COMPLEMENTARITY AND SUBSTITUABILITY IN MULTIPLE-RISK INSURANCE MARKETS*

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We study imperfect competition between insurers in a multiple-risk environment. In the absence of asymmetric information, equilibria are efficient, and we determine the degrees of specialization under which the specialized insurers are able or unable to capture the surplus. We show in contrast that under adverse selection, specialization systematically prevents second-best efficiency. Concluding on the role of our notions of strategic complementarity/substituability on the tradeoff between efficiency and fairness of the allocation, we give indications on the desirable structure of the insurance industry.

1. INTRODUCTION

It is generally assumed in standard insurance modeling that consumers are covered by a *unique grand contract* purchased at one company only. This is not an accurate image of reality. In sectors like life insurance, exclusivity clauses are rarely implementable, and customers can easily bypass certain forms of nonlinear pricing (which is the most efficient way to mitigate adverse selection or to extract monopolistic rents) by underwriting several contracts conditioned on the same set of events. Comprehensive contracts are not observed either; in practice, insurance firms are specialized, for legal or strategic reasons, and even when they are not, they tend to offer contracts for different risks separately rather than real bundles. Whatever the prevailing form of competition, this results *a priori* in a loss in the strength of self-selection mechanisms.

In the adverse-selection setting of Rothschild and Stiglitz (1976), equilibria without exclusivity have been studied by Jaynes (1978) and Hellwig (1988). Jaynes showed in particular that there is no pure-strategy equilibrium without additional restrictions and that existence may be restored when a form of communication between firms is allowed.

In this article we consider the fact that insurers are specialized. In principle, the total monetary risk borne by the agent can always be broken up into independently

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insurable parts (housing, health, automobile, liability, etc.). In this view, elementary risks have not to be statistically independent; all that is needed are standard types of information (of lower quality than the full description of the state of the world but easily observable) on which it is agreed to condition insurance contracts.² Owing to the simplicity (in terms of indemnity rules) of such insurance contracts, design costs are spared. As for the agents, they may prefer avoiding the complexity of searching for and comparing comprehensive contracts.

Gollier and Schlesinger (1995) propose a model where insurance markets are perfectly competitive, but each insurer is specialist of one of two risks. Insurance contracts have to take into account administrative costs, as in Arrow's model. Their main results are based on the efficiency loss due to the fact that optimal contracts in that context being nonlinear, specialized contracts cannot duplicate the optimal comprehensive policy.

In our model, the sources of risk are independent, and a monopoly operates in each branch. In an industry where intellectual property rights are not protected and imitation is quite easy, one can doubt whether monopolistic power can be maintained over time. Our model suggests an explanation for durable specialization based on strategic considerations. Starting from a stylized form of specialization, we conclude that it can be seen, when adequately designed *ex ante*, as an implicit way to sustain collusion between insurers. Intuitively, specialization limits the conflict between competitors, and the role it plays in the distribution of the surplus induces the insurance industry to enforce certain forms of specialization rather than others.

The common customer sets up a link between firms that are not directly competitors. In some respects, the model is a variety of common agency models à la Bernheim and Whinston (1986). Contrary to Martimort (1992), we do not assume that agents are forced to contract with all the principals or none. It appears in fact that the threat that a consumer rejects one offer may be strong enough for him or her to extract part of the surplus. Depending on the degree of conflict between insurers, we show that the consumer may benefit from the specialization in the insurance industry even when risks are independent and that each monopoly makes less profit than if he or she were the only insurer in the market. Conversely, when the interests of the insurers are aligned, the agent does not benefit from the game, and each monopoly takes advantage of the other's presence.³

Existence, multiplicity, efficiency of the equilibria, and the distribution of welfare are also investigated. Participation constraints being endogenous, the principal

² The literature on optimal insurance with background risk shows to what extent unobserved losses can be insured provided they are correlated, in a broad sense, with some observable. See Doherty and Schlesinger (1983).

³ Laussel and Le Breton (1996, henceforth LLB) present a model inspired of Bernheim's and Whinston's (1986) common agency notion. The analogy between LLB's results and ours comes from their characterization of the different equilibrium regimes based on properties of the nonadditive measure that maps all possible coalitions of the principals into the set of the maximal sum of payoffs. There are two differences, however. The first is that LLB use quasi-linear utility, whereas we insist on risk aversion. The second is more substantive: In LLB, the actions of the individual are perfectly observable and can be used in contracts, whereas in our model, "principals" cannot observe all the the "agent's" actions (i.e., choice of insurance coverage for the other risks).

analytic difficulty comes from the discontinuous nature of the payoffs of the game played between the insurers. We propose a methodology, different from the standard fixed-point arguments, that proves the existence and provides a characterization of pure-strategy equilibria.

Section 2 sets up the model, and Section 3 introduces an auxiliary game serving for a first selection of candidate equilibria. Section 4 gives the principal results: Existence is proved, and the relationship between complementarity/substituability and the configuration of the exogenous parameters is discussed. Section 5 extends the analysis to adverse selection. Section 6 concludes.

2. THE MODEL

A risk-averse consumer endowed with wealth w faces two independent risks $\tilde{x}$ and $\tilde{y}$; u denotes his or her continuous, increasing, and strictly concave Von Neumann–Morgenstern utility function. The total wealth of the consumer that remains uninsured is the random variable $w + \tilde{x} + \tilde{y}$. Without loss of generality, we shall assume that $E\tilde{x} = E\tilde{y} = 0$.

The two active insurers are specialized in the sense that proposed reimbursement policies can only be conditional on the realization of the loss of which the insurer is a specialist. Monopolies are risk-neutral, and they maximize their expected profits. Monopoly X 's offer (or menu) is denoted by $\mathcal{O}_X \equiv [\pi_X^k, I_X^k(\tilde{x})]_{k \in K}$, where π_X^k is an insurance premium and $I_X^k(\tilde{x})$ is the indemnity paid when the realization of the loss is $\tilde{x}$ (K is a sufficiently large set for indexing all offers); offers with different indices may be identical; to avoid identification problems, we assume throughout that $E I_X^k(\tilde{x}) = 0$, the premium being directly interpretable as the profit. We allow stochastic contracts (i.e., the reimbursement can be a stochastic function of the observed loss); this will play a role in the adverse-selection case only (Section 5). Monopoly Y 's offer $\mathcal{O}_Y$ is similarly defined. The noncooperative game between the insurers (via the agent's choice) runs as follows:

In the first step, the two monopolies make simultaneous firm take-it-or-leave-it offers of contracts. Offers cannot depend on the offer of the opponent or on the action of the agent concerning insurance of the other risk; in particular, offers cannot be withdrawn or added on observation of the other insurer's move.

In the second step, the agent's problem is to make the best choice. He or she chooses either to purchase one contract from each insurer, or to reject them all, or to purchase one contract from one insurer only; what the agent does when he or she is indifferent, at the optimum, between several of these choices, is endogenous. The utility derived from his or her choice is therefore

$$(1) \quad \sup_{k_X, k_Y \in K} \left\{ Eu \left[w - \pi_X^{k_X} - \pi_Y^{k_Y} + \tilde{x} + I_X^{k_X}(\tilde{x}) + \tilde{y} + I_Y^{k_Y}(\tilde{y}) \right], Eu(w + \tilde{x} + \tilde{y}), \right. \\ \left. Eu \left[w - \pi_X^{k_X} + \tilde{x} + I_X^{k_X}(\tilde{x}) + \tilde{y} \right], Eu \left[w - \pi_Y^{k_Y} + \tilde{x} + \tilde{y} + I_Y^{k_Y}(\tilde{y}) \right] \right\}$$

We look at the Nash equilibria of this game. Assuming that the agent does not play strategically amounts to examining sequential equilibria only of a game where

he or she would be a strategic player. A natural justification of this mild restriction is that each individual being a negligible fraction of the insurers' clientele, he or she is not in a position to influence the monopolies' decisions with a nonperfect strategy.

