Timed Unfoldings For Networks of Timed Automata

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Motivations

The general framework is this of **model-checking** :

Motivations

Our objective:

→ Apply partial order techniques to timed systems.

In this work:

- Partial order techniques = **unfoldings**
- Timed systems = **Networks of timed automata (NTA)**

Classical difficulties:

- merge time and concurrency
- handle urgency (invariants in TA)

Related Work

Partial order methods for NTA:

- ▶ Local semantics of time [Yi et al 98, Minea 99]
- ▶ Mazurkiewicz traces [Niebert et al 04, 06]

Unfoldings for time Petri nets:

- ▶ Unfolding of the underlying Petri net [Lilius 98]
- ▶ Proved to be “locally infeasible” [Aura & Lilius 00]
- ▶ Unfolding with discrete time [Fleishback & Steino 02]
- ▶ General case, introducing read arcs [Chatain & Jard 06].

- 1 Generalities
 - Networks of timed automata
 - Unfolding of discrete-event systems
- 2 Discrete Structure of our Unfolding
 - Clocks as shared variables
 - Invariants
 - An example
- 3 Adding Time
 - Timed non-branching process
 - A first timed unfolding
 - Computation via local zones
- 4 Finite and complete prefix
- 5 Conclusion

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Network of Timed Automata (NTA)

A NTA is a finite set of processes, defined as TA, together with a synchronization function. We allow:

- invariants,
- shared clocks,
- n -ary synchronizations.

Configuration = a vector of discrete locations + a clock valuation

Example:

$$f : \begin{cases} (a_1, \perp, \perp) \mapsto a_1 \\ (\perp, a_2, \perp) \mapsto a_2 \\ (\perp, \perp, a_3) \mapsto a_3 \\ (b?, b!, \perp) \mapsto b \end{cases}$$

Network of Timed Automata (NTA)

$$\begin{matrix} x \\ y \end{matrix} \mid \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

Example:

$$f : \begin{cases} (a_1, \perp, \perp) \mapsto a_1 \\ (\perp, a_2, \perp) \mapsto a_2 \\ (\perp, \perp, a_3) \mapsto a_3 \\ (b?, b!, \perp) \mapsto b \end{cases}$$

Network of Timed Automata (NTA)

$$\begin{matrix} x \\ y \end{matrix} \left| \begin{pmatrix} 0 \\ 0 \end{pmatrix} \xrightarrow{(1)} \begin{pmatrix} 1 \\ 1 \end{pmatrix} \right.$$

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Network of Timed Automata (NTA)

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Unfolding of discrete-event systems

Unfolding of a system = net describing its concurrent behaviors.

Usually **infinite**,

but: existence of a **finite** and **complete** prefix.

→ Well known for discrete-event systems such as Petri Nets, or networks of automata (McMillan, Esparza et al, ...)

→ we can decide reachability, transition enabling, deadlocks...

Remark1: Very efficient for safe Petri Nets

Remark2: Networks of Automata give safe Petri Nets!

→ it remains to handle **time** !

Example of the unfolding of a discrete system

$$f : \begin{cases} (a_1, \perp) \mapsto a_1 \\ (\perp, a_2) \mapsto a_2 \\ (\perp, a_3) \mapsto a_3 \\ (b?, b!) \mapsto b \end{cases}$$

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Example of the unfolding of a discrete system

Some definitions:

Causal relation $<$

if $p \in \bullet t$, then $p < t$

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subnet corresponding
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Nbp associated with an event

minimal causal past

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Cut of a nbp C

$(Min \cup C^\bullet) \setminus \bullet C$

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Clocks as shared variables

- ▶ Clocks induce dependencies betw. **a priori indep.** transitions
- ▶ Examples :

a and b are **dependent**

- ▶ Consider clocks as shared variables:
 - Test as reading
 - Reset as writing

Remark: related to [ICALP'06], [Srba], [Lugiez et al]

Clocks as shared variables

- ▶ Clocks induce dependencies betw. **a priori indep.** transitions
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a and b are **independent**

- ▶ Consider clocks as shared variables:
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Remark: related to [ICALP'06], [Srba], [Lugiez et al]

Test as reading

$$p \in t_1^\bullet \wedge p \in {}^\circ t_2 \Rightarrow t_1 < t_2$$

$$p \in {}^\circ t_2 \wedge p \in {}^\bullet t_3 \Rightarrow t_2 < t_3$$

→ Leads to another notion of unfolding: [Vogler et al, Winkovski]

- ▶ More difficult to define
- ▶ Increase significantly the concurrency relation

→

a and *b* are **concurrent**

Reset as writing

- ▶ a and b are **dependent**,
- ▶ there are two occurrences of the transition b .

