Flexi-security stocks, a new approach for semi-raw materials used in blending

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Abstract. The objective of this paper is the control of risks related to unforeseen changes in output demand at the end of a continuous-process supply chain. A dynamic blending approach has been developed to simultaneously define the optimal blends of inputs according to the order book and the inputs available in the blending area or which can be conveyed there from the mine. In the context of an uncertain universe, the risk of output stockout due to unpredictable demand is addressed by building up security stocks for some critical inputs along with the flexibility offered by dynamic blending. The optimization model for dynamic blending is described and a real-life example of the behavior of risk management through flexi-safety stocks is provided.

Keywords: Safety Stock, Dynamic Blending, Risk, Uncertain Universe.

1. Introduction

1.1. Industrial context

OCP group, Morocco's leading firm in terms of annual sales, is the exclusive operator of the Moroccan Phosphate deposit, which accounts for 73% of the world's known phosphate reserves. OCP operates across the entire value chain of the Phosphate industry from ore extraction to the export of fertilizers to many countries worldwide. It is also world No 1 exporter of phosphate, phosphoric acid and fertilizers. OCP's global logistics supply chain comprises three independent axes: the north, center and southern axes. This paper focuses on the center axis which produces, mainly to order, 5 varieties of ores (called merchantable ores) obtained by blending 14 inputs obtained by extraction from the mine (called source ores).
On average, 30 to 50 phosphoric acid ships are exported each year. Each ship contains 2 to 3 acid tanks with a capacity of 8,000T each, which corresponds to 5 to 9 average production days of merchantable ores. The ores are exported in 20,000T or 50,000T ships, which correspond to average merchantable ores production of 3 to 8 average production days. Orders for phosphoric acid and minerals are in relation to annual framework and spot market contracts, the latter of which are inherently unpredictable. Thus, the production master plan has to be regularly updated for spot demand orders involving the rescheduling of master plan contract demand (impacting the short-term rolling plan). For these reasons, demand for merchantable ores cannot be probabilized. The risk that we are trying to address here is related to the occurrence of spot orders or short-term forecasting of acid or merchantable ores shipments, planned under annual framework agreements. Due to this uncertainty we must be able at any time to manufacture the equivalent of about one week's output of any merchantable ore.
This paper deals with the improvement of risk control through the flexibility delivered by blending and security stocks of source ores. After a brief review of the risks to be addressed and how to do so (§1.2), we turn to discussing the differentiation of (§1.3) the concept of safety stocks used in stochastic universe from that of *security stocks,* which we suggest to be more relevant to an uncertain universe. Finally we note that the specific context of this mine's supply chain leads to a new approach to risk management by stocks (§1.4).

1.2. Risk and risk awareness

The risk can be defined as the occurrence of an unforeseen and unwanted event that significantly changes the characteristics of a system (a logistic system, in this case). Such event leads to significant changes in demand flow characteristics (in terms of volume and composition) produced or exchanged in the supply chain or may result from unavailability of equipment (as a result of breakdowns, for example) or of human or material (stock shortages) resources. In a supply chain, the propagation of disturbances generally occurs crescendo upstream and downstream (bullwhip effect): a local undesirable effect becomes a source of risk elsewhere in the SC. This observation means that one is addressing a rather wide spatial scope of risk analysis. The need to control the risk is due to the fact that the consequences of these disturbances can significantly impair overall business performance.

To avoid the risk, or at least mitigate its effect, management can leverage three sets of measures:

- Improve the quality of factual and procedural information available for decision-making. Concerning factual information, areas for improvement include the relevance, reliability and speed of availability as part of an efficient and secure information system. Management procedures, backed up by an information system used in structured or semi-structured decision-making [5] must improve production system responsiveness and flexibility.
- Flexibility of human or material resources. Versatility and resource capacity surplus increase system responsiveness and flexibility.
- The stockpiling of finished or intermediate products along the supply chain helps to increase the flexibility of the system to cope with unforeseen demand fluctuations and forecast errors.

Both dynamic blending and specific security-stocks (section 2) pertain to management procedures that mitigate risk and also help avoid resource capacity surplus.

1.3. Safety versus security stocks

In a stochastic environment, a safety stock is the difference between a level of completion (of a stock or, by extension, a production) defined to face random demand over a reference period, and its mathematical expectation. In this context, the risk of non-response to demand can be defined as the probability of observing a stock shortage at the end of this reference period (classically determined by using the probability distribution of demand). Supply management models define optimal analytical solutions where the optimal risk is a function of the order costs, carrying costs and shortage costs. This approach works well for a finished product if we can estimate its stockout probability. It also applies to the components used by discrete products, the distribution of demand for these components being deduced from that of the finished products by combining BOM explosion and lead time offset mechanisms (classical in MRP). An important number of papers show the importance of this topic when demand is stochastic and the production process is fixed (for example, [3]; [4]; [6]).