3. PAYOFF STRUCTURE: PROBLEMS AND SOLUTION

3.1. *Ill-Defined Best Responses.* The profits of the monopolies (taking into account the customer's choice) are denoted by $P_X(\mathcal{O}_X, \mathcal{O}_Y)$ and $P_Y(\mathcal{O}_X, \mathcal{O}_Y)$. Unfortunately, they are not upper semicontinuous (u.s.c.) everywhere,⁴ which raises the problem of the definition of the best responses. To see this, note that in the case of one offer per menu in pure strategies, monopoly X 's offer solves

$$(2) \quad \max_{\pi_X, I_X(\tilde{x})} \pi_X - EI_X(\tilde{x})$$

under the rationality constraint

$$(3) \quad \begin{aligned} &\sup \{Eu[w - \pi_X + \tilde{x} + I_X(\tilde{x}) + \tilde{y}], \\ &\quad Eu[w - \pi_X - \pi_Y + \tilde{x} + I_X(\tilde{x}) + \tilde{y} + I_Y(\tilde{y})]\} \\ &\geq \sup \{Eu[w + \tilde{x} + \tilde{y}], Eu[w - \pi_Y + \tilde{x} + \tilde{y} + I_Y(\tilde{y})]\} \end{aligned}$$

Monopoly Y 's program is similar. Clearly, profit maximization amounts to making the agent exactly indifferent between accepting the offer and contenting himself or herself with the best outside opportunity. In the conventional case of the pure monopoly, this difficulty is easily solved: The insurer could approximate the optimal profit as close as he or she wants and ensure participation with certainty. Rigorously, this reasoning consists of weakening the equilibrium concept and examining the convergence of the sequence of ϵ equilibria when ϵ goes to 0. Fortunately, the limit of these ϵ equilibria is also a Nash equilibrium.

As long as the discontinuity of payoffs is only due to a simple participation constraint, applying this method is sufficient. In our model, the agent's participation constraint imposed on each monopoly is endogenous, and confronting ϵ -best responses is not simpler than tackling the initial problem. In the systematic analysis of the equilibria, we will meet cases where, for example, it may be optimal for the agent either to accept $\mathcal{O}_X$ and not $\mathcal{O}_Y$ or to accept $\mathcal{O}_Y$ and not $\mathcal{O}_X$, whereas it is not optimal to accept or reject both. In this case, insurers cannot both consistently assume that their offers will be chosen with certainty: Either one or the other will be taken, and at least one of the optimal offers is ill-defined.⁵

Payoffs not being u.s.c., using the simplest existence theorems is impossible. As long as solely existence proofs are sought, Simon and Zame (1990) propose a powerful

⁴ For continuity to be defined, we assume here that the space of random variables is endowed with a Banach space structure; it implies that the space of offers is endowed with the same structure.

⁵ Discontinuity of profit at one's best action is inherent to duopolistic competition (e.g., the location game, or the Bertrand-Edgeworth game with capacity constraints). Here the nature of our problem gives a particular structure to the set where payoffs are discontinuous.

method. The fact that what consumers do when they are indifferent between several choices is endogenous typically suffices to prove existence in full generality. However, another line of argument should be found to prove that equilibria in *pure strategies* exist.

3.2. The Auxiliary Game. With the help of an auxiliary game, we obtain certain simplifications that will prove instrumental in the determination of the pure-strategy market equilibria. The profit functions are first slightly changed: For all $\mathcal{O}_Y$, $\bar{P}_X(\cdot, \mathcal{O}_Y)$ will be the u.s.c. envelope of $P_X(\cdot, \mathcal{O}_Y)$

$$(4) \quad \bar{P}_X(\mathcal{O}_X, \mathcal{O}_Y) = \limsup_{\mathcal{O} \rightarrow \mathcal{O}_X} P_X(\mathcal{O}, \mathcal{O}_Y)$$

Similar notations are used for monopoly Y . The two functions P_X and $\bar{P}_X$ can differ only for pairs of menus involving at least two optimal choices; this difference, however, matters very much because, given the structure of optimal strategies, a series of indifferences is systematically involved in equilibrium. These transformed profits define an auxiliary game (hereafter the AG) where insurers' optimal actions are well defined everywhere. The proposition that follows links the pure strategy Nash equilibria of the initial game to the AGs:

PROPOSITION 1. $(\mathcal{O}_X^*, \mathcal{O}_Y^*)$ is a Nash equilibrium of the initial game if and only if

- (i) $(\mathcal{O}_X^*, \mathcal{O}_Y^*)$ is a Nash equilibrium of the AG.
- (ii) $P_X(\mathcal{O}_X^*, \mathcal{O}_Y^*) = \bar{P}_X(\mathcal{O}_X^*, \mathcal{O}_Y^*)$ and $P_Y(\mathcal{O}_X^*, \mathcal{O}_Y^*) = \bar{P}_Y(\mathcal{O}_X^*, \mathcal{O}_Y^*)$.

The proof is immediate. The first point is convenient because, by definition, the AG does not require any hypothesis on the agent's selection of the optimal choice when he or she is indifferent. The second point exhibits insightful features: It requires that the best responses of the initial game be both well defined; in other words, it imposes that real profits be consistent *in equilibrium* with each insurer's (optimistic) expectation, in the AG, according to which he or she will be selected if his or her offer is optimal.

3.3. Equilibria of the Auxiliary Game. The well-known result that equilibrium insurance contracts offer full coverage is preserved in this context. The strategy spaces are initially very large because menus may contain an arbitrary number of totally useless offers. The following proposition clarifies the properties that really matter: *essentially* means that an offer that cannot be removed from a menu without affecting the equilibrium has to satisfy the property.

PROPOSITION 2. *In the AG*

- (i) *The support of the optimal strategies is essentially a set of contracts such that $I_X(\tilde{x}) = -\tilde{x}$ and $I_Y(\tilde{y}) = -\tilde{y}$ (full insurance).*
- (ii) *The optimal response to a pure strategy is essentially a unique pure strategy.*

PROOF. The proof of (i) is easy, and the details are left to the reader; basically, any “optimal” offer that is not a complete insurance can always be improved to maximize profit without ceasing to be attractive to the consumer. The second point is a consequence of the first: In pure-strategy equilibria, two optimal full insurance offers cannot be different. $\square$

We search for the pure-strategy equilibria of the AG. Proposition 2 provides a substantial simplification because we can reason in the sequel on the remaining strategic variables π_X and π_Y . To be more precise, the best-response correspondence (of which we search for a fixed-point) mapping *essentially* the initial strategy space into the set of full insurance contracts, this simplification is innocuous for the determination of the equilibrium. Menus will make a reappearance in the section on adverse selection.

For each pair (π_X, π_Y) , the agent has four possible choices, each being symbolized by a mnemonic letter. He or she can indeed choose to take no contract (move N), monopoly X ’s contract only (move X), monopoly Y ’s contract only (move Y), or both monopoly X ’s and monopoly Y ’s offers (move B). We denote by $\preceq$ the weak order of preferences of the agent over the set of moves $\{B, X, Y, N\}$ when monopolies make offers π_X and π_Y ($\approx$ denotes indifference).