Invariants

- ▶ An invariant of any process may affect the behavior of other processes:

- ▶ For every clock x appearing in an invariant, and for every event t , we require:

$$x \in {}^\circ t \cup {}^\bullet t$$

- ▶ This may have bad consequences on the causal relation.

Invariants (2)

- ▶ What happens if a transition modifies an invariant?

- ▶ considered as a writing on clock x .
- ▶ To a condition encoding a clock x corresponds informally a set of constraints satisfied by x .

Example

○ y ○ ℓ_0 ○ ℓ'_0 ○ x ○ ℓ''_0

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Timed Non-branching Process

We attach timing informations to nodes:

- For transitions:
 - ▶ d : date of firing
- For locations:
 - ▶ d_b : date of birth (= production)
 - ▶ d_e : date of end (= consumption)
- For clocks:
 - ▶ d_b : date of birth (= production)
 - ▶ d_e : date of end (= consumption)
 - ▶ d_r : date of last reset (to compute clock's value)

→ These are **absolute dates**

From timed sequences to timed nbp

A timed sequence $\nu = \nu' \cdot (d, a)$

We define dates as follows:

$$\mathbf{d}(t_1) = d$$

$$\mathbf{d}_e(p_1) = \mathbf{d}_e(p_3) = d$$

$$\mathbf{d}_b(p_4) = \mathbf{d}_b(p_5) = \mathbf{d}_r(p_5) = d$$

Feasible Timed Non-branching Process

for any clock x , let us denote:

$$v(x) = \mathbf{d}(t) - \mathbf{d}_r(p_x^-)$$

$$v'(x) = \mathbf{d}(t) - \mathbf{d}_r(p_x^+)$$

Causal (in)equations:

- $\forall p \in t^\bullet, \mathbf{d}_b(p) = \mathbf{d}(t)$
- $\forall p \in \bullet t, \mathbf{d}_e(p) = \mathbf{d}(t)$
- $\forall p \in {}^\circ t, \mathbf{d}_b(p) \leq \mathbf{d}(t) \leq \mathbf{d}_e(p)$
- $\forall p \in \text{Min}, \mathbf{d}_b(p) = \mathbf{d}_r(p) = 0$

Timed (in)equations:

- $g(t)[\{x \leftarrow v(x)\}_{x \in X}]$
- $\bigwedge_{\ell \in L(t)} \text{Inv}(\ell)[\{x \leftarrow v(x)\}_{x \in X}]$
- $\bigwedge_{x \in R(t)} v'(x) = 0$
- $\bigwedge_{x \in \text{Redefined}(t)} v'(x) = v(x)$

A first timed Unfolding

- ▶ A timed non-branching process is *feasible* iff it satisfies previous equations.

Proposition

- (i) *The timed nbp associated with a timed sequence is feasible,*
- (ii) *If a timed nbp is feasible, then it admits a timed sequence.*

- ▶ If we consider the previous equations as zones (symbolic representation for valuations), then we get:

Theorem

The resulting timed unfolding contains exactly all the timed sequences of the NTA : it defines a symbolic unfolding for NTA.

Computation via local zones

- ▶ Drawback of the previous unfolding: the size of zones increases while unfolding.
- ▶ Idea: keep only the part of the zone related to the cut
→ zones of bounded size ($2n + 3|X|$).
- ▶ Interest: sufficient to extend a nbp.
- ▶ Drawback: not sufficient to “merge” two nbps.

▷ Details

Computation via local zones (2)

- ▶ Using a topological sort, we compute local zones “slice by slice” (slice = set of concurrent events).
- ▶ We obtain a way to compute a timed unfolding of an NTA, in which zones attached to events are of bounded size.

