integrating this result, and plugging it into the no-arbitrage condition yields: $\bar{v} = \int_0^t e^{(\rho+g)t} \bar{\pi} dt$. Integrating yields the steady-state value of a typical firm:

$$\bar{v} = \frac{\bar{\pi}}{\rho + \bar{g}} \quad (9)$$

where $\bar{g}$ is either the common steady-state growth rate in an interior equilibrium, or the northern growth rate in the Core-in North case.

Let us now express the steady-state value of nominal spending as a function of s_K . We have the following definition: $E + w a_i \dot{K} = Y$. In the steady-state, using (2), (9) and $v = w a_i$, it becomes $\bar{E} = \bar{w}L + \bar{w} \bar{s}_K (\rho + \bar{g}) \bar{K} / \bar{A} - \bar{K} \bar{g} \bar{w} \bar{s}_K / \bar{A}$. Finally, we get $\bar{E} = \bar{w}(L + \rho \bar{s}_K / \bar{A})$.

In the spirit of Turnovski (1996), we use Tobin's q to find the optimal level of firms' investment. Originally, Tobin's q is a static approach (see Tobin, 1969) which says that it is optimal for firms to invest until q reaches 1 where q is defined as the ratio market share value over replacement cost of capital. In our setting, this is simply v/F , and $q = 1$ actually amounts to the free entry condition of the I-sector. Hence, we now have a very intuitive way of characterizing the two kinds of long-run equilibria:

$$\begin{cases} 0 < \bar{s}_K < 1 \Leftrightarrow \bar{q} = 1 \Leftrightarrow \bar{g} = g^* \\ \bar{s}_K = 1 \Leftrightarrow \bar{q} = 1 \text{ and } \bar{q}^* < 1 \Leftrightarrow \bar{g} = g \text{ and } g^* = 0 \end{cases} \quad (10)$$

It is useful to develop the first line of (10). From the definition of q and F, we get $\bar{q} = \bar{\pi}(K^w \bar{A}) / \bar{w}(\rho + g)$ which, using (5), yields: $\bar{w} = b \bar{B} \bar{E}^w \bar{A} (\rho + \bar{g})^{-1}$. For the South, we find an analogous expression: $\rho + g = b \bar{B}^* \bar{E}^w \bar{A}^*$. Finally, combining the two expressions yields the condition of existence of an interior equilibrium, i.e. one so that both nations accumulate capital at equilibrium:

$$\bar{w} = \frac{\bar{A} \bar{\pi}}{\bar{A}^* \bar{\pi}^*} \quad (11)$$

The intuition behind this relationship is straightforward: indeed, for both nations to find it profitable to innovate, it must be that a higher cost of innovation (i.e. a lower A) is compensated by higher profits. Consider as a thought experiment that wages are equal to 1 in both

15. A bar above a variable denotes the steady-state value of this variable.

countries: the relationship then becomes $\bar{A}\bar{\pi} = \bar{A}^*\bar{\pi}^*$. We thus understand that a positive wage differential is an additional cost since I-sector's firms have to pay their researchers a higher wage; this has to be compensated, either by higher externalities that reduce costs (through A), or by higher profits. Otherwise, firms won't have the necessary incentives to innovate.

Finally, it is useful for what follows to have the steady-state values of the endogenous at both the symmetric and CP equilibrium. Setting $\dot{E}/E = \dot{E}^*/E^* = \dot{s}_K = 0$, $s_K = 1/2$ and $w = 1$ in (8) leads to $\bar{A} = \bar{A}^* = \frac{1}{2}(1 + \lambda)$, $\bar{E}/\bar{E}^* = L + \frac{\rho}{1 + \lambda}$, $\bar{B} = \bar{B}^* = 1$.

Likewise, at the CP equilibrium (i.e. $s_K = 1$), we get: $\bar{A} = 1$, $\bar{A}^* = \lambda$, $\bar{E} = L + \rho$, $\bar{E}^* = L$, $\bar{B} = \bar{B}^* = 1$.

Equal-wage trajectories: $w = w^* = 1$

In this section, we shall focus exclusively on the case where wages are equalized between countries. We follow the standard procedure of growth models, by studying first the existence of the long-run equilibrium (a) before pondering on the stability of this (these) equilibrium (a). As we have seen, a long-run equilibrium is a triplet $(\bar{E}, \bar{E}^*, \bar{s}_K)$ such that $\dot{E} = \dot{E}^* = \dot{s}_K = 0$. Considering the symmetry of the model, we can already infer that a symmetric equilibrium will always exist. Nevertheless, we have to answer two critical questions: is this symmetric equilibrium unique? And is it stable?

It is useful for what follows to derive two relationships between s_E and s_K . Indeed, thanks to the law of motion of s_K , we already know that *a priori*, some corner outcomes also exist. It is possible to show formally that, actually, there exist multiple equilibria: to this end, we need two relationships between s_E and s_K . We get the first one setting $w = 1$ in (11), and using (1) and (5) yields:

$$s_K = \frac{1}{2} + \frac{(s_E - 1/2)(1 - \phi^2)(A(1 - s_K) + A^*s_K)}{\lambda(1 + \phi^2) - 2\phi} \quad (12)$$

Using the definition of s_E yields the second relationship:¹⁶

$$s_E = \frac{w \left(L + \rho \left(\frac{s_K}{A} \right) \right)}{L(w + 1) + \rho \left[w \left(\frac{s_K}{A} \right) + \frac{1 - s_K}{A^*} \right]} \quad (13)$$

Baldwin *et al.* (2001) show that the interplay of these two relationships (with $w = w^* = 1$) yields the bifurcation diagram for equal-wage trajectories of FIGURE 1.¹⁷

16. Northern expenditure is just income minus spending on new capital. Using $q = 1$, we obtain $\pi K = (\rho + g)FK$ and with (1) and (2) we get $L_I = gFK$. Thus, E equals $L + \rho FK$. Plugging these results in the definition of s_E leads to the relationship of the text.