Comparing the four possible choices of the agent in pairs, we obtain six indifference curves in the plane (π_X, π_Y) , where, for example, C_{BN} is the set of offers such that move B and move N are equivalent.

$$(5) \quad C_{BN} = \{(\pi_X, \pi_Y) | Eu(w - \pi_X - \pi_Y) = Eu(w + \tilde{x} + \tilde{y})\}$$

$$(6) \quad C_{XN} = \{(\pi_X, \pi_Y) | Eu(w - \pi_X + \tilde{y}) = Eu(w + \tilde{x} + \tilde{y})\}$$

$$(7) \quad C_{YN} = \{(\pi_X, \pi_Y) | Eu(w + \tilde{x} - \pi_Y) = Eu(w + \tilde{x} + \tilde{y})\}$$

$$(8) \quad C_{XB} = \{(\pi_X, \pi_Y) | Eu(w - \pi_X + \tilde{y}) = Eu(w - \pi_X - \pi_Y)\}$$

$$(9) \quad C_{YB} = \{(\pi_X, \pi_Y) | Eu(w + \tilde{x} - \pi_Y) = Eu(w - \pi_X - \pi_Y)\}$$

$$(10) \quad C_{XY} = \{(\pi_X, \pi_Y) | Eu(w - \pi_X + \tilde{y}) = Eu(w + \tilde{x} - \pi_Y)\}$$

These curves depend on the agent’s characteristics only (his or her utility function and the risks he or she incurs). They share the plane (π_X, π_Y) in parts where his or her preferences are known. Due to the fact that preferences are transitive, they cross three by three (see Figures 1 and 2). They include (but are strictly larger than) the best response curves of the monopolies. For any expected π_Y , insurer X ’s best response is denoted $\hat{\pi}_X(\pi_Y)$; $\hat{\pi}_Y(\pi_X)$ is defined similarly.

Now we give our first existence proposition.

PROPOSITION 3. *The AG has an equilibrium in pure strategies.*

PROOF. Applying the theorem of the maximum to monopoly X ’s program shows that $\hat{\pi}_X(\pi_Y)$ is a well-defined continuous function. Indeed, for a fixed π_Y , $\hat{\pi}_X(\pi_Y)$ is a solution of

$$(11) \quad \max \pi_X$$

under the constraint (continuous in π_Y)

$$(12) \quad \sup\{X; B\} \geq \sup\{N; Y\}$$

By symmetry, $\hat{\pi}_Y$ is also continuous with respect to π_X .

The best-response curves must cross at least once: $\hat{\pi}_X$, which is defined over $\mathbb{R}$, and continuous, takes its values in a compact set included in $\mathbb{R}$. Indeed, zero is a lower bound (profits must be nonnegative), and the risk premium in case the agent rejects monopoly Y 's proposal and the risk premium in case it is accepted are both bounded. The same reasoning applies to $\hat{\pi}_Y$. This ends the proof. $\square$

The following proposition provides us with indications on the constraints that are binding in a Nash equilibrium of AG. The AG has two types of equilibria: Either (1) $B \approx N$, and the agent is indifferent between purchasing both contracts and remaining uninsured, or (2) $X \approx Y$, and he or she is indifferent between contracts:

PROPOSITION 4. *The three following assertions are equivalent:*

- (i) (π_X, π_Y) is a Nash equilibrium of the AG.
- (ii) $(\pi_X, \pi_Y) \in C_{XY} \cup C_{BN}$ and $\pi_X = \hat{\pi}_X(\pi_Y)$.
- (iii) $(\pi_X, \pi_Y) \in C_{XY} \cup C_{BN}$ and $\pi_Y = \hat{\pi}_Y(\pi_X)$.

PROOF. We first prove that (i) $\Rightarrow$ (ii). If (π_X, π_Y) is a Nash equilibrium of the AG, then $\pi_X = \hat{\pi}_X(\pi_Y)$. In addition, monopoly X 's profit maximization implies that constraint (12) binds. Therefore, either $X \approx N$, or $X \approx Y$, or $B \approx N$, or $B \approx Y$. An analogous result is obtained exchanging X and Y . In all these cases, directly or by transitivity, it appears that $X \approx Y$ or $B \approx N$.

We prove now that (ii) $\Rightarrow$ (i). It is sufficient to show that if $(\pi_X, \pi_Y) \in C_{XY} \cup C_{BN}$ (i.e., if $X \approx Y$ or $B \approx N$) and $\pi_X = \hat{\pi}_X(\pi_Y)$, then $\pi_Y = \hat{\pi}_Y(\pi_X)$. From $\pi_X = \hat{\pi}_X(\pi_Y)$, we have $\sup\{X, B\} \approx \sup\{N, Y\}$. We reason by contradiction. If $\pi_Y \neq \hat{\pi}_Y(\pi_X)$, then either $\sup\{Y, B\} > \sup\{N, X\}$, and monopoly Y charges too low a price, or $\sup\{Y, B\} < \sup\{N, X\}$, and monopoly Y charges too high a price. Whether $X \approx Y$ or $B \approx N$, exchanging the two equivalent choices in the last two strict inequalities yields $\sup\{X, B\} \not\approx \sup\{N, Y\}$, a contradiction.

We can prove in a similar way that (i) $\Leftrightarrow$ (iii). $\square$

4. COMPLEMENTARITY, SUBSTITUABILITY, AND EQUILIBRIUM CONSEQUENCES

4.1. *Equilibrium Participation of the Agent.* We search now for equilibria of the AG that pass the test of condition (ii) in Proposition 1, i.e., which are equilibria of the initial game.

PROPOSITION 5. *In an equilibrium of the initial game, the agent purchases both contracts, although he or she always has alternative optimal possibilities.*

PROOF. We reason by contradiction: Assume that (π_X, π_Y) is an equilibrium of the AG and that the agent acts in such a way that monopoly X makes a profit π_X

with probability $p < 1$ and else 0 (if $p = 1$, then replace X by Y). By undercutting π_X by a rate of ϵ ($0 < \epsilon < 1$), monopoly X obtains participation with certainty, and his or her expected profit becomes $(1 - \epsilon)\pi_X$. If $\epsilon < 1 - p$, the profit is increased, and the equilibrium is broken. $\square$

In consequence, an equilibrium of the initial game that satisfies $X \approx Y$ but not $B \approx N$ is possible only if B is as good as X and Y and strictly preferred to N ($X \approx Y \approx B \succ N$). It must be stressed that it is *part of the equilibrium* that B is played by the agent. The proposition gives exactly the actions of the agent that are consistent with the existence of equilibria of the initial game in pure strategies: The agent must be expected to buy both contracts in the corresponding equilibrium of the AG, whereas his or her off-equilibrium strategy is irrelevant.

4.2. *The Two Competitive Situations.* Three parameters are of capital importance for the intuitions of the conflict between the insurers and for discriminating the different types of equilibria. We will denote them $\bar{\pi}_X$, $\bar{\pi}_Y$, and $\bar{\pi}_B$, where

$$(13) \quad Eu(w - \bar{\pi}_X + \tilde{y}) = Eu(w + \tilde{x} - \bar{\pi}_Y) = Eu(w - \bar{\pi}_B) = Eu(w + \tilde{x} + \tilde{y})$$

They are the upper bounds of the premia that, respectively, monopoly X alone, monopoly Y alone, and both insurers together can extract from the agent. Risk aversion implies that $\bar{\pi}_X < \bar{\pi}_B$ and $\bar{\pi}_Y < \bar{\pi}_B$ for nontrivial risks. Note that $C_{XN} = \{(\pi_X, \pi_Y) \mid \pi_X = \bar{\pi}_X\}$, $C_{YN} = \{(\pi_X, \pi_Y) \mid \pi_Y = \bar{\pi}_Y\}$, and $C_{BN} = \{(\pi_X, \pi_Y) \mid \pi_X + \pi_Y = \bar{\pi}_B\}$ (these curves are straight lines). As for the other three curves (see above), the main general properties are the following: In the plane (π_X, π_Y) , C_{XY} is increasing; if u is DARA (decreasing absolute risk aversion), then C_{XB} and C_{YB} are nondecreasing.⁶

If $\bar{\pi}_X + \bar{\pi}_Y \leq \bar{\pi}_B$, there is obviously room for cooperation between insurers for realizing a complete capture of the surplus. Conversely, if $\bar{\pi}_X + \bar{\pi}_Y \geq \bar{\pi}_B$, whatever the sharing of the surplus when insurers cooperate, at least one insurer is worse off than if he or she had been the only insurer (though specialized) in the market. The following definition plays a major role:

DEFINITION 1. *Risk aversion with respect to $\tilde{x}$ and $\tilde{y}$ exhibits complementarity (respectively, substitutability) if and only if $\bar{\pi}_X + \bar{\pi}_Y < \bar{\pi}_B$ (respectively, $\bar{\pi}_X + \bar{\pi}_Y > \bar{\pi}_B$). We also will say that firms experience strategic complementarity (respectively, substitutability).*

To figure out the geometric equivalent of the two competitive situations, see Figure 1, where $\bar{\pi}_X + \bar{\pi}_Y < \bar{\pi}_B$, and Figure 2, where $\bar{\pi}_X + \bar{\pi}_Y > \bar{\pi}_B$. The corresponding best-response curves are also given. In the case where $\bar{\pi}_X + \bar{\pi}_Y < \bar{\pi}_B$, they are not monotonic.