To our knowledge, no work analyses the case where demand cannot be approximated in a stochastic environment or where a large unforeseen order appears, which can be regarded as an outlier from the probability point of view. In that case, the size of the safety stock cannot be determined from any cost parameter. As explained in section 1.1, that is the case for OCP whose final demand for phosphoric acid and merchantable ores cannot be described by a probability distribution or by a breakdown in trend, cycle and random components. Moreover, the ores sold by OCP are obtained by blending extracted ores in variable composition depending on production time. This implies that one may not rely on any BOM to derive demand for extracted ores from that of merchantable ores. Preventing risk by managing stocks of extracted ores is less costly than managing stocks of acids or merchantable ores that can be produced to order.

Thus, the risk problem does not arise in a stochastic universe. We will show that to be able to fulfill an unexpected order for any merchantable ore, amounting to a week's output, one must be able to draw on minimal stocks of different inputs, to be determined by specific rules. These shall be discussed below as they deviate from the stochastic approach. To avoid any ambiguity, we have coined the term of "*security stock*" to refer to it.

1.4. The problem at hand

The five merchantable ores sold or used by OCP to produce phosphoric acid are outputs obtained by mixing inputs taken from a set of I=14 source ores. The traditional approach of risk control by safety stocks of these outputs and inputs faces two difficulties. Each output j, $(j=1,...,J)$ is linked to a quality chart with an admissible min-max target $\{\beta_c^{Min};\beta_c^{Max}\}$ of five components c, $(c=1,...,5)$; each component c is available in a proportion α_{ic} in each input i, $(i=1,...,I)$ (see Tables 1).

Table 1 : Composition α_{ci} of inputs and specifications $\{\beta_c^{Min} - \beta_c^{Max}\}$ of composition of some outputs

Share α_{ci} (%) of component c in the weight of input i | Constraints on the share β_{cj} (%) of component c in the weight of output j

Component c		Input i														Output j				
		$i=1$ C3 sup	$i=2$ SA2	$i=3$ C3G	$i=4$ C1	$i=5$ C0	$i=6$ C4	$i=7$ C5	$i=8$ C2 sup	$i=9$ SB	$i=10$ SX	$i=11$ C3 inf	$i=12$ C1 Exp	$i=13$ C2 Exp	$i=14$ C6	$j=1$ Tess	$j=2$ Stand	$j=3$ MT	$j=4$ BT	$j=5$ TBT
$c=1$	BPL	50.0	54.9	56.0	59.5	59.5	61.0	59.0	60.0	61.5	63.0	64.0	65.5	65.5	65.7	$57,9<\beta_{11}<61$	$57,9<\beta_{12}<61$	$\beta_{13}>64$	$57<\beta_{14}<60$	$54<\beta_{15}<56$
$c=2$	CO_2	3.7	7.7	5.4	4.5	5.2	4.8	7.7	5.1	5.2	5.5	5.0	5.9	4.6	5.0	$5,6<\beta_{21}<7$	$5,6<\beta_{22}<7$	$5<\beta_{23}<7$	$6<\beta_{24}<8$	$7<\beta_{25}<9$
$c=3$	MGO	1.0	0.7	0.9	1.2	1.2	1.5	1.7	0.9	0.8	0.8	1.1	0.8	0.7	1.2	$\beta_{31}<1,4$	$\beta_{32}<1,4$	$\beta_{33}<1$	$\beta_{34}<1,4$	$\beta_{35}<2$
$c=4$	SIO_2	18.0	8.0	17.2	9.5	8.5	11.7	9.8	11.5	8.0	11.0	10.0	7.5	8.0	6.0	$\beta_{41}<13,5$	$\beta_{42}<13,5$	$\beta_{43}<8$	$\beta_{44}<13,5$	$\beta_{45}<15$
$c=5$	Cd/B (ppm)	24	16	8	11	8	10	14	12	10	10	5	12	13	9	$\beta_{51}<10$	$\beta_{52}<12$	$12<\beta_{53}<18$	$\beta_{54}<20$	$\beta_{55}<26$

The optimal blend of inputs required to produce a given output varies according to the available inputs (initial stocks supplemented by supplies to be determined, conveyed by 2 conveyors from the mine) and the structure of the production program [1]. The problem posed here is the Center axis' ability to immediately reschedule production to meet immediately an unforeseen demand amounting to around one week of blending output, as explained in section 1.1. Outputs 1 and 4 are used to produce phosphoric acid and the remaining outputs are exported. To this end, the procedural flexibility offered by dynamic blending [1] must be backed by security stocks (defined in an uncertain environment).

2. The dynamic blending model and related experimentation

In this paper, risk management is limited to the ability to instantly adjust to a change in demand to be met in the following week, in a weekly rolling programming context. We will show that this implies building security stocks for some inputs (§2.1) which, combined with the flexibility provided by dynamic blending, allows to address a sudden change in the production schedule for the following week (§2.2), while providing no guarantee that such demand will be met.