17. See also APPENDIX 1 where we carry a short analysis of the stability of the symmetric and CP equilibria.

Figure 1 - The bifurcation diagram for equal-wage trajectories

Dashed lines represent unstable equilibria while full lines represent symmetric equilibria. Hence, the symmetric equilibrium is stable until trade costs reach a critical value, i.e. the break point, ϕ^B . Then, for trade costs between the break and the sustain point (where the CP outcomes becomes stable, i.e. ϕ^S), there are two stable non-symmetric interior equilibria as symbolized by the two branches of the pitchfork. The choice between the upper branch (i.e. agglomeration in the North) or the lower branch (i.e. agglomeration in the South) is decided outside of the model and, for instance, by initial conditions on s_K . The crux of this analysis is that the break point occurs before the sustain point, entailing that the symmetric equilibrium becomes unstable *before* the CP structure is sustainable. Hence, agglomeration takes a more realistic pattern in this model since it is non-catastrophic.¹⁸ Finally, note that both the break point and the sustain point are independent of $b \equiv \alpha/\sigma$.

In this model, there is no relocation *per se* since capital is actually immobile. Nevertheless, agglomeration occurs through another channel, namely, the one of accumulation of capital: the North might temporarily accumulate capital faster than the South, in such a way that a new steady-state where the North owns more firms than the South is reached. In a setting where firms are homogeneous, this is tantamount to the relocation one can find in standard NEG models.

18. This is so because the bifurcation is a supercritical pitchfork, which, by definition, is continuous, while for instance, a subcritical one is discontinuous, which entails a discrete jump of the state variable. We show in a working paper version that the bifurcation is a supercritical one, something which was not done in Baldwin *et al.*

As usual, there are two opposite forces that shape the pattern of agglomeration. The first one is an agglomeration force that tends to destabilize the symmetric equilibrium: it is the standard *market-access effect*, i.e. production shifting leads to expenditure shifting since capital-earnings are spent locally, which in turn raises firms' operating profit in one country and induces more agglomeration there (because it raises the expected value of firms and as such prompts the I-sector to innovate more), which in turn raises local spending... and the circular causality goes on. The second one is a dispersion force, the so-called *market-crowding effect*, i.e. as the number of firms in one country grows, the share of demand firms can appropriate shrinks, thus reducing operating profits. Moreover, there is an additional force stemming from the addition of growth into an otherwise standard NEG framework. Indeed, when spillovers are not entirely global, it is straightforward that an increase in the number of firms in one country reduces the cost of carrying research there more than it does in the other country. This, in turn, induces more agglomeration (i.e. more innovation) in that country, which, decreases once again the cost of doing research and so on: this is the so-called *localized spillovers effect*.

A critical feature of these models is that the strength of the agglomeration force decreases more slowly than the strength of the dispersion force as trade gets freer: this usually explains why firms agglomerate in a single country. A new feature of this particular model, however, is that the third force, i.e. the localized spillovers effect, is *independent of trade costs*, and depends instead on the degree of localization of the spillovers. In standard NEG models, recall that when trade is totally free, location is meaningless (since the strength of the two forces is nil), whereas here one force remains in operation: at the limit, when trade is totally free, the symmetric equilibrium is not stable provided the spillovers are not global (i.e. $\lambda \neq 1$).

Unequal-wage trajectories

In this subsection, we remove the standard hypothesis of New Economic Geography and Growth models entailing that the two countries are never fully specialized, so that both of them own an active T-sector which pins down the wage to 1 in both countries. The interest of removing this assumption is that it allows us to study a full general equilibrium model¹⁹ by allowing wages both to differ between countries and to vary over time: in that case, the remaining northern sectors compete with each other for inputs. Moreover, when nominal wages are equalized between countries, one gets the impression that the two countries have already converged towards each others which is unappealing in a growth setting. Removing this assumption has one clear drawback however, since we will not be able to get analytical results: indeed, the dynamics of capital construction/deconstruction now depends on the wage rate through the profit functions; unfortunately, it appears in a non-linear way in these

19. When the T-sector is active, the wage rate is determined by the equilibrium conditions of this sector. Moreover, since labor is solely used as a variable cost in the M-sector, the wage of industrial workers must equal the wage of traditional workers. In essence, all workers earn the same wage, which is determined exclusively in the T-sector, independently of the rest of the model. This gives a strong partial equilibrium flavor to the model.

profits. Hence, in what follows, we will have to resort to numerical analysis to get most of the main results. Some of the questions that we will have to answer are the following: do wages diverge before the CP outcome is reached? And in that case, do the CP outcome still exist in the second regime? Finally, if the answer to the first question is “no”, and the answer to the second question is “yes”, can wages diverge after the CP equilibrium is reached? We discuss the “Non-Full Specialization” assumption that holds true at all times in most NEG models, and which we are to remove.

The assumption under scrutiny

The main role of the T-sector in this kind of model is to permit international wages equalization to get a simple expression for the price of the industrial goods. This is true as long as both countries do possess this sector. Now, as the industrial structures of the countries diverge as trade gets freer (implying that more and more workers move from the traditional sector into the industrial sector), it is highly likely that, at some point, the northern T-sector disappears: in that case, the equilibrium must involve $w > w^*$.