⁶ The first property comes from the monotonicity of the utility function; the second from the preservation of DARA under expectation in the presence of a background risk (Kihlstrom et al. 1981).

FIGURE 1

INDIFFERENCE CURVES, BEST RESPONSES, AND EQUILIBRIA WHEN $\bar{\pi}_X + \bar{\pi}_Y < \bar{\pi}_B$.

The CARA case is atypical. Then, $C_{XN} = C_{YB}$, whereas $C_{YN} = C_{XB}$. Straight lines C_{BN} and C_{XY} are, respectively, of slopes -1 and 1 . Monopoly X 's best response (respectively, monopoly Y 's) is constant and equal to $\bar{\pi}_X$ (respectively, $\bar{\pi}_Y$). The equilibrium is necessarily unique and reached for these values. The allocation is Pareto optimal, all the risks are exchanged, and the surplus is captured by insurers. This case of strategic independence strongly relies on the specification of the utility function, for which $\bar{\pi}_X + \bar{\pi}_Y = \bar{\pi}_B$.

4.3. Main Theorem

THEOREM 1. Let $\mathcal{E}_1 = \{(\pi_X, \pi_Y) \mid \pi_X + \pi_Y = \bar{\pi}_B; \pi_X \geq \bar{\pi}_X; \pi_Y \geq \bar{\pi}_Y\}$ and $\mathcal{E}_2 = \{(\pi_X, \pi_Y) \mid B \approx X \approx Y \succeq N; \pi_X \geq 0; \pi_Y \geq 0\}$.

- (i) The Nash equilibria of the game are exactly supported by $\mathcal{E}_1 \cup \mathcal{E}_2$.
- (ii) Risk aversion exhibits complementarity if and only if $\mathcal{E}_1 \neq \emptyset$.
- (iii) If risk aversion exhibits substitutability, then $\mathcal{E}_2 \neq \emptyset$.
- (iv) $\mathcal{E}_1$, if not empty, is a continuum, whereas $\mathcal{E}_2$, if not empty, is generically not a continuum.

PROOF. (i) At every point of $\mathcal{E}_1$, the two insurers bind their constraints and are making optimal offers (indeed, such a point satisfies $B \approx N$, $X \preceq B$ and $Y \preceq B$). Every point of $\mathcal{E}_2$ is an equilibrium because it satisfies $B \approx X \approx Y \succeq N$; no profitable deviation is possible. The converse stems immediately from Proposition 4 and from the fact that B must be optimal in an equilibrium.

- (ii) Obvious.

FIGURE 2

INDIFFERENCE CURVES, BEST RESPONSES AND EQUILIBRIA WHEN $\bar{\pi}_X + \bar{\pi}_Y > \bar{\pi}_B$.

(iii) Let us prove that if $\bar{\pi}_X + \bar{\pi}_Y > \bar{\pi}_B$, then $\mathcal{E}_2$ is nonempty [in case of equality, $(\bar{\pi}_X, \bar{\pi}_Y)$ is obviously an equilibrium]. Let (π_X, π_Y) be an equilibrium of the AG, the existence of which coming from Proposition 3. We first prove that, necessarily, $X \approx Y$. If for example, $X < Y$, from Proposition 4, we must have $B \approx N$. But at the equilibrium $\sup\{X, B\} \approx \sup\{Y, N\}$, which implies $Y \leq N$, i.e., $\pi_Y \geq \bar{\pi}_Y$. Since $B \approx N$, $\bar{\pi}_X + \bar{\pi}_Y > \bar{\pi}_B = \pi_X + \pi_Y$. In consequence, $\pi_X < \bar{\pi}_X$, i.e., $X > N$, a contradiction. Therefore, $X \approx Y$.

The aversion toward risk of the agent ensures that monopoly X 's best response when $\pi_Y = 0$ lies in C_{YB} . Moreover, the existence of an equilibrium of the AG in which $X \approx Y$ ensures that at least one best response of monopoly X lies in C_{XY} . The indifference curves and monopoly X 's best-response curve being continuous, there must exist one best response of monopoly X lying in $C_{XY} \cap C_{YB}$ ($X \approx Y \approx B$). From the transitivity of the preferences, this point also corresponds to monopoly Y 's best response and therefore lies in $\mathcal{E}_2$.

(iv) See Appendix. □

The elements of $\mathcal{E}_1$ will be called *type 1 equilibria*, and the elements of $\mathcal{E}_2$ will be called *type 2 equilibria*. At all these points, B is an optimal choice and must be taken as the actual action of the agent (Proposition 5). Figures 1 and 2 provide an illustration. Notice that we do not exclude that the two types coexist for certain parameters when $\bar{\pi}_X + \bar{\pi}_Y < \bar{\pi}_B$.

As argued before Definition 1, the determinants of the sharing of the surplus can be anticipated from the comparison of $\bar{\pi}_X + \bar{\pi}_Y$ with $\bar{\pi}_B$. Our statements, however, are more precise now: When “collusion” is attractive for both insurers (complemen-

tarity), then the total extraction of the surplus by the insurers is *actually* made for certain *noncooperative* equilibria (each monopoly benefits from the presence of the other); when we can anticipate that at least one insurer will lose from the presence of the other (substituability), in fact, both monopolies lose.

Type 1 equilibria correspond to the case of complementary risk aversion. The set of equilibria is connected and compact. These equilibria are all equivalent for the individual (his or her participation constraint is binding), but they are ordered in opposite orders by the insurers. The difference between equilibria only relies on the sharing of the surplus between monopolies. Interestingly, the sharing cannot be too unequal for the stability of the equilibrium: The profit made by each insurer must exceed the profit he or she would make alone. In other words, monopoly X gets, at least, $\bar{\pi}_X$, and monopoly Y , at least, $\bar{\pi}_Y$. The only “free” surplus that can be shared without constraint is $\bar{\pi}_B - \bar{\pi}_X - \bar{\pi}_Y \geq 0$.

In contrast, the type 2 equilibria correspond to the case of substitutable risk aversion. Generically, these equilibria are separated (an equilibrium is in general not arbitrarily close to an other equilibrium). They are identically ordered by the insurers (their profits either increase or decrease both from one equilibrium to another) whereas, quite obviously, the consumer’s utility decreases when profits increase. They all leave a strictly positive rent to the agent.