2.1. Evaluation of the minimum level of input stocks to build a security stock

While there are infinite alternative blends to produce a given output, minimal amounts of some inputs may be required to produce that output [1]. This implies that, in an uncertain environment, the stock of these inputs must remain above a certain level in order to be able to face any unexpected change of the production plan at the end of the current week the day before the next week begins (ie Monday). The approach adopted to define security stocks of inputs blended to manufacture the five relevant outputs is based on the results yielded by the blending model in its static variant [1]. Let x_{ij} be the quantity of input i $\left(i=1,\ldots,\mathrm{I}\right)$ used to manufacture the quantity D_j of output j $\left(j=1,\ldots,\mathrm{J}\right)$. Our aim here is to minimize x_{ij}/D_j successively for each input of a single output, and also to see if any input may be used in the production of any output. The result of 140 independent optimizations (2 types of optimizations combined with 5 outputs and 14 inputs) is given in Table 2.

Table 2: Minimal and Maximal % of inputs *i* in each output *j*

Output *j*	Input *i*: $i=1$	$i=2$	$i=3$	$i=4$	$i=5$	$i=6$	$i=7$	$i=8$	$i=9$	$i=10$	$i=11$	$i=12$	$i=13$	$i=14$
$j=1$	0-11	0-15	10-57	0-32	0-43	0-48	10-46	0-27	0-20	0-28	0-49	0-19	0-15	0-32
$j=2$	0-20	0-22	0-57	0-43	0-43	0-60	10-62	0-44	0-26	0-35	0-49	0-24	0-22	0-32
$j=3$	0-10	0-15	0-12	0-26	0-26	0-18	0-22	0-27	0-40	0-34	0-33	12-90	0-88	0-60
$j=4$	0-0	22-80	0-10	0-33	0-50	0-18	0-22	0-21	0-70	0-21	0-26	0-48	0-48	0-47
$j=5$	0-18	0-90	0-31	0-23	0-24	0-18	0-49	0-22	0-17	0-15	0-12	0-16	0-11	0-12

This static analysis shows that input 7 is required to produce outputs 1 (10% minimum) and 2 (10% minimum). Therefore, in order to manufacture 40,000 tons (weekly average production) to cater to an urgent unforeseen order for any product, we need to have 4,000 tons (10% of 40,000) of input 7 in stocks. The ceiling level for minimal required amounts of each input can be analyzed as a security stock in uncertain environment. The lack of such a security stock for input 7 prevents any production of both outputs 1 and 2. Table 3 shows the corresponding level of security stock for each input. We can add that any input may be used to produce any output, as the maximum yielded by optimization is higher than 0. Blending flexibility implies absence of a BOM linking merchantable ores to extracted ores, knowing that any extracted ore may be used to produce any merchantable ore. Thus it is more efficient and effective to manage risk at extracted ore level.

Table 3: Minimal stock S_i^{Min} of input *i*

	Input *i*: $i=1$	$i=2$	$i=3$	$i=4$	$i=5$	$i=6$	$i=7$	$i=8$	$i=9$	$i=10$	$i=11$	$i=12$	$i=13$	$i=14$
S_i^{Min}	-	8800	4000	-	-	-	4000	-	-	-	-	4800	-	-

This risk strategy, designed to address an urgent and unexpected order, relies on: *i*) security stocks for indispensable inputs to manufacture certain outputs and *ii*) the flexibility provided by dynamic blending to adjust output composition according to demand characteristics, combined with immediately available input stocks and availability of feeding stocks from source ores ready to be removed from the mine over the next few weeks (formulation in §2.2.a). For all these reasons, we coined the term "*flexi-security stocks*" that reflects both types of risk coverage in uncertain environment. Nevertheless, note that there remains a risk that one will not be able to fulfill an order as other (optional) inputs may not be available in sufficient quantity to produce one of the possible alternative blends.

2.2. Dynamic blending with security stocks

The risk of being unable to fulfill an unforeseen order of any merchantable ore in an amount corresponding to one week's output, is partly addressed by dynamic blending (§2.2.1) and by security stocks, as defined above. In §2.2.2, we describe a protocol for an illustration of the proposed approach. Please note that we do not need to prove the superiority of managing risk with security stocks under managing without them. Where available quantity of an input is under the security stock, it will not be possible to satisfy the corresponding urgent and unexpected order for some outputs based on information given in Table 2. If these quantities are available, it may be possible that the substitutability offered by the blending in using available extracted ores is not adequate to meet all urgent and unexpected orders for all of the outputs.

2.2.1. Dynamic blending modelling

To evaluate the benefits of security stocks, we examine the blending problem in its dynamic form based on splitting time into periods $(t=1,...,T)$ and taking into account feeding of inputs as a decision to make [1]. The problem concerns the fulfilment of a set of K production orders $(k=1,...,K)$ already scheduled on parallel blending processors. Each order will be satisfied by a blend of inputs available in a secondary stock of source ores in the blending zone. This stock is supplied from a primary stock of source ores in the mine (Figure 1). Each order *k* consists of an amount D_k of output $j=\lambda_k$ $(1 \le \lambda_k \le J)$. We note x_{ik} as the decision variable defined as the amount (in tons) of input *i* used to produce D_k tons of the output

λ_k. Accordingly the blend verifies $\sum_i x_{ik} = D_k, \forall k$. The schedule authorizes withdrawal of inputs to fulfill order *k* during periods *t* only where $\delta_{kt} = 1$ (otherwise, this boolean equals 0); the withdrawal time of order *k* is noted ν_k $(\sum_t \delta_{kt} = \nu_k)$.
To satisfy component specifications (Table 1), the relative weight $\beta_{c\lambda_k}$ of component *c* in the output obtained by blending must verify $\beta^{Min}_{c\lambda_k} < \beta_{c\lambda_k} < \beta^{Max}_{c\lambda_k}$, where $\beta_{c\lambda_k} = \sum_c \alpha_{ci} \cdot x_{ik} / D_k$.