Let us investigate the condition over the parameters which is necessary for wages not to diverge. The answer to this question is greatly eased if we consider a steady-state where all firms are agglomerated in a single country, say, the North. In that case, all research activity and all M-good production is located in the North, while the South only produces the T-good. The question is whether the North also produces the T-good. Since the South is fully specialized, we know that T-good production equals L^* . On the other hand, world demand for the T-good is $(1 - \alpha)\bar{E}^W$. For the two countries to produce the traditional good, it is necessary that southern production cannot satisfy world demand all by itself. Formally, it is that: $(1 - \alpha)\bar{E}^W \geq L^*$. With the definition of $\bar{E}^W$ evaluated around $s_K = 1$, we find a condition over α , so that both countries own a traditional sector:

$$\alpha \leq \frac{L + \rho}{2L + \rho} \quad (14)$$

Given that the right-hand side of equation (14) is bounded to the left by one-half²⁰, we know that at a core-periphery equilibrium, wages won't diverge if α is less than one-half. Moreover, wages are more likely to diverge the more firms are concentrated (because more firms means more people working in the industrial sector, which increases the likelihood that the traditional sector disappears). The following proposition concludes:

Proposition 1. *If the share of consumer expenditure on the M-good is less than one-half, wages won't diverge even if all M-firms are concentrated in a single country. Since the more firms are concentrated in a single country, the more wages are likely to diverge, we are certain that wages will never diverge if $\alpha < 1/2$.*

20. Note that, given the respective magnitude of L and ρ ($\rho \ll 1$), it is nonetheless very close to one-half.

Proposition 1 gives us an inkling of the necessary condition for wages to diverge: α has to be at least higher than one-half. That point being made, we can now turn to a more formal analysis of the second regime, beginning by finding the frontier between the two regimes.

Looking for the frontier

Let us now assume that (14) is not verified: it opens the possibility of a second regime, one with unequal-wage trajectories. In that case, our first task is to find the frontier between the two regimes. The key to find this frontier lies in equation (7), i.e. the northern current account. Indeed, the trade pattern is very different from one regime to the other. In the first regime, $Y_T = L_T > 0$ and we can distinguish two stages: (i) the first stage exists as long as we are at the symmetric equilibrium, which implies that the two countries only trade the differentiated goods such that $Y_T = C_T$ and $Y_T^* = C_T^*$; (ii) the second stage is when the North starts attracting firms and becomes a net exporter of M-goods and $Y_T < C_T$. On the other hand, the second regime is characterized by $Y_T = L_T = 0$ since only the South produces the T-good. The trade pattern is mostly similar to the one in the second stage of the first regime except that the North now imports an amount of M-good equal to its demand, such that $Y_T^* = C_T + C_T^*$. With (4), we can rewrite the current-account in the following way:

$$L_T[s_E, s_K, w] = \frac{s_E}{1-s_E} \left(1 - \frac{\alpha s_K}{s_K + \phi(1-s_K)\omega} \right) - \alpha \left(1 - \frac{1-s_K}{1-s_K + \phi\omega s_K} \right) \quad (15)$$

The frontier between the two regimes is such that $L_T[s_K, s_E, 1] = 0$. With (12) and (13) and setting $w = 1$ in (15), we can express the frontier for an interior equilibrium as a function of the parameters:

$$\alpha = \frac{\psi [L\lambda Z(1-\phi^2) + \chi]}{\lambda(1-\phi^2)[L\lambda(Z + \phi(2\lambda + \phi - \lambda^2)) - 2L\phi + \chi]} \quad (16)$$

where:

$$\chi \equiv \sqrt{Z\{4\rho^2\phi^2(\phi\lambda - 1)^2 + L^2Z\psi + 4L\rho\phi[2\phi - \lambda(1+3\phi^2) + \lambda^2\phi(1+\phi^2)]\}}$$

$$\psi = \phi(\phi\lambda - 2) + \lambda, \text{ and } Z = 1 - \lambda^2.$$

Equation (16) implicitly defines the *switch point*, i.e. the value of the trade costs for which the model undergoes a regime-change. Numerical analysis shows that for every value of α there exists a single economically relevant value of ϕ so that there is a switch from the first regime to the second regime.²¹ This value is called ϕ^{SW} . As trade gets freer, firms agglomer-

21. Note that we also get an acceptable value of ϕ for values of α less than one-half. In that case however, numerical analysis shows that $\phi^{SW} > \phi^S$, which means that the CP equilibrium becomes stable before the switch occurs. Hence, since equation (16) is only valid for an interior equilibrium, the switch point has no economic meaning. It also shows that $\phi^{SW} > \phi^S$ for values of α slightly higher than one-half. This is not surprising if one recalls that the right-hand side of equation (14) is actually slightly higher than one-half if $\rho > 0$. Nevertheless, in what follows, when we will allude to this condition, we will always say "when the share of consumer expenditure on industrial goods is higher than *one-half*" for the sake of brevity. The formal condition is that $\phi^{SW} > \phi^S$, which, with (16) yields the minimum value of α associated with a given constellation of parameters such that the model undergoes a regime-change.

ate in the North so that the northern industrial sector keeps getting bigger by drawing workers from the agricultural sector. In FIGURE 2, using (4), (7) and the northern labor market clearing, we have plotted²² the evolution of the northern workforce as trade gets freer, for an interior non-symmetric equilibrium.

Figure 2 - Evolution of the Northern workforce

In both cases, there comes a moment when labor devoted to the traditional sector becomes nil: this defines the switch point. Numerical analysis of this switch point shows that, quite intuitively, the switch occurs all the sooner (i.e. for high trade costs) that α is high because world demand for the T-good is low in that case, entailing that the South will be able to satisfy the whole world demand by itself "sooner". Now that we know under what conditions the second regime will appear, we can turn to the analysis of the behavior of the model under this regime. Thus, in the next section we will derive the static expression of the wage rate as a function of the other endogenous variables.

22. Unless specified differently, in all the graphics that follow, the following values of the parameters were used: $\sigma = 3$, $\rho = 0.3$, $\lambda = 0.3$, $L = 1$. This combination of values was chosen mainly because, it yields a realistic range of values for the growth rates (see *infra*). Also, note that some econometric evidence (i.e. Brakman *et al.*, 2001; Combes and Overman, 2004; Mion, 2004) points out to a value of σ between 2 and 8.