4.4. Theoretical Support and Empirical Relevance. Whether monopolistic insurance companies experience strategic complementarity or substitutability depends on the consumer’s utility function: Complementarity (respectively, substitutability) means that insurance for one risk increases (respectively, decreases) his or her willingness to pay for insurance of the other. It would be natural to search for characterizations of u that predict the market regime. At first sight, we only have to borrow the adequate criterion from the huge research effort, initiated by Kihlstrom et al. (1981) and Doherty and Schlesinger (1983), that has been devoted to the comparison of the willingness to pay (or willingness to accept) for one risk in the presence and in the absence of a background risk. Unfortunately, refinements of risk aversion such as proper risk aversion of Pratt and Zeckhauser (1987) and risk vulnerability of Gollier and Pratt (1996) cannot be exploited in our context in a powerful way.⁷

In the background-risk-oriented literature, one compares risk aversions toward risk $\tilde{x}$ with and without uninsured risk $\tilde{y}$; in our model, we compare risk aversions toward risk $\tilde{x}$ with uninsured risk $\tilde{y}$ and with risk $\tilde{y}$ exchanged for its certainty equivalent

⁷ A utility function u exhibits proper risk aversion if and only if any undesirable risk remains undesirable when the agent is submitted to an additional independent undesirable risk:

$$(14) \quad \forall w, \tilde{x}, \tilde{y}: Eu(w + \tilde{x}) \leq u(w) \quad \text{and} \\ Eu(w + \tilde{y}) \leq u(w) \Rightarrow Eu(w + \tilde{x} + \tilde{y}) \leq Eu(w + \tilde{x})$$

A utility function is risk vulnerable if and only if adding an unfair background risk makes risk-averse agents behave in a more risk-averse way:

$$(15) \quad \forall w, \tilde{x}: E\tilde{x} \leq 0 \Rightarrow -\frac{Eu''(w + \tilde{x})}{Eu'(w + \tilde{x})} \geq -\frac{u''(w)}{u'(w)}$$

value. To see why the two experiments are not comparable, note that using the definitions of $\bar{\pi}_X$ and $\bar{\pi}_B$, we get

$$(16) \quad Eu(w - \bar{\pi}_X + \tilde{y}) = u[w - \bar{\pi}_X - (\bar{\pi}_B - \bar{\pi}_X)]$$

$\bar{\pi}_B - \bar{\pi}_X$ is therefore the risk premium of $\tilde{y}$ for the utility function $u(w - \bar{\pi}_X + \cdot)$. Under DARA (respectively IARA), we get $\bar{\pi}_B - \bar{\pi}_X \geq$ (respectively, $\leq$) π_Y , where π_Y is the ordinary equivalent risk premium defined by $u(w - \pi_Y) = Eu(w + \tilde{y})$. The comparison between $\bar{\pi}_Y$ and π_Y is also easy: Under risk vulnerability, $\bar{\pi}_Y \geq \pi_Y$. The difficulty is now apparent: We can compare $\bar{\pi}_X + \bar{\pi}_Y$ with $\bar{\pi}_B$ using these results only in the case where the utility is risk vulnerable *and* IARA. Risk vulnerability implying DARA, these two conditions are met for CARA utility functions only. In the other cases, the ambiguity remains. In short, vulnerability effects and wealth effects draw in opposite directions.

In addition to the fact that the refinements of risk aversion above (generally seen as very likely) are theoretically compatible with complementarity as well as with substitutability,⁸ there is some experimental evidence that neither case is systematically in force. Indeed, the comparison of $\bar{\pi}_X + \bar{\pi}_Y$ to $\bar{\pi}_B$ is reminiscent of Schoemaker and Kunreuther (1979). The experiment consisted of submitting volunteers to several imaginary risks. They were asked to evaluate how much they would be willing to pay for complete insurance of their global risk first for the comprehensive contract and second by giving in detail the premium for each contract covering one of the risks. In general, the two answers differed. This difference is paradoxical at first sight (people do not realize they give different values to the same thing) but could be explained by a misinterpretation of the second question by the respondents. It is likely that part of them reasoned as if they were asked to give the value of insurance of *one risk at once*. If our interpretation is correct, the results of the experiment could be written in our terms as follows: For about half the people, $\bar{\pi}_X + \bar{\pi}_Y > \bar{\pi}_B$; for about a quarter, $\bar{\pi}_X + \bar{\pi}_Y = \bar{\pi}_B$; and for the remaining quarter, $\bar{\pi}_X + \bar{\pi}_Y < \bar{\pi}_B$.

5. ADVERSE SELECTION

In the preceding section we were able to establish a classification of the equilibria based on the structure of the insured's risk premia. One of the important—and quite unexpected—conclusions was that splitting a monopoly into two firms does not necessarily drive the overall profit of the industry down. However, this result was proved with one type of agent only, i.e., without asymmetric information, and we want to question now whether the same effect can mitigate the impact of adverse selection due to imperfect classification techniques. The global monopoly, observing all the information revealed *ex post* (the realization of the two losses), is able to implement the most powerful self-selective scheme; intuitively, it is not necessarily replicable by addition of the specialized monopolies' schemes. The strength of the main result of the section is that the loss due to specialization is *generically strict*.

⁸ Examples are available on request to the authors.

In consequence, when adverse selection is considered, even if the market exhibits a form of complementarity that limits the conflict between the insurers, there are efficiency losses due to specialization, and the profits at the industry level are strictly decreased. We come up with a useful distinction. Specialization has two effects: First, it determines the purely strategic interaction between the insurers (Sections 3 and 4); second, by allocating the information-bearing signals, it determines the extent to which self-selection tools are weakened. In general, the net effect on the reallocation of the surplus is ambiguous.

5.1. *Equilibria.* Let us assume now that the monopolies confront two types of agents, a and b , in normalized numbers λ_a and λ_b ($\lambda_a + \lambda_b = 1$), who are incurring, with obvious notations, risks $\tilde{x}_a$ and $\tilde{y}_a$ and risks $\tilde{x}_b$ and $\tilde{y}_b$. Their utility functions u_a and u_b are not assumed to be identical. Specifically, insurer X (and analogously insurer Y) is able to observe the value taken by $\tilde{x}_a$ or $\tilde{x}_b$ but not whether the individual is of type a or b , i.e., the tastes of the client and the statistical distribution of his or her loss. The game between the insurers is similar to the previous one: The strategy spaces are the same (a menu of contract), but the payoffs are modified to take care of adverse selection. Stiglitz (1977) clarified the important features of the equilibrium solution for a monopoly confronting a similar adverse selection restricted to the probability of a given accident: The high-probability type is fully insured, and the low-probability type remains with his or her reservation utility; moreover, the high probability type is indifferent between his or her contract and the contract offered to the low-probability type.

In this subsection we propose to adapt Stiglitz' results using a methodology close to Landsberger and Meilijson (1999, henceforth LM). In LM, an insurer confronts a general adverse-selection problem with two types (preferences and distributions are type-specific and unobservable); types have to be separated in equilibrium only if they differ in their certainty equivalents of their original risks; in other words, there are no good and bad risks, the correct discrimination is based on a synthesis of risk and attitude toward risk. In our model, however, the way in which the consumers respond to one monopoly's offer is not independent of the offer made by the other monopoly; from a monopoly's viewpoint, types are endogenous. Our procedure has to deal with this difficulty.

If there were *ex post* perfectly revealing realizations, adverse-selection-free contracts could be designed easily; for example, if there is an event (measurable with respect to monopolies' information) of strictly positive probability that reveals a for sure, by setting sufficiently strong a penalty in the contract assigned to b in case of this event, the selection constraint of a can be totally loosened.⁹ Note, though, that even in this situation, each type would be a market wherein the conflict between insurers exposed in the preceding sections would persist. In order to address more realistic situations, we assume that $\tilde{x}_a$ and $\tilde{x}_b$, on the one hand, and $\tilde{y}_a$ and $\tilde{y}_b$, on the other hand, are continuous one with respect to the other.¹⁰ We need additional nota-

⁹ If we assume that penalties are not limited.

¹⁰ In the discrete case, it simply means that any event that occurs with probability 0 for one type, occurs with probability 0 for the other.

tions: For $i = a, b$, $l_x^i(x)$ [respectively, $l_y^i(y)$] denotes the probability (or the density in the continuous case) that agent i incurs loss x (respectively, y); $l_b^i(x, y)$ stands for the joint probability; note that given independence $l_b^i(x, y) = l_x^i(x)l_y^i(y)$.