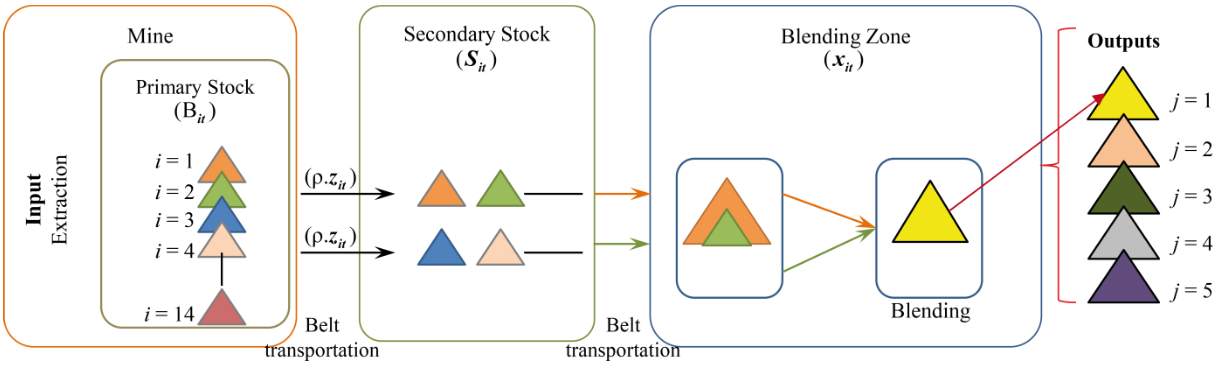

Figure 1: Center Axis Blending Process

Stock S_{it} of input *i*, defined at the end of period *t* is initialized on *t*=0 (initial stocks S_{i0}). Its supply is linked with the binary variable z_{it}, which equals 1 only if S_{it} is fed during period *t.* Stock feeding is constrained by the number of conveyors R_t available at each period *t*, enforced by relation (1), which operate at a constant flow rate ρ. Assuming that a conveyor can only supply one input during a single period *t*, the amount that can be conveyed and stored in S_{it} is $\rho \cdot z_{it}$. Furthermore, supplies have to comply with available source ores on the mine. In the programming horizon retained (four to six weeks), there is no constraint on removing ores lying on site in order to access lower layers [2]. The possible existence of such constraints leads to additional relations (not included in this paper) to force a minimal use of certain source ores at a particular term. That said, considering such availabilities and their daily change, we define at the beginning of the period t, availability accumulation B_{it} of source ore *i* stacked on the mine (without considering withdrawals), which leads to relation (2). It is assumed here that all ores transported on trucks from stocks of source ores ready to be stacked during a period feed stocks S_{it} during this period (no intermediate stock). Therefore, flow conservation constraints are represented in relations (3) and (4).

$$\sum_i z_{it} \le R_t;\ t = 1,\dots,T \qquad (1)$$

$$\rho.\sum_{t'=1}^{t'=t} z_{it'} \le B_{it};\ t = 1,\dots,T;\ i = 1,\dots,I \qquad (2)$$

$$S_{it} = S_{it-1} + \rho \cdot z_{it} - \sum_{k=1}^{K} \delta_{kt} . (x_{ik} / \tau_k);\ i = 1,\dots,I;\ t = 1,\dots,T \qquad (3)$$

$$S_{it} \ge 0;\ i = 1,\dots,I;\ t = 1,\dots,T \qquad (4)$$

Stocks feeding is also bound with a maximal storage capacity S^{Max} regarding all input stocks (5) and a security stock S_i^{Min} for each input *i* (6), as defined in §2.1.

$$\sum_i S_{it} \le S^{Max};\ t = 1,\dots,T \qquad (5)$$

$$S_{it} \ge S_i^{Min};\ i = 1,\dots,I;\ t = 1,\dots,T \qquad (6)$$

(6) can be replaced with (7) which penalizes the non-fulfilment of security stock levels for some inputs with a relative cost w_i (not taken into consideration in the objective-function and in the numeric illustration).

$$w_{it} \geq 0, \forall i,t$$
$$w_{it} \geq S_i^{\text{Min}} - S_{it}, \forall i,t \qquad (7)$$

The model used here enables outputs composition to be as close as possible to specifications at upper and lower boundaries (according to technical considerations). The general objective of this model is to stabilize the composition of merchantable ores used to manufacture phosphoric acid in order to avoid additional costs created by frequent setups of these lines (which is related to variability of upstream output composition). Relations (8) and (9) enable to determine the gap between order composition and the upper or lower structure boundaries regarding component *c* of output λ_k (only component *c*=1 has to be evaluated for its minimal threshold).