The static expression of w

In the first regime, the wage rate is determined by the equilibrium condition of the T-sector. Now, in the second regime, this is not the case anymore: the steady-state value of w can be found thanks to the Northern current account. That is, we can inject the value of s_K as defined in (12) and the value of s_E as defined in (13) into the definition of L_T i.e. equation (15), to get an equation linking w to the other two endogenous. In the second regime, $L_T[s_E, s_K, w] = 0$. Solving this yields two roots: we check numerically that one root is negative, while the other (i.e. the one that subtracts the radical) is always greater than one. Hence, the latter is the static value of w as a function of the endogenous and we label it w^{stat} . We are now ready to simulate the behavior of the model in the second regime: indeed, the system composed of w^{stat} , (11) and (13) is all we need to find the closed form of the model. This is what we do in the next section. Before that however, we address the question of the existence of the CP equilibrium in the second regime in the next subsection.

The CP equilibrium in the second regime

The CP equilibrium in the second regime is easily characterized. Indeed, it is straightforward to find the steady-state values of s_E and w , since solving (15) evaluated around $s_K = 1$ for s_E yields the simple $\bar{s}_E = \alpha$ whereas we still have $\bar{A} = 1$, $\bar{A}^* = \lambda$, $\bar{B} = \bar{B}^* = 1$. Hence, in the second regime, when all firms are concentrated in the North, the share of Northern expenditure is exactly equal to the share of expenditure in the industrial good. The value of w then follows directly from the definition of s_E evaluated around the CP equilibrium:

$$\bar{w} = \frac{\alpha L}{(1 - \alpha)(L + \rho)} \quad (17)$$

We check that this value is indeed higher than one: for the second regime to occur, equation (14) implies that α has to be at least higher than $(L + \rho)/(2L + \rho)$. In that case, $\bar{w}$ is higher than one.

Let us now study the existence of the CP outcome in the second regime by finding the sustain point. The test is once again that $\bar{q} = 1$, $\bar{q}^* < 1$, when evaluated around the CP equilibrium. Southern Tobin's q is: $\bar{q}^* = \lambda \bar{w}^\sigma ((1 - \bar{s}_E)\phi^{-1} + \phi \bar{s}_E)$.

Solving $\bar{q}^* = 1$ with $\bar{s}_E = \alpha$ yields two solutions, the economically relevant one²³ is the sustain point for the second regime:

$$\phi^{s_2} = \frac{1 - \sqrt{1 - 4\bar{w}^{2\sigma}\alpha\lambda^2(1 - \alpha)}}{2\alpha\lambda\bar{w}^\sigma} \quad (18)$$

where $\bar{w}$ is defined in (17). Inspection of the term under the square root shows that it is decreasing in α (i.e. its derivative with respect to α being $4\lambda^2\bar{w}^{2\sigma}[2(\alpha - \sigma) - 1]$ which is

23. The other one is higher than one which entails that the CP exists for all values of the trade costs higher than the sustain point.

negative since $\alpha < \sigma$). Hence, when the share of industrial expenditure is high enough, the term under the square root becomes negative, and the sustain point is imaginary. Hence,

Proposition 2. *In the second regime, the core-periphery equilibrium does not always exist. In particular, it ceases to exist when α is too high.*

Note that, as in the equal-wage trajectories case, the CP equilibrium is stable as long as it exists (i.e. for $\phi \in [\phi^{S2}, 1]$). Intuitively, it means that, for low trade costs, southern R&D remains unprofitable despite the lower cost of labor there: it amounts to the idea that the Northern innovative advantage stemming from the knowledge spillovers outweighs the Southern cost-advantage. This is an idea we shall now discuss.

■ PUTTING IT ALL TOGETHER

The pattern of agglomeration

Combining the ideas from the previous sections we can draw the pattern of agglomeration. We start in the first regime, which is only valid for $L_T[s_E, s_K, 1] > 0$. The first regime is itself composed of two stages, depending on whether or not the symmetric equilibrium is stable. As trade gets freer, we move to the second stage (i.e. when the non-symmetric equilibria are stable). Then, as trade gets even freer, there comes a moment when the Northern industrial sector is so large that there is not enough labor left for the agricultural sector (i.e. the second regime exists when $L_T[s_E, s_K, w] = 0$) and we switch to the second regime as wages start to diverge. Our goal is to investigate how the pattern of agglomeration changes for different values of the share of consumer expenditure on manufactured goods. The numerical procedure we follow is described in the APPENDIX 2. The pattern of agglomeration crucially depends on the value of α : three cases have to be considered depending on whether α is lower than one-half, slightly higher, or higher. These three cases are depicted in FIGURE 3: the leftmost panel corresponds to the bifurcation diagram for the first regime, whereas the other two panels correspond to the second regime. In all cases, note that *the value of the break point is identical since it does not depend on α* .²⁴

24. See APPENDIX 1 for a derivation of the break and sustain points in the first regime.

Figure 3 - Bifurcation diagrams for three values of α

FIGURE 4 sums up our results in a stylized diagram: we have plotted the upper branch of the bifurcation diagram for different values of α . The uppermost branch is for a value of α lower than one-half, so that wages don't diverge. Then, each branch underneath represents a higher value of α : the switch between regimes occurs all the faster that α is high. Also, we see that for values of α close to one-half, the CP outcome is reached all the same at some point; furthermore, the sustain point occurs all the faster that α is close to one-half.²⁵ Then, for higher values of α , we notice that there is never full agglomeration in the North. In that case, when trade costs are very low, firms stop agglomerating in the North, and even slightly start spreading back in the South. Also, note that as soon as we switch to the second regime, the symmetric equilibrium ceases to exist: this is symbolized by a grey line in the figure.

25. The FIGURE suggests that $\phi^S < \phi^{S2}$. Our simulations show that it is not true for all values of the parameters, i.e. if α is close to one-half for instance. However, in that case, ϕ^{SW} is higher than the sustain point, entailing that the second regime does not arise. Hence, $\phi^S < \phi^{S2}$ for all relevant values of the parameters.