Theorem

We can compute the timed unfolding $\mathcal{T}(\mathcal{A})$ with local zones $(Z_t)_t$ verifying:

- *t appears in $\mathcal{T}(\mathcal{A})$ iff there exists a timed sequence whose nbp is the one of t ,*
- *Z_t characterizes the set of values reachable by timed sequences along the nbp of t*

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Objective

- ▶ Limit the unfolding to a **finite prefix** which contains full information about the reachable states.
- ▶ In the untimed framework:

$$e \text{ cut-off} \iff \exists e' \prec e \text{ s.t. } \begin{cases} \lambda(e) = \lambda(e') \\ \lambda(\text{Cut}(e)) = \lambda(\text{Cut}(e')) \end{cases}$$

Then prune the subtree rooted in e .

- ▶ In the timed framework, we have to compare zones! (inclusion)

How to compare zones

- ▶ Compute **relativization** of zone Z_t w.r.t. date $\mathbf{d}(t)$ of t :
(operation : $\forall \mathbf{d}, \mathbf{d} := \mathbf{d}(t) - \mathbf{d}$)
- ▶ **But** these zones may be unbounded
→ does not ensure termination!
- ▶ **Favorable case:** “bounded” TA = TA such that :
 - clock values are bounded,
 - time elapsed in any location is bounded.→ finitely many zones $\Rightarrow$ Termination!
- ▶ **General case:** Enforce global synchronization to reinitialize the unfolding, and use extrapolation on “clock zones”.

Synchronized events

Define the clock zones $Test_t$ obtained from Z_t by performing:

- Relativization w.r.t. variable $\mathbf{d}(t)$
- Projection on variables $\mathbf{d}_r(.)$
- Extrapolation

Synchronized events: (chosen among unavoidable edges)

Perform global synchronization, and reinitialize the unfolding (consume all places of the cut, and produce them back)

Lemma (Forgettable Past)

Let $t \in SE$.

It is equivalent to extend the nbp of t and to build a nbp from $Cut([t])$ which initially satisfies $Test_t$.

Algorithm for finite prefix

- ▶ We consider an adequate order $\prec$ on SE (see [Esparza et al])
- ▶ Cut-off event :

$$e \in SE \text{ cut-off} \iff \exists e' \prec e \text{ s.t. } \begin{cases} \lambda(e) = \lambda(e') \\ \lambda(\text{Cut}(e)) = \lambda(\text{Cut}(e')) \\ \text{Test}_e \subseteq \text{Test}_{e'} \end{cases}$$

Theorem

This algorithm terminates and produces a finite and complete prefix.

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Conclusion

- ▶ Definition of a timed unfolding for NTA
- ▶ Computation using “local zones”
- ▶ Construction of a finite and complete prefix

Further work:

- ▶ Reduce dependencies due to invariants
- ▶ Reduce the number of variables ($\mathbf{d_b}$, $\mathbf{d_e}$, $\mathbf{d_r}$)
- ▶ Try to avoid synchronized edges
- ▶ Extend the set of properties we can verify
- ▶ Implementation

A last remark

There were two papers at ATVA'06:

- “Timed Unfoldings For Networks of Timed Automata” (BHR)
- “Symbolic Unfoldings For Networks of Timed Automata” (Cassez, Chatain, Jard)

The two contributions are close but there are important differences:

- they propose a way to define the unfolding, but do not consider its “efficient” construction,
- this allows them to define a “smaller” object,
- they make some stronger assumptions on the system (no deadlocks, no shared clocks).

→ We are currently trying to bring closer the two approaches.

Thanks for your attention!

Extend the unfolding

Firing e_1 implies $\mathbf{d}(e') < 1$

Firing e_2 implies $\mathbf{d}(e') > 1$

But e' does not appear in $\text{Cut}(e_1) \cup \text{Cut}(e_2)$!

→ Taking $Z_{e_1} \wedge Z_{e_2}$ is not enough precise...

Computing by slices

- 1 Compute the starting event e'
- 2 Sort by slices (sets of concurrent events) the elements of $([e_1] \cup [e_2]) \setminus [e']$
- 3 Extend $Z_{e'}$, slice by slice.

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