$$\Delta_{ck}^{\inf} = \sum_i \alpha_{ci} \cdot x_{ik} - \beta_{c\lambda_k}^{\inf} \cdot \mathrm{D}_k, \forall k \mid \beta_{c\lambda_k}^{\inf} \neq 0 \ ; \ c = 1; \ k = 1,\ldots,\mathrm{K} \qquad (8)$$

$$\Delta_{ck}^{\sup} = \beta_{c\lambda_k}^{\sup} \cdot \mathrm{D}_k - \sum_i \alpha_{ci} \cdot x_{ik}, \forall k \mid \beta_{c\lambda_k}^{\sup} \neq 0 \ ; \ c = 2,\ldots,\mathrm{C}; \ k = 1,\ldots,\mathrm{K} \qquad (9)$$

The optimization criterion chosen here is the minimization of the sum of those deviances related to orders that concern outputs for internal use (10). We consider that each component's deviance has the same marginal cost θ. The objective function is derived as follows:

$$Min\left(\sum_k \sum_c \theta.(\Delta_{ck}^{\sup} + \Delta_{ck}^{\inf})\right) \qquad (10)$$

with $c = 1,\ldots,5$ and $k \in K_0$, K_0 the subset of orders produced for acid lines.

2.2.2. Test protocol analysis

We now turn to the possibility of adjusting the production schedule by changing the order of the following week just before its beginning. We compare two cases of dynamic blending practice, one without security stocks and the other with security stocks, using real programming values over one month. This comparison serves to illustrate our proposed approach. It does not amount to a demonstration by simulation to prove the superiority of the use of security stocks, which is obvious, since without them, certain unexpected orders may be impossible to produce. Our example shows the impact of unexpected orders to be fulfilled in the two cases being compared. Note again, that safety security stocks do not guarantee that all unexpected orders can be satisfied.

The problem at hand addresses the need to meet some production orders to be scheduled over a few weeks in a rolling programming approach. The schedule can be revised weekly given a one-month visibility. Two scenarios are proposed: *case I* is the benchmark case, where initial stocks of inputs S_{i0} are the actual stocks and the constraints of security stocks (6) are not taken into account. In *case II*, we assume that initial stocks are at least equal to the security stocks level (which corresponds to $\mathrm{Max}(S_{i0}; S_i^{\text{Min}})$), and that optimization is performed with a view to maintaining these levels (hence (6) is taken into account). In other words, *case I* reflects current OCP management, using dynamic blending, and *case II* assesses the value of security stocks. The idea is to show the relative performance of each case in a risk situation.

In both cases, the optimal solution is given in terms of inputs consumption and stocks feeding based on a daily time bucket. Leaving production risks aside, we assume that at some point in the planning (after one or two weeks to avoid excessive dependency of the scenario on the initial state), we receive an unexpected order to be satisfied immediately. For example, if the emergency order arrives at the end of week 2, then the order portfolio is updated and the new order is scheduled in week 3 while the order scheduled for that week is cancelled or postponed along with the following ones. To this end, we have to retain inputs stock situation S_{it} at the end of week 2 as well as the availability accumulation B_{it} on site at the beginning of week 3 in order to address another problem starting from week 3. Two variants of unexpected situations, the most likely ones, have been reviewed in this study:

- Variant A: replacing an order for an output intended for a spot market by another meant for internal use;
- Variant B: postponing orders by a few days to include an order for an output intended for the spot market.

In all variants, we do not consider any new information beyond the horizon adopted, even if shifting orders implies exceeding it. In fact, we do not extend availability accumulation B_{it} on the site as our extraction process is a LIFO system [2]. In section §3 we compare the two scenarios (with and without security stocks) in terms of structure deviance regarding outputs for internal use.

3. Numerical analysis

In this section, we illustrate the flexibility offered by the availability of security stocks for the blending of 4 orders manufactured on a single blending unit over 28 days (planning horizon T). The input stocks are supplied by two conveyors with a flow rate (ρ) of 4,000 units per day. The deviance is in tons and the marginal cost in the objective function is θ =100.

Reference problem data (before urgent unexpected order) is given in Table 4 and optimal solutions are shown in Table 5, the source ore feeding being different in the two cases at hand.

Table 4 : Reference problem data

Period t	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28
Order k	1							2							3							4						
Output λ_k	2							1							3							2						
Qunatity	40000							40000							40000							40000						

Input	i =1	i =2	i =3	i =4	i =5	i =6	i =7	i =8	i =9	i =10	i =11	i =12	i =13	i =14
Initial Inventory $S_{i\,0}$ Cases I & II	0	0	0	0	4620	5032	4086	15823	7811	0	0	6956	0	7576
Safety Stock	0	8800	4000	0	0	0	4000	0	0	0	0	4800	0	0