Let $\mathcal{O}_Y \equiv [\pi_Y^k, I_Y^k(\cdot)]_{k \in K}$ be the menu offered by insurer Y , where K is sufficiently large for indexing all possible offers of monopolies X and Y . The offer of a monopoly could *a priori* contain more contracts than types, contracts not taken in equilibrium deterring the opponent to make some prejudicial offers; in other terms, we cannot invoke the revelation principle.¹¹ Nevertheless, for simplicity but without loss of generality, the following program characterizes interesting properties of the contracts offered by insurer X that are actually taken in equilibrium (symbols in braces are for Lagrange multipliers):

$$(17) \quad \max_{\pi_X^a, I_X^a(\cdot), \pi_X^b, I_X^b(\cdot)} \lambda_a [\pi_X^a - E_a I_X^a(\tilde{x}_a)] + \lambda_b [\pi_X^b - E_b I_X^b(\tilde{x}_b)]$$

such that identification restrictions

$$(18) \quad E_a I_X^a(\tilde{x}_a) = 0 \quad \{\mu_a\}$$

$$(19) \quad E_b I_X^b(\tilde{x}_b) = 0 \quad \{\mu_b\}$$

are satisfied, as well as incentive compatibility and participation constraints according to which $\exists k_a, k_b \in K$ s.t. $\forall i \in K$

$$(20) \quad \begin{aligned} E_a u_a [w - \pi_X^a + I_X^a(\tilde{x}_a) - \pi_Y^{k_a} + I_Y^{k_a}(\tilde{y}_a)] \\ \geq E_a u_a [w - \pi_X^b + I_X^b(\tilde{x}_a) - \pi_Y^i + I_Y^i(\tilde{y}_a)] \end{aligned} \quad \{\nu_a^i\}$$

$$(21) \quad \begin{aligned} E_b u_b [w - \pi_X^b + I_X^b(\tilde{x}_b) - \pi_Y^{k_b} + I_Y^{k_b}(\tilde{y}_b)] \\ \geq E_b u_b [w - \pi_X^a + I_X^a(\tilde{x}_b) - \pi_Y^i + I_Y^i(\tilde{y}_b)] \end{aligned} \quad \{\nu_b^i\}$$

$$(22) \quad \begin{aligned} E_a u_a [w - \pi_X^a + I_X^a(\tilde{x}_a) - \pi_Y^{k_a} + I_Y^{k_a}(\tilde{y}_a)] \\ \geq E_a u_a [w + \tilde{x}_a - \pi_Y^i + I_Y^i(\tilde{y}_a)] \end{aligned} \quad \{\tau_a^i\}$$

$$(23) \quad \begin{aligned} E_b u_b [w - \pi_X^b + I_X^b(\tilde{x}_b) - \pi_Y^{k_b} + I_Y^{k_b}(\tilde{y}_b)] \\ \geq E_b u_b [w + \tilde{x}_b - \pi_Y^i + I_Y^i(\tilde{y}_b)] \end{aligned} \quad \{\tau_b^i\}$$

¹¹ The theoretical issue is partially solved in a specific context by Martimort's (1992) equivalence principle. However, it is not necessarily true that more contracts than types are proposed in multi-principal equilibria. The related literature developing since has left many questions open.

The following propositions generalize Stiglitz' and LM's results:

PROPOSITION 6. *In an equilibrium in pure strategies:*

- (i) *For each insurer, at least one of the types is completely insured.*
- (ii) *For each insurer, the self-selection constraint of the completely insured type is binding.*
- (iii) *For each insurer, if one type is partially insured, the rationality constraint of this type is binding, and if both types are fully insured, the rationality constraint of at least one of the types is binding.*

PROOF. See Appendix. □

PROPOSITION 7. *Up to a null measure set, the optimal offers allocate to the partially insured agent a contract that exchanges his or her initial position for a transfer that depends only on the likelihood ratio of the observed loss [i.e., $l_X^a(x)/l_X^b(x)$ for monopoly X 's offer].*

PROOF. See Appendix. □

The preceding proposition states that efficient separations are performed only by using the informational contents of the loss (here simply the *ex post* relative probabilities of possible types), not the size of the loss, nor its probability. This interpretation was not possible in Stiglitz' simple model.

To prove existence of an equilibrium, we can apply the main theorem in Simon and Zame (1990).¹² It suffices, as done in the first sections, to treat the way in which consumers decide when they are indifferent between several optimal solutions as endogenous variables. The proof is a direct application when the loss support is countable: In this case, compactness of the strategy spaces is clear; upper hemicontinuity of the payoff correspondence is also guaranteed; this correspondence has nonempty, convex, compact values. With continuous loss distributions, technicalities beyond the scope of this article arise.

5.2. *Efficiency losses.* In the following, *profit* will mean expected (with respect to the uncertainty on the type of the consumer) *per capita* profit. With the superscript standing for adverse selection, the comparison between the profit of the global monopoly ($\bar{\pi}_B^{AS}$) and the sum of the profits each monopoly would make in the absence of the other ($\bar{\pi}_X^{AS} + \bar{\pi}_Y^{AS}$) still measures the conflict that specialization introduces, although these numbers are no longer interpretable as equivalent risk premia. In particular, it remains obvious that when there is substitutability ($\bar{\pi}_B^{AS} < \bar{\pi}_X^{AS} + \bar{\pi}_Y^{AS}$), the conflict is such that the equilibrium under adverse selection leads to strictly less than maximal a profit.

Even in the case of complementarity ($\bar{\pi}_B^{AS} > \bar{\pi}_X^{AS} + \bar{\pi}_Y^{AS}$), we are able to show, by excluding nongeneric situations, that the insurers are unable to coordinate their offers so as to extract the maximal rent. This contrasts with the one-type case.

¹² In contrast, Dasgupta and Maskin (1986) seems inapplicable.

PROPOSITION 8. *Whenever there is adverse selection, the total equilibrium profit of the specialized monopolies is generically strictly lower than that of a global monopoly. By generically, we mean with probability one in the space of preferences, in the space of loss distributions, and in the space of type distributions.*

PROOF. See Appendix. □

The second-best allocation that is most favorable to the insurance industry (implementable by a global monopoly) generically entails the offer of a nontrivial menu of contracts. But such a menu is generically not replicable by the summation of two independent contracts offered by specialized monopolies. Therefore, in the case of (generalized) complementarity as well as in the case of (generalized) substitutability, the insurance industry loses from being split into separate, specialized, entities.

To summarize, specialization (almost) systematically implies second-best efficiency losses. For consumers, this loss has to be balanced with the advantages of the diminished monopoly power of the insurer.¹³ Of course, insurers see the situation differently, but a form of specialization that minimizes the strategic conflict (say, that provokes complementarity, i.e., $\bar{\pi}_B^{AS} > \bar{\pi}_X^{AS} + \bar{\pi}_Y^{AS}$) may be, on net, disastrous because, as a consequence of the nonreplicability result, the observable variables allocated to monopoly X on the one hand and monopoly Y on the other hand may lead to an equilibrium total profit substantially lower than $\bar{\pi}_B^{AS}$ (and even lower than $\bar{\pi}_X^{AS} + \bar{\pi}_Y^{AS}$). If adverse selection is mild (types are easily separated) and substitutability strong, specialization remains an attractive alternative to more severe, inapplicable, regulations.

6. CONCLUSION

In this conclusion, we leave apart the effects due to adverse selection in view of drawing lessons on the pure effects of specialization as compared with global monopolization. According to our analysis, a limited scope being associated with monopolistic profits, a firm in the insurance industry has an incentive to spend design and marketing expenses on contracts insuring only particular hazards. This rent-seeking attitude, which can be seen at certain times as the desirable effect of competition (markets are completed), also can be seen as the result of a form of invisible collusion.

In the case of one type of agent, provided risks are independent, the specialization of the insurers remains compatible with Pareto efficiency, i.e., total exchange of the risks. Depending on the preferences and on the nature of the specialization, competition exhibits a form of strategic substitutability or complementarity. These notions measure the degree of conflict between the monopolies, and they directly influence the qualitative and quantitative allocation of the surplus between the insurers, on the one hand, and the insured, on the other hand.

Suppose there are two insurance companies in the market. Given the structure of the game, no specialization would imply a Bertrand competition. This situation is the most favorable to the consumer. In our analysis, specialization appears to be a means

¹³ With perfect competition under adverse selection, specialization also leads systematically to welfare losses (Fluet and Pannequin 1997), and there is no argument for its defence.

of limiting the conflict between the insurers. One of the lessons of our study is that the insurers' common interest is to seek strategic complementarity *ex ante*. Provided some feasibility constraints be taken into account (for observability reasons, the total risk can be split only into certain pairs), there exists an optimal degree of specialization.