Figure 4 - Bifurcation diagrams for the full-blown model

Wage-gap versus localized spillovers

The explanation of FIGURE 4 lies in what happens when we switch regimes. Indeed, as soon as wages are not pinned down to one in both countries, Northern wages start to rise since the supply of labor is bounded while demand rises (because the Northern industrial sector grows as trade gets freer). In turn, this discourages firms from agglomerating in the North, *everything else being equal*. To disentangle these different effects, let us come back to the definition of the profits. It is straightforward that the Northern profit diminishes with s_K (i.e. the local-competition effect), and increases with s_E (i.e. the market-access effect). Finding the impact of the wage rate is more involved however since, on the one hand, a higher wage increases firms' costs, but on the other hand, it also increases the revenues of Northern workers which, in turn, raises demand. To get some insights, we plot the evolution of the wage-gap (FIGURE 5).²⁶

26. Since the Southern wage rate is constant, the evolution of the wage-gap indeed follows the evolution of Northern wages. Note that, for a given value of ϕ , the *real* wage-gap is even higher than the nominal wage-gap: indeed, as usual, the Northern price index is lower than its Southern counterpart thanks to the presence of trade costs. Of course, this discrepancy tends to disappear as trade gets freer, and, at the limit, when trade is totally free, the two coincide.

Figure 5 - Evolution of the Northern wage rate as trade costs decrease

There are two distinct cases, depending on whether the CP equilibrium is reached (case I) or not (case II). The heavy dark curve (i.e. $\alpha = 0.6$) and the thin greyish one (i.e. $\alpha = 0.65$) correspond to the former case. The main difference between the two lies in the “speed” at which the CP outcome is reached, that is, the range of values of ϕ necessary to reach the CP equilibrium: when α is very close to one-half, we already know that wages start to diverge when agglomeration in the North is already almost complete (see the second branch in FIGURE 4). Consequently, the Northern wage rate rises quickly, because the supply of traditional workers is scarce and all the more scarce that s_K gets closer to one, but not necessarily by much since the CP equilibrium is quickly reached: then, northern wages remain constant since $s_K = 1$. All the other curves depict situations where full agglomeration in the North will never occur (case II). In that case, the higher α is, the smaller the rise of the wage rate: if α is high, the switch occurs when firms are hardly agglomerated so that, on the one hand, the supply of traditional workers is still important and the tension in the labor-market is low, and on the other hand, the localized spillovers are weak. This entails that the wage-gap does not rise much, but this rise is sufficient to slow agglomeration down, which, in turn, eases the pressure in the labor market. All in all, the pattern of the wage-gap roughly follows the pattern of agglomeration.²⁷

27. FIGURE 5 suggests that there might be a non-monotonic relationship between α and w for a given ϕ . This is so because in case I, the wage rate increases more quickly than in case II, so that it is first higher whatever the value of ϕ ; then, in case I, the wage rate remains constant when the CP equilibrium is reached, whereas it keeps increasing as trade costs fall in case II so that there might come a moment when it is higher. See for instance the curves associated with $\alpha = 0.65$ and $\alpha = 0.7$ on the figure.

It is well known in standard NEG models that the magnitude of both the force of agglomeration (i.e. the market-access effect) and the force of dispersion (i.e. the local competition effect) decreases as trade gets freer. Moreover, the strength of the latter decreases faster than the strength of the former. Here, as we have seen in section 1, this is also true. We have argued however that the localized spillovers effect is not trade cost-dependent. Hence, when trade tends to become costless, the balance between the different forces has definitely shifted towards agglomeration since the localized spillovers effect is likely to be the sole force in operation in the first regime. Nevertheless, when one opens up the possibility that wages can diverge, there is an additional dispersion force (henceforth labelled the *wage-gap effect*) stemming from the increasing wage differential that arises as firms agglomerate in one country. The critical point is that this force is also independent of the level of trade costs.²⁸ Hence, when trade costs are low, everything else being equal, firms prefer to be where labor is cheaper, i.e. in the South. The balance between agglomeration and dispersion forces thus shifts once again, but this time in the opposite direction. At the limit, when trade costs are nil, the only two forces that interact with each other are the localized spillovers effect and the wage-gap effect. Here, the tension between those two opposite forces yields the pattern of agglomeration observed in FIGURE 4.

Wrapping up

We integrate the insights of the previous sections to describe a typical pattern of agglomeration.²⁹ Things happen as follows. First, when trade is prohibitive, firms are equally distributed between countries. Indeed, agglomeration is not a sustainable equilibrium for high trade costs: to prove this, let us consider that trade costs are infinite (i.e. $\phi = 0$); the value of sales of a firm located in the North is thus E/s_K (since wages are equal in the first regime) whereas the value of sales of a firm located in the South is E^*/s_K^* . Let us now consider that all firms are agglomerated in the North (i.e. $s_K = 1$). In that case, the value of sales of a firm relocating in the South would be infinite (even though the North is a bigger market) since it would be the only firm selling there. As such, its profit would also be infinite: Northern firms would find it profitable to relocate in the South. The argument still holds when trade costs are not infinite but large.³⁰ Hence, agglomeration of firms in a single country is not an equilibrium when trade costs are high and an even distribution of firms between countries is the

28. This formulation might seem surprising since the wage-gap varies as trade gets freer as evidenced in FIGURE 5. This is so because the wage-gap depends on the concentration of firms which in turn depends on trade costs. Actually, this is also the case for the localized spillovers: indeed, the more firms are concentrated in a single region, the stronger the localized spillovers effect is. However, our point is that these two forces do not depend directly on the level of the trade costs, so that, even when trade costs are nil, these forces still exist. However, they cease to exist when firms are equally distributed between countries.

29. In this subsection, we use the term "relocation" to bolster intuition even though, as we have already argued, firms do not actually move.