	Accumulation of the prepared ores B_{it}																
Period	t =1	t =2	t =3	t =4	t =5	t =6	t =7	t =8	t =9	t =10	t =11	t =12	t =13	t =14	t =15	…	t =27	t =28
i =1	893	1786	2679	3571	4464	5357	6250	7143	8036	8929	9821	10714	11607	12500	13393	…	24107	25000
i =2	0	0	0	0	0	0	0	1214	2429	3643	4857	6071	7286	8500	9714	…	24286	25500
i =3	1254	2507	3761	5014	6268	7521	8775	10029	11282	12536	13789	15043	16296	17550	18804	…	33846	35100
i =4	1421	2843	4264	5686	7107	8529	9950	11371	12793	14214	15636	17057	18479	19900	21321	…	38379	39800
i =5	1429	2857	4286	5714	7143	8571	10000	11429	12857	14286	15714	17143	18571	20000	21429	…	38571	40000
i =6	839	1679	2518	3357	4196	5036	5875	6714	7554	8393	9232	10071	10911	11750	12589	…	22661	23500
i =7	1776	3551	5327	7103	8879	10654	12430	14206	15981	17757	19533	21309	23084	24860	26636	…	47944	49720
i =8	1068	2136	3204	4271	5339	6407	7475	8543	9611	10679	11746	12814	13882	14950	16018	…	28832	29900
i =9	0	0	0	0	0	0	0	929	1857	2786	3714	4643	5571	6500	7429	…	18571	19500
i =10	714	1429	2143	2857	3571	4286	5000	5714	6429	7143	7857	8571	9286	10000	10714	…	19286	20000
i =11	954	1907	2861	3814	4768	5721	6675	7629	8582	9536	10489	11443	12396	13350	14304	…	25746	26700
i =12	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1446	…	18804	20250
i =13	1227	2454	3680	4907	6134	7361	8588	9814	11041	12268	13495	14721	15948	17175	18402	…	33123	34350
i =14	1293	2585	3878	5170	6463	7755	9048	10340	11633	12925	14218	15510	16803	18095	19388	…	34898	36190

Table 5 : Optimal solution of the reference problem

	x_{ik} (tonne)								x_{ik} / D_k (%)							
	k = 1		k = 2		k = 3		k = 4		k = 1		k = 2		k = 3		k = 4	
	Case I	Case II	Case I	Case II	Case I	Case II	Case I	Case II	Case I	Case II	Case I	Case II	Case I	Case II	Case I	Case II
i =1	0	0	3189	2398	91	0	720	720	0%	0%	8%	6%	0%	0%	2%	2%
i =2	0	0	0	0	0	1937	0	0	0%	0%	0%	0%	0%	5%	0%	0%
i =3	0	4954	16000	11046	0	0	13782	13782	0%	12%	40%	28%	0%	0%	34%	34%
i =4	0	0	0	0	0	0	0	0	0%	0%	0%	0%	0%	0%	0%	0%
i =5	11622	4240	6989	19110	2009	1270	0	0	29%	11%	17%	48%	5%	3%	0%	0%
i =6	7013	8806	0	0	0	226	0	0	18%	22%	0%	0%	0%	1%	0%	0%
i =7	7000	12000	7826	7109	0	521	24371	24371	18%	30%	20%	18%	0%	1%	61%	61%
i =8	10000	10000	0	0	0	0	0	0	25%	25%	0%	0%	0%	0%	0%	0%
i =9	1366	0	0	0	6445	7811	0	0	3%	0%	0%	0%	16%	20%	0%	0%
i =10	0	0	0	0	6873	0	1127	1127	0%	0%	0%	0%	17%	0%	3%	3%
i =11	0	0	5995	338	2005	0	0	0	0%	0%	15%	1%	5%	0%	0%	0%
i =12	3000	0	0	0	0	8000	0	0	8%	0%	0%	0%	0%	20%	0%	0%
i =13	0	0	0	0	5467	2736	0	0	0%	0%	0%	0%	14%	7%	0%	0%
i =14	0	0	0	0	17111	17500	0	0	0%	0%	0%	0%	43%	44%	0%	0%
Sum	40000	40000	40000	40000	40000	40000	40000	40000	100%	100%	100%	100%	100%	100%	100%	100%

Case I

$\rho^* z_{it}$		Period $t=1$	$t=2$	$t=3$	$t=4$	$t=5$	$t=6$	$t=7$	$t=8$	$t=9$	$t=10$	$t=11$	$t=12$	$t=13$	$t=14$	$t=15$	$t=16$	$t=17$	$t=18$	$t=19$	$t=20$	$t=21$	$t=22$	$t=23$	$t=24$	$t=25$	$t=26$	$t=27$	$t=28$	Sum
Input	$i=1$	0	0	0	0	0	0	0	4000	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	4000
	$i=3$	0	0	0	0	0	4000	4000	0	0	0	4000	0	4000	0	0	0	0	0	0	0	0	4000	4000	4000	0	0	0	4000	32000
	$i=5$	0	0	4000	0	0	4000	0	0	4000	0	0	4000	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	16000
	$i=6$	0	0	0	0	4000	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	4000
	$i=7$	0	0	4000	0	4000	0	0	0	4000	0	4000	0	0	0	0	0	0	0	0	0	4000	4000	4000	4000	4000	4000	4000	0	44000
	$i=8$	0	0	0	4000	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	4000
	$i=10$	0	0	0	0	0	0	0	0	0	0	0	0	0	4000	0	0	0	4000	0	0	0	0	0	0	0	0	0	0	8000
	$i=11$	0	0	0	0	0	0	0	4000	0	0	0	4000	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	8000
	$i=13$	0	0	0	0	0	0	0	0	0	0	0	0	0	0	4000	0	0	0	0	4000	0	0	0	0	0	0	0	0	8000
	$i=14$	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	4000	4000	4000	4000	4000	0	0	0	0	0	0	0	0	20000
																														148000