This analysis is easily extended to more than two sectors. Determining the sharing of the surplus between the monopolies and the agent depends in a nontrivial way on the structure of the risk premia for all possible partial risk bundles. Nevertheless, a simple prediction can be stated: If the sum of the partial risk premia (calculated for one risk at once, all the other risks remaining incurred) is larger than the total risk premium (we can call this case *global substitutability*), then all equilibria give a rent to the agent.

Another important extension is the following. Suppose risks are not independent. If the two risks were perfectly correlated, it is clear that insurers could pretend they insure one risk only, whereas they would insure in fact the entirety of the risks (observing one loss gives all the information on the total loss). The degree of conflict between insurers would be maximal, and competition would allocate all the surplus to the agent. When the correlation is not perfect, specialized insurance contracts violate one of the Borch properties according to which indemnities should depend on the total loss only, and this results in an efficiency loss. In consequence, if it were impossible for the insurers to find independent risks, the optimal specialization, from their viewpoint, would be to force certain efficiency losses provided they would imply benefits by limiting (imperfect) competition. Alternatively, breaking monopolies may lead to imperfect competition; however, this partial success may be desirable provided specialities be defined to maximize substitutability; the benefits may be larger for the consumer than the cost in terms of second-best efficiency.

APPENDIX

A.1. *Proof of Theorem 1 (End).* Type 2 equilibria necessarily belong to $C_{XY} \cap C_{YB}$. They are therefore necessarily solutions of

$$(A.1) \quad Eu(w - \pi_X + \tilde{y}) = Eu(w + \tilde{x} - \pi_Y)$$

$$(A.2) \quad Eu(w + \tilde{x} - \pi_Y) = u(w - \pi_X - \pi_Y)$$

Equation (A.2) defines a continuous function $\pi_X(\pi_Y)$ that is independent of $\tilde{y}$. Thus any type 2 equilibrium corresponds to a pair (π_X, π_Y) where π_Y is a zero of

$$(A.3) \quad f(\pi_Y, \tilde{y}) = Eu[w - \pi_X(\pi_Y) + \tilde{y}] - Eu(w + \tilde{x} - \pi_Y)$$

Several nongenericity notions are possible; we propose the following: Let us denote by $Z(\tilde{y})$ the set of zeros of $f(\cdot, \tilde{y})$. Genericity is meant relatively to random variable $\tilde{y}$ in the following sense.¹⁴ Let I be a closed interval of $\mathbb{R}$, and let $\{\tilde{y}_i\}_{i \in I}$ be a set of random variables that are strictly ordered with respect to the second-order stochastic

¹⁴ An analogous construction could be made with $\tilde{x}$.

dominance (for fixing ideas, the dominant random variable corresponds to the lower index): The subset of I such that $Z(\tilde{y}_i)$ contains a nondegenerate interval is a null measure set.

We need to prove the following: Let $g(w, \lambda)$ be a continuous function mapping $W \times L \subset \mathbb{R}^2$ into $\mathbb{R}$ and strictly decreasing with respect to the second variable. Let $z(\lambda)$ be the set of zeros of $g(\cdot, \lambda)$. The subset of L over which $z(\lambda)$ contains a nondegenerate interval is a null measure set. Indeed, to each λ for which there exists a nondegenerate interval of zero, we associate this interval. Note that these intervals do not overlap, thanks to the strict monotonicity of the function with respect to λ . But each of these intervals contains at least one rational number. The set of rational numbers being countable, therefore of null measure, the result follows.

We apply the result to $\tilde{f}$, where $\tilde{f}: \mathbb{R} \times I \rightarrow \mathbb{R}$ and $\tilde{f}(x, t) = f(x, \tilde{y}_t)$. Any increase of t is a degradation of the risk in terms of the second-order stochastic dominance; $\tilde{f}$ is therefore strictly decreasing with respect to the second variable. The generic separation of the solutions of the system (A.1) and (A.2) proves the generic separation of the equilibria. $\square$

A.2. Proof of Proposition 6. Properties of monopoly X 's best response will *a fortiori* be satisfied by his or her equilibrium offer. Given monopoly Y 's offer, monopoly X designs his or her own optimal list of contracts $(C_X^k)_{k \in K}$ including (C_X^a, C_X^b) (contracts taken in equilibrium by a and b , respectively). Type a consumers have a weak preference order over this menu that depends on monopoly Y 's offer; C_X^i is (weakly) preferred to C_X^j if and only if there exists k such that for all h , choosing (C_X^i, C_Y^k) gives him or her at least as much expected utility as choosing (C_X^j, C_Y^h) . Let us denote by $K_{X,i}^a \subset K$ the set of k such that, conditional on choice C_X^i , choosing C_Y^k is optimal for a . Symmetric conventions are set for b . For all choices C_Y^k with $k \in K_{X,i}^a$, take the contract fully insuring $\tilde{x}_a$ providing the same utility to a type a consumer as choice C_X^i . The most profitable of these full insurance policies corresponds to a generalized certainty equivalent that we denote by $G_{X,i}^a$.

(i) If no offer actually chosen by the agents is a complete insurance, we show that the monopoly's profit can be increased. (1) Assume there is one type, say, b , who strictly prefers the contract he or she takes C_X^b , C_X^a being the offer chosen by a ; we substitute the mixture contract $\epsilon G_{X,a}^a + (1 - \epsilon)C_X^a$ for C_X^a .¹⁵ By taking a small positive ϵ , continuity of the preferences ensures that b continues to prefer C_X^b . The new offer is now the best choice for a ; it is as good as the former; hence he or she cannot be better off by choosing C_X^b because this choice, already available to him or her before the change, was not preferred to C_X^a .¹⁶ Finally, $G_{X,a}^a$ itself is strictly more profitable for monopoly X than C_X^a , and therefore, so is the mixture contract. The initial offer was not optimal. (2) Assume that each type is indifferent between

¹⁵ Remember that stochastic contracts are allowed. The interpretation of the mixture contract is that the indemnity policy of contract $G_{X,a}^a$ is enforced with probability ϵ , whereas contract C_X^a is enforced with probability $1 - \epsilon$; the indemnity is a stochastic function of the realization of $\tilde{x}_a$. Note also that a mixture contract is not a mixed strategy because randomization here is done after acceptance.

¹⁶ His or her choice of the mixture contract may induce a deviation on the insurance of $\tilde{y}_a$ —a question that does not matter for monopoly X .

the contract he or she picks and the contract picked by the other type. Let a be the type whose equivalent complete contract as defined above charges a higher (in the weak sense) premium. Monopoly X simply replaces the offer picked by a by this complete insurance contract. Whatever b does, the profit made on him or her is not lower, and the profit made on a is strictly increased. Therefore, the total profit of the monopoly is strictly increased.

(ii) When a monopoly does not differentiate the contracts he or she offers, both types of agents are fully insured. Now, if no agent were stuck to his or her rationality constraint, the insurer could safely rise the premium charged for full insurance so as to increase his or her profit with certainty. Therefore, one of the types at least would have a binding participation constraint, and the other would have a (trivially) binding self-selection constraint. We check now that binding constraints are the same in the case that remains, where one type, say, a , is fully insured, whereas the other, say, b , is strictly partially insured. Assume that a strictly prefers C_X^a , the contract he or she picks, to C_X^b , the contract picked by b . A reasoning parallel to the one given above allows the insurer to improve his profit.