30. The difference being that the Southern firm would not be the only one to serve the South; still, it would be so shielded from competition that its sales would still be more important than if it were located in the North:

$$E^*/(\phi s_K + 1 - s_K) + \phi E/(s_K + (1 - s_K)\phi) > \phi E^*/(\phi s_K + 1 - s_K) + E/(s_K + (1 - s_K)\phi).$$

only stable equilibrium. As trade costs decrease, the increase in competition induced by a movement of firms (i.e. $ds_K > 0$) diminishes quickly (that is, it does not lower sales, i.e. $E/(s_K + \phi(1-s_K)) + \phi E^*/(\phi s_K + 1-s_K)$, and as such, output, as much as it used to) whereas the incentives to agglomerate due to the localized spillovers are still present. There comes a point when the agglomeration forces overcome the sole dispersion force and firms start agglomerating in the North. The critical feature to notice here is that, for a given decrease in the trade costs, the competition effect is still strong enough to ensure that a full-fledged circular causality does not occur: as firms agglomerate, the incentives for further agglomeration are reduced so that not all firms would want to relocate as soon as the break point is crossed. In other words, there is no catastrophic agglomeration.

Then, depending on the value of the share of industrial spending, two things can happen: either the countries become fully specialized, or not. In the latter case (i.e. the so-called first regime), only three forces battle out until all firms are agglomerated in the North. It is obvious that, contrary to standard NEG models where the location of firms is irrelevant when trade is totally free, this is not the case here, because of the spillovers. Indeed, consider that trade is totally free (i.e. $\phi = 1$) and that firms are all agglomerated in the North; a firm that would move to the South would not be shielded from competition and would not benefit from a better access to a larger market (when trade is perfectly free, all firms have an equal access to all markets, whatever their location). Moreover, it would not benefit from any local spillovers so that the cost of opening a new plant (i.e. the cost to build a new unit of capital) in the South would be higher, whereas its profit would be identical in the two locations: relocating in the South is not profitable and agglomeration in the North is a stable equilibrium. The reasoning extends to finite but very low trade costs.

If the share of industrial spending is high, overall demand for the traditional good is low enough to be satisfied by a single country. As more and more firms agglomerate in the North, labor demand in the M and I sectors increases whereas supply is bounded: the Northern T-sector shrinks as a consequence. There comes a moment when it ceases to exist. As soon as it happens, Northern wages are not pinned down to one anymore, and they start to rise. The effects of this rise are twofold: (i) it increases firms' marginal costs (in the M and I sectors) which tends to deter firms from locating in the North; (ii) it also increases workers revenues, and as such total income in the North. However, this does not translate fully into a higher industrial demand since a fraction of income is devoted to spending on the traditional good. As trade is liberalized further, firms agglomerate in the North mainly because of the presence of the spillovers that help overcome the higher cost of labor in the North. Meanwhile, recall that competition in the North increases, as does the Northern market-size (i.e. $ds_E > 0$). However, since trade is becoming freer, these two effects matter relatively less than the other two forces. Because of the increasing tension in the labor-market, the Northern wage rate keeps increasing. From that point, the pattern of agglomeration can follow two paths, depending, once again, on the value of α . If it is close to one-half (case I in

FIGURE 5), we know that the divergence in wages only occurs when firms are already very agglomerated in the North; in that case, even though the Northern wage rate increases, the cost of innovating in the North is so low compared to the one in the South ($A \rightarrow 1 \gg A^*$ whenever $s_K \rightarrow 1$ such that $a_l \ll a_l^*$), that firms keep agglomerating in the North until they are all agglomerated there: the wage differential comes too late to prevent the emergence of a Core-Periphery structure. It should be noted that a higher λ reduces the strength of the localized spillovers effect which entails that for a given α , the higher λ is, the less likely full agglomeration is.

Otherwise, when the share of industrial spending is very high (case II, FIGURE 5), wages diverge when firms are only mildly agglomerated in the North. Therefore, as trade gets even freer, firms first keep agglomerating in the North, but as the wage rate keep rising also, there comes a moment when it is strong enough to discourage more creation of firms in the North: firms relocate in the South where labor is cheaper. As firms agglomerate back in the South, the tension in the Northern labor-market is eased entailing that the Northern wage rate decreases: relocating in the South becomes less profitable. This explains why firms are partially agglomerated in the North when trade is close to costless. We sum up our result in the following proposition:

Proposition 3. *Opening up the possibility that wages can diverge yields results that differ from the existing literature. Indeed, contrary to Baldwin et al. (op. cit.), the presence of an additional force of dispersion which is independent from trade costs, i.e. the wage-gap effect, leads to a less concentrated world since firms do not necessarily fully agglomerate in a single country for very low trade costs. Moreover, the relationship between agglomeration and trade liberalization might be non-monotonous.*

Growth rates

The analysis of the growth rates of the first regime is identical to the one in Baldwin et al. That is, we find the steady-state value of L_t and plugging into (2) yields:

$$\bar{g}_{|s_K=1/2} = \bar{g}_{|s_K=1/2}^* = \frac{\alpha(1+\lambda)L - \rho(\sigma - \alpha)}{\sigma} \quad (19)$$

We now turn to the growth rate of real income: by definition, real income equals nominal income Y divided by the perfect consumption price index, i.e. $P \equiv P_T^{1-\alpha} P_M^\alpha$. In steady-state, Y is constant but, as is standard in expanding product variety models, P decreases under the pressure of the ever-increasing number of varieties (i.e. K^w rises at the rate $\bar{g}$ in steady-state). Hence, real-income grows at $\alpha\bar{g}/(\sigma - 1)$, which yields:

$$\bar{g}_{\frac{Y}{P}} = \bar{g}_{\frac{Y^w}{P^w}}^* = \frac{\alpha^2(1+\lambda)L - \rho\alpha(\sigma - \alpha)}{\sigma(\sigma - 1)} \quad (20)$$

For the core-periphery equilibrium, we obtain:

$$\bar{g}_{|s_k=1}^{1st} = 2bL - \frac{\rho}{\sigma}(\sigma - \alpha) \quad \bar{g}_{|s_k=1}^* = 0 \quad (21)$$

Note the absence of λ in (21): indeed, if all firms are concentrated in a single country, the magnitude of the spillovers is irrelevant.