Case II

$\rho^* z_{it}$		Period $t=1$	$t=2$	$t=3$	$t=4$	$t=5$	$t=6$	$t=7$	$t=8$	$t=9$	$t=10$	$t=11$	$t=12$	$t=13$	$t=14$	$t=15$	$t=16$	$t=17$	$t=18$	$t=19$	$t=20$	$t=21$	$t=22$	$t=23$	$t=24$	$t=25$	$t=26$	$t=27$	$t=28$	Sum
Input	$i=1$	0	0	0	0	0	0	0	4000	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	4000
	$i=2$	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	4000	4000	0	0	0	0	0	0	0	8000
	$i=3$	0	0	0	0	0	4000	4000	0	0	0	0	4000	4000	0	0	0	0	0	0	0	0	0	0	4000	4000	4000	4000	0	32000
	$i=5$	0	0	0	0	0	0	0	4000	4000	4000	0	4000	0	4000	0	0	0	0	0	0	0	0	0	0	0	0	0	0	20000
	$i=6$	0	0	0	0	4000	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	4000
	$i=7$	0	0	4000	0	4000	0	4000	0	0	0	4000	0	0	4000	0	0	0	0	0	0	0	4000	4000	4000	4000	0	4000	4000	44000
	$i=8$	0	0	0	4000	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	4000
	$i=10$	0	0	0	0	0	0	0	0	0	0	0	0	4000	0	0	0	0	0	0	0	0	0	4000	0	0	0	0	0	8000
	$i=11$	0	0	0	0	0	4000	0	0	4000	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	8000
	$i=12$	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	4000	0	4000	0	0	0	0	0	0	0	8000
	$i=13$	0	0	0	0	0	0	0	0	0	0	4000	0	0	0	0	4000	0	0	0	0	0	0	0	0	0	0	0	0	8000
	$i=14$	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	4000	4000	4000	4000	0	0	0	0	0	0	0	0	16000
																														164000

Case	Constraint on Security Stock	Internal orders	$c=1$ Internal Output Deviance	$c=2$ Internal Output Deviance	$c=3$ Internal Output Deviance	$c=4$ Internal Output Deviance	$c=5$ Internal Output Deviance	Internal Output Total Deviance
I	No	$k=1$	967.51	560.00	71.13	1419.32	492.43	3510.38
		$k=2$	0.00	560.00	93.96	115.74	0.00	769.70
		$k=4$	0.00	91.22	0.00	391.38	0.00	482.60
II	Yes	$k=1$	586.21	454.26	36.34	829.36	303.88	2210.05
		$k=2$	0.00	560.00	78.48	715.59	0.00	1354.07
		$k=4$	0.00	91.22	0.00	391.38	0.00	482.60

The analysis of the optimal solution for *case I* reveals that for critical input 2, we started the simulation with a no initial inventory. Therefore, this input has largely fed our inputs stocks (for *case I* 32,000 and for *case II* only 8,000) and was used in excess of the necessary minimal quantities, thus building excess security stock. The comparison of total ore feeding between the two cases shows that *case II* required 16,000 tons more than *case I*. This is explained by: *i*) in *case I*, we started with an initial inventory that we didn't try to maintain security stock, unlike in the second case; *ii*) the total deviance of internal output for *case I* is somewhat better than in *case II*, which is allowed by an increase of input feeding.

We consider that, in period 14, we received these urgent orders after completing the first two orders. We reviewed the possibility of satisfying orders at the beginning of period 15 (scenarios A and B). So, our initial inventory for both scenarios was equal to the inventory at the end of period 14 $(S_{i,t=14})$. As explained in section §2.2.b, $\mathrm{B}_{i,t>28} = \mathrm{B}_{i,t=28}$ for the scenario B.

Concerning the determination of the aggregated available ores B_{it}, we have to subtract from period 15 the amount of prepared ores used to feed our stock of inputs. The new B'_{it} becomes $\mathrm{B}'_{it} = \mathrm{B}_{i,t+14} - \rho \sum_{\tau=1}^{\tau=14} z_{i\tau}$, with $\mathrm{T} = 28 - 14 = 14$ (scenario A) or $\mathrm{T} = 35 - 14 = 21$ (scenario B). Table 6 shows the new composition of prepared ores B'_{it}, for both cases and a summary of the scenarios reviewed.