(iii) No insurance of $\tilde{x}$ and $\tilde{y}$, respectively, will be assimilated to *bona fide* contracts and denoted by C_X^0 and C_Y^0 . Assume that b is the partially insured agent, and suppose that he or she strictly prefers C_X^b to C_X^0 and C_X^a . Let us find a contract C_X that provides any individual with an aggravation of his or her residual risk in terms of first-order stochastic dominance relatively to his or her initial contract C_X^b . There are many possible C_X ; for example, the monopoly cuts by ϵ the indemnities larger than, say, 2ϵ , with ϵ small enough for those indemnities to exist and small enough for the new offer to remain attractive for b . The first-order stochastic dominance aggravation ensures that a does not switch. The fact that C_X is strictly more profitable than the initial offer proves that the profit of the insurer is strictly improved. In conclusion, type b individuals cannot in equilibrium strictly prefer their contract to alternatives. Let us check now that C_X^0 , and not C_X^a , is the contract that is equivalent to C_X^b for b ; indeed, if C_X^a were as good as C_X^b , it would be more profitable to propose C_X^a only because C_X^a provides full insurance and, consequently, is more profitable on any type than any other equivalent contracts. $\square$

A.3. Proof of Proposition 7. In the case where the loss has a continuous support, we can easily write the first-order conditions of insurer X 's program. Let us assume that b is the partially insured agent; for a given loss x , the first-order condition with respect to $I_X^b(x)$ of monopoly X 's program is

$$\begin{aligned}
 \text{(A.4)} \quad 0 &= -(\lambda_b + \mu_b)I_X^b(x) - I_X^a(x) \\
 &\quad \times \sum_{i \in K} \nu_a^i E_a u'_a [w - \pi_X^b + x + I_X^b(x) - \pi_Y^i + \tilde{y}_a + I_Y^i(\tilde{y}_a)] + I_X^b(x) \\
 &\quad \times \left[\sum_{i \in K} (\nu_b^i + \tau_b^i) \right] E_b u'_b [w - \pi_X^b + x + I_X^b(x) - \pi_Y^{k_b} + \tilde{y}_b + I_Y^{k_b}(\tilde{y}_b)]
 \end{aligned}$$

The key remark is that this equation is homogeneous with respect to $[I_X^a(x), I_X^b(x)]$. Marginal utility being decreasing, the equation implies that any increase of

$I_X^a(x)/I_X^b(x)$ can only be accommodated by the decrease of $x + I_X^b(x)$ (calculate the derivative of I_X^b with respect to the likelihood ratio using the implicit function theorem, sign using the second-order condition).

The proof can be extended to any distribution at the cost of straightforward but tedious technical precautions. See Landsberger and Meilijson (1999); in the case of discrete loss distributions, the optimum may entail nondegenerate stochastic contracts, which means that the transfer may be a lottery at a finite number of points. Nevertheless, the transfer and the likelihood ratio are comonotonic. $\square$

A.4. Proof of Proposition 8. Assume first that loss distributions are continuous. The proof is in two parts. First, we prove that the following conditions are sufficient for types to be separated by the global monopoly: (1) equivalent consumptions are not equal (to fix ideas, a 's equivalent consumption is strictly lower than b 's), (2) the likelihood ratio $[I_B^a(x, y)]/[I_B^b(x, y)]$ differs from one with nonnegative probability (types are not statistically equivalent), (3) the distribution of types is nondegenerate ($\lambda_a, \lambda_b > 0$). This situation is generic because conditions above are independent one of another, and each is generically true. Second, we prove that generically, the optimal separating menu of the global monopoly cannot be replicated by the specialists.

First part. We reason by contradiction. Assume that both types are offered a full insurance contract by the *global* monopoly (note that given that each distribution is assumed to be continuous with respect to the other, incentive compatibility ensures that they pay the same premium). Given (1) above, there is generically one type, say, a without loss of generality who draws an informational rent from the situation. The *net* compensation takes the following structure: $I_B(x, y) = -x - y + I_B$, $I_B \in \mathbb{R}$. Let $\Omega = \{(x, y) | [I_B^a(x, y)]/[I_B^b(x, y)] \leq 1\}$ and $\bar{\Omega} = \{(x, y) | [I_B^a(x, y)]/[I_B^b(x, y)] > 1\}$. Let ϵ, η , and φ be positive numbers. Now, suppose that instead of this unique offer, the monopoly offers two contracts, one providing full insurance $I_B^a(x, y) = -x - y + I_B - \epsilon$ and the other providing $\forall (x, y) \in \Omega, I_B^b(x, y) = -x - y + I_B + \eta$ and $\forall (x, y) \in \bar{\Omega}, I_B^b(x, y) = -x - y + I_B - \varphi$ such that

$$(A.5) \quad u_a(w + I_B - \epsilon) = I_B^a(\Omega)u_a(w + I_B + \eta) + I_B^a(\bar{\Omega})u_a(w + I_B - \varphi)$$

$$(A.6) \quad u_b(w + I_B) = I_B^b(\Omega)u_b(w + I_B + \eta) + I_B^b(\bar{\Omega})u_b(w + I_B - \varphi)$$

Type a 's participation is secured by taking small enough an ϵ . Incentive compatibility for both types, and b 's participation, are imposed by construction. Given that $I_B^a(\Omega)/I_B^b(\Omega) < I_B^a(\bar{\Omega})/I_B^b(\bar{\Omega})$, the theorem of implicit functions proves that $\eta(\epsilon)$ and $\varphi(\epsilon)$ are defined and positive in a neighborhood of 0. Moreover, η and φ are of second-order in terms of ϵ ; therefore, with the new menu, the average profit is increased of $\lambda_a \epsilon$ and decreased of $\lambda_b [I_B^b(\Omega)\eta - I_B^b(\bar{\Omega})\varphi]$ (a quantity of the order of ϵ^2). In conclusion, given (3) above, for small enough an ϵ , separating types improves profits.

Second part. Now we know that, generically, there is a partially insured type. As proved in Proposition 7, the optimal consumption of the partially insured type is only a function of the likelihood ratio of what is observed. In consequence, the transfer

$I_B(x, y)$ takes the form $-x - y + J_B[l_B^a(x, y)/l_B^b(x, y)]$, where J_B is not a constant. We look whether this optimal allocation can still be implemented after the monopoly has been split into two specialists. In other words, whether this structure can be replicated by the summation of the transfers of the specialized monopolies, which have the form $I_X(x) = -x + J_X[l_X^a(x)/l_X^b(x)]$ and $I_Y(y) = -y + J_Y[l_Y^a(y)/l_Y^b(y)]$.

To see this, we search whether it is possible to find functions J_X and J_Y such that, for all x, y , $J_B[l_X^a(x)/l_X^b(x) \cdot l_Y^a(y)/l_Y^b(y)] = J_X[l_X^a(x)/l_X^b(x)] + J_Y[l_Y^a(y)/l_Y^b(y)]$. Define $\alpha = \ln l_X^a(x)/l_X^b(x)$, $\beta = \ln l_Y^a(y)/l_Y^b(y)$, $f = J_B \circ \exp$, $g = J_X \circ \exp$, $h = J_Y \circ \exp$. The functional restriction, after the changes of variables, becomes $f(\alpha + \beta) = g(\alpha) + h(\beta)$, where the three functions are defined on intervals. Given that f is $\mathcal{C}^1$ (the indemnity in the partial insurance optimal contract is regular with respect to the likelihood ratio), the same is true for g and h ; therefore, we can easily conclude that f, g , and h are all linear with the same slope. Now let $f(z) = K_1 z + K_2$, with $K_1, K_2 \in \mathbb{R}$, hence $I_B(z) = K_1 \ln z + K_2$. Independently (apply the proof of Proposition 7 to a global monopoly), we know that in equilibrium $u'_b[w + J_B(z)] = K_3 + K_4 z u'_a[w + J_B(z)]$, $K_3, K_4 \in \mathbb{R}$, where a is the fully insured type, and b the partially insured type. After a change of variables ($\tau = w + K_1 \ln z + K_2$), we get $u'_b(\tau) = K_3 + K_4 \exp(\tau - (K_2/K_1)) u'_a(\tau)$ over an interval. That is obviously not true for generic pairs of utility functions.

For noncontinuous distributions, our argument of nonreplicability works, although its development is more tedious to manage. The basis of the argument remains that the restrictions implied by the functional equation above are non-generic.

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