In the second regime, we can only derive an analytical solution for the growth rate in the CP equilibrium. Indeed, using (2), (17) and $\bar{s}_E = \alpha$ yields:

$$\bar{g}_{|s_k=1}^{2nd} = \frac{L - \rho(\sigma - 1)}{\sigma} \quad \bar{g}^* = 0 \quad (22)$$

A striking feature appears when one compares (21) and (22): indeed, it is straightforward to see that $\bar{g}^{1st} < \bar{g}^{2nd}$ (recall that the first regime only prevails for $\alpha < 1/2$). Hence, the growth rate in the CP equilibrium in the second regime is higher than the one in the first regime: indeed, in the second regime, Northern labor is solely divided between the M-sector and the I-sector, while in the first regime, some resources are diverted towards the “non growth-enhancing” T-sector. Moreover, recall that wages are constant in steady-state, so that if the rate of growth of varieties is higher in the CP equilibrium in the second regime, the rate of growth of real income also is.

Comparing (19) and (22) reveals another puzzling feature: the growth rate of the CP outcome in the second regime is not necessarily higher than the symmetric growth rate. It is higher if and only if:

$$\lambda < \frac{(L + \rho)(1 - \alpha)}{\alpha L}$$

where rather startlingly, we notice that the right-hand side term is exactly equal to the inverse of the steady-state value of w at the CP equilibrium as defined in (17) and as such is lower than one. Actually, this is not that surprising. Indeed, recall that two opposite forces independent of trade costs battle out: the wage-gap effect and the localized spillovers effect. Now, the strength of the former increases with w , while the strength of the latter decreases with λ . Hence, if $\lambda = 1/\bar{w}$, the two forces cancel out at the CP equilibrium, which entails that the growth rates are identical.³¹ A very critical corollary of this result is presented in the following proposition:

Proposition 4. *Contrary to the existing literature that shows that agglomeration is always beneficial to both countries in terms of steady-state real income growth rate, we argue that all is not that simple. Actually, if (i) spillovers are not too localized, and (ii), the share of expenditure that falls on the industrial good is not too much higher than one-half, the concentration of firms in a single country leads to slower growth and both countries are worse off.*

31. Indeed, at the symmetric equilibrium, those two forces don't exist (since firms are equally distributed between countries), such that they have no impact on the symmetric growth rate.

The intuition behind this proposition is clear: the less localized the spillovers are, the less the concentration of firms in a single country is beneficial for growth; in that case, if the Northern wage rate is high enough (an example would be the curve associated with $\alpha = 0.65$ in FIGURE 5), the spillovers might not be strong enough to offset the lower cost of labor.

Incorporating the previous calculations allows us draw the “pattern” of the growth rate as trade costs decrease. The results are depicted in FIGURE 6, for different values of α higher than one-half. Two of the four curves represent situations where the CP equilibrium is reached.

Figure 6 - Evolution of the growth rate as trade costs decrease

We notice that in the upper left panel of FIGURE 6, the growth rate in the non-symmetric interior equilibrium of the second regime increases as trade gets freer while in the upper right panel, the growth rate decreases. While this may seem peculiar, the explanation is actually straightforward and lies in the evolution of the wage rate: indeed, when α is close to one-half, we know that wages don't increase much. Moreover, on the one hand, the growth rate of the number of varieties tends to decrease as the wage rate increases, but on the

other hand, it increases as firms tend to be more concentrated (via the localized spillovers effect). Hence, in the upper left panel of FIGURE 6, since the wage rate has not increased by much, the wage-gap effect is weak and the growth rate increases as trade gets freer (and as firms agglomerate in the South). Conversely, in the upper right panel, the wage rate is large enough to slow agglomeration down which tends to reduce the growth rate. Note that, as α does not appear in (22), the growth rate in the CP equilibrium is the same in both panels: this is so because what determines the growth rate in the CP outcome is simply the amount of labor in the I-sector, which is independent of both α and the wage rate. The lower panels represent a situation where the CP equilibrium is never reached: as such, the growth rate now always depends on the level of trade freeness. Quite understandably, the growth rate follows the inverse pattern of the wage rate (and as such, the same pattern as the agglomeration of firms) which, as we have seen in FIGURE 5, first increases as trade gets freer, then decreases.

The crux of this analysis is that, liberalizing trade does not necessarily increase the growth rate and might even be detrimental. At least, as FIGURE 6 shows, there might come a moment when further trade liberalization lowers the growth rate of both countries.

■ CONCLUSION

Starting from the premise that factor price differences are certainly bound to be an important part of any convergence or divergence process, we have extended the model of Baldwin *et al.* (2001) by removing the standard "Non-Full Specialization" assumption which ensures that both countries always own a traditional sector. We have argued that, as trade gets freer, there comes a point when the Northern traditional sector disappears and the Northern wage rate increases as a consequence. This rise is another dispersion force militating against agglomeration in the North counterbalancing the lower cost of carrying research there. Crucially, this force is independent of trade costs. Hence, it slows agglomeration in the North, yielding a more realistic pattern of agglomeration, that is, as long as the share of expenditure that falls on the industrial goods is high enough, both countries always own some firms, even for zero trade costs (contrary to most of the existing literature). This result might allay European policymakers' fear that a growing integration within the EU might result in even more polarization and inequalities between member states. Finally, we have also argued that, contrary to what previous studies have found, the concentration of firms in a single country might actually lead to a slower growth rate for both countries. All in all, contrary to existing literature, the model seems to imply that full agglomeration is neither desirable, nor inevitable.

As is usual in the literature, the endogenous growth engine used in our model is based on Grossman and Helpman (1991). Nevertheless, this is clearly not the only endogenous growth model we could use. Hence, a natural question to ask is whether the results we have obtained here would hold if we were to use another setting. For instance, what about using

a Schumpeterian model of growth in the spirit of Aghion and Hewitt (1992)? Especially as Aghion and Hewitt (2005) argue convincingly that Schumpeterian theory provides a paradigm more suited to studying the policy of growth than standard expanding product variety models of growth. What new insights would we get about the link between location of firms and growth?

F. A.³²

APPENDIX 1

A note on stability

Investigating the stability of the symmetric equilibrium consists in finding the value of ϕ for which the symmetry breaks, i.e. the so-called "Break Point". Two different methods yield the desired result: (i) the first one is informal and used in most standard NEG models and consists in investigating a slight exogenous perturbation of the symmetric equilibrium and checking the sign of the profit-gap; (ii) the second one is the standard procedure in growth models of forming the Jacobian of the system (8) and evaluating it around the symmetric equilibrium. We have used the latter method. Let us consider this system: there are two variables forward (i.e. E and E^*) and one backward (i.e. s_K). Hence, this dynamic system will be stable if and only if there are as many variables that can "jump" as there are positive eigenvalues. Let us form the Jacobian matrix composed of the nine partial derivatives of the system (8); for instance, c_{11} (i.e. the term on the first row and the first column of the Jacobian) is the partial derivative of the first expression in (8) with respect to E , evaluated around the symmetric equilibrium:

$$\frac{\partial \dot{E}(E, E^*, s_K)}{\partial E} = \frac{\phi[\sigma(\rho v - L\phi) - \lambda[\lambda a v + (\alpha v(L + \rho) + \sigma L\phi)]}{\alpha(1 + \lambda)\phi}$$

where $\phi \equiv 1 + \phi$ and $v \equiv 1 - \phi$.

The associated characteristic polynomial is then $(e - \xi)(ae^2 + be + c) = 0$ where $\xi \equiv L(1 + \lambda) + \rho$,

$$a \equiv \sigma(1 + \lambda), \quad b \equiv \sigma[\xi(1 - \lambda) + 2\lambda\rho] + 4\frac{\alpha\phi(1 - \lambda\phi)\xi}{\phi^2} \quad \text{and} \quad c \equiv -2\alpha\xi\left[\xi\left(\lambda - 2\frac{\phi(1 + \lambda)}{\phi^2}\right) - \frac{\lambda\rho v}{\phi}\right].$$

Solving for e yields the three eigenvalues. Inspection of the eigenvalues shows that the first one is always positive, while if $b^2 - 4ac > 0$, the other two are real. Hence, the eigenvalue that adds the radical is always positive, whereas the one that subtracts the radical changes sign for $c = 0$. Solving for ϕ yields the break point:

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All figures were drawn with the help of Adobe Illustrator, and the whole numerical analysis was done with Mathematica. All these files are available upon request. Some technical details can be found in a slightly dated working paper version available at www.dauphine.fr/eurisco/eur_cr.html.

$$\phi^B = \frac{L(1+\lambda) + \rho - \sqrt{(1-\lambda^2)[L(1+\lambda) + \rho] + \lambda^2 \rho^2}}{\lambda(L(1+\lambda) + 2\rho)} \quad (23)$$

If $\phi > \phi^B$, there are three positive roots, entailing that the symmetric equilibrium is not stable, while for $\phi < \phi^B$, there are two positive and one negative eigenvalue, implying that the symmetric equilibrium is a saddle-point. Hence, this system is non-linear, in the sense that it undergoes a change in its qualitative behavior around a specific value of one parameter, i.e. ϕ : this is called a *bifurcation*.

By definition, in the CP outcome only one region (the North for instance) innovates and breeds new varieties, i.e. $\bar{q} = 1$, $\bar{q}^* < 1$. Let us find $\bar{q}^*$ when $s_K = 1$:

$$q^*|_{s_K=1} = \frac{b\lambda(\phi(L+\rho) + L\phi^{-1})}{\rho + \frac{2\alpha L - \rho(\sigma - \alpha)}{\sigma}}$$

Thus, if this value is less than 1, southern firms have no incentives to innovate, and the core-periphery outcome exists. Solving $q^*|_{s_K=1} = 1$ yields the frontier of existence of the agglomerated equilibrium.³³

$$\phi^S = \frac{2L + \rho - \sqrt{(2L + \rho)^2 - 4\lambda^2 L(L + \rho)}}{2\lambda(L + \rho)} \quad (24)$$

The CP equilibrium exists if and only if $\bar{q}^* < 1$, that is, if Southern firms don't have incentives to innovate. If they don't innovate (whereas Northern firms do), new firms are set up solely in the North and the configuration $s_K = 1$ is bound to be stable. As such, the CP equilibrium is stable whenever it exists, that is for $\phi \in [\phi^S, 1]$.

APPENDIX 2

The five steps to solve the model numerically

Step 1. In the first step, using (23) we calibrate³⁴ the value of the break point (which does not depend on α) and with (24), we find the sustain point for the first regime.

Step 2. With (16), we get the switch point and we check that it is not greater than the sustain point.

Step 3. If it is not, we use (12) and (13) (and $w = w^* = 1$) to get the steady-state values of s_K for each $\phi \in [\phi^B, \phi^{SW}]$ which yields the upper branch of the pitchfork, for the first regime. Then, we check that those interior equilibria are stable by computing the derivative of the profit-gap with respect to n , and evaluating it around the said equilibrium: a negative sign indicates stability.

33. The second root is always higher than 1 which means that the CP equilibrium will exist for all values of the trade costs higher than ϕ^S .

34. Unless specified otherwise, all the calibrations use the following values of the parameters: $\rho = 0.3$, $\lambda = 0.3$, $L = 1$.

Step 4. We simulate the system made of w^{Stat} , (11) and (13) for $\phi \in [\phi^{SW}, 1]$ to get the upper tail of the upper branch of the bifurcation, i.e. the second regime.

Step 5. Finally, using (18) we calibrate the sustain point in the second regime, if it does exist. Then, the rest of the CP equilibrium is easily found using (17) and $\bar{s}_E = \alpha$.

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