Table 6 : Scenario problems data

		t	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35
Scenario A	Replace an external order by an internal one	Order k	5							4													
		Output λ_k	1							2													
		Demand D_k	40000							40000													
Scenario B	Add an exceptional order	Order k	5							3							4						
		Output λ_k	4							3							2						
		Demand D_k	40000							40000							40000						

Case I

$B'_{it'}$	$t'=1$	$t'=2$	…	$t'=14$	$t'\geq15$
	$t=15$	$t=16$	…	$t=28$	$t\geq29$
$i=1$	9393	10286	…	21000	21000
$i=2$	9714	10929	…	25500	25500
$i=3$	2804	4057	…	19100	19100
$i=4$	21321	22743	…	39800	39800
$i=5$	*5429*	*6857*	…	*24000*	*24000*
$i=6$	8589	9429	…	19500	19500
$i=7$	*10636*	*12411*	…	*33720*	*33720*
$i=8$	12018	13086	…	25900	25900
$i=9$	7429	8357	…	19500	19500
$i=10$	6714	7429	…	16000	16000
$i=11$	6304	7257	…	18700	18700
$i=12$	1446	2893	…	20250	20250
$i=13$	*18402*	*19629*	…	*34350*	*34350*
$i=14$	19388	20680	…	36190	36190

Case II

$B'_{it'}$	$t'=1$	$t'=2$	…	$t'=14$	$t'\geq15$
	$t=15$	$t=16$	…	$t=28$	$t\geq29$
$i=1$	9393	10286	…	21000	21000
$i=2$	9714	10929	…	25500	25500
$i=3$	2804	4057	…	19100	19100
$i=4$	21321	22743	…	39800	39800
$i=5$	*1429*	*2857*	…	*20000*	*20000*
$i=6$	8589	9429	…	19500	19500
$i=7$	*6636*	*8411*	…	*29720*	*29720*
$i=8$	12018	13086	…	25900	25900
$i=9$	7429	8357	…	19500	19500
$i=10$	6714	7429	…	16000	16000
$i=11$	6304	7257	…	18700	18700
$i=12$	1446	2893	…	20250	20250
$i=13$	*14402*	*15629*	…	*30350*	*30350*
$i=14$	19388	20680	…	36190	36190

The differences observed for $B'_{it'}$ between *cases I* and *II* are due to the difference in total ore feedings for the first 14 periods. The optimal solutions for each scenario considering the two cases are presented in Table 7.

Table 7 : Optimal solutions x_{ij}

	Scenario A				Scenario B					
Case	I	II	I	II	I	II	I	II	I	II
Order k	5		4		5		3		4	
Output λ_k	1		2		4		3		2	
$i=1$	3397	3816	720	7725	9036	882	371	0	7404	720
$i=2$	0	0	0	0	11200	15008	2548	1792	0	0
$i=3$	0	10667	13782	9333	0	2031	0	0	7320	13782
$i=4$	0	0	0	0	0	0	0	0	0	0
$i=5$	14009	4445	0	824	259	0	0	0	5750	0
$i=6$	1215	0	0	0	1740	0	0	0	0	0
$i=7$	9707	9128	24371	11417	49	0	0	520	11125	24371
$i=8$	0	0	0	0	0	0	0	0	0	0
$i=9$	0	0	0	0	6445	0	0	2322	0	0
$i=10$	0	0	1127	0	0	0	4000	2873	0	1127
$i=11$	11673	11944	0	10701	1363	18272	0	0	8402	0
$i=12$	0	0	0	0	0	3807	0	993	0	0
$i=13$	0	0	0	0	0	0	12928	14000	0	0
$i=14$	0	0	0	0	9907	0	20153	17500	0	0
Demand D_k	40000	40000	40000	40000	40000	40000	40000	40000	40000	40000

The results obtained by the optimization model concerning our criteria (10) for the different scenarios and cases are grouped in Table 8. In *case I*, the total deviance of order 2 concerning internal output 1 in the reference solution is of approximately 770T (Table 5) while the deviance of order 5 concerning the same internal output in scenario A is approximately 2,755T (Table 8). This difference is explained by the stock level of indispensable inputs at the end of period 14 and the cumulated availability $B'_{it'}$ for these critical

inputs. On the other hand, the deviance of the same order in *case II* is approximately 779T due to the presence of security stock for the indispensable inputs. The analysis of the total deviance shows that the max optimal deviance for internal output (j=1) is 432T. The max deviance obtained by introducing both scenarios in the absence of security stock (*case I*) for the same output is approximately 2,755 T, while this value becomes 1,444 T for *case II* where we consider the constraint on safety stocks. For the other internal output (j=2), the max optimal deviance is 567 T. In *case I* the max deviance achieved is 1,017 T, unlike *case II* where this value was below 799 T.
We note that the absence of security stock (*case I*) greatly increases the max optimal value previously observed, whereas with *case II* the where security stocks enables to approximate the optimal deviances.
This example demonstrates that reliance on security stocks combined with the use of the dynamic blending approach enables coping with unexpected situations with maximum flexibility. Remember that this example is provided for illustration purposes and does not amount to a demonstration of the superiority of a solution with security stock, which, as noted above, is logically obvious ().

Table 8 : Total internal deviance

Scenario	Case	Constraint on Secutity Stock	Internal orders	c =1 Internal Output Deviance	c =2 Internal Output Deviance	c =3 Internal Output Deviance	c =4 Internal Output Deviance	c =5 Internal Output Deviance	Internal Output Total Deviance
A	I	No	k = 5	812.62	560.00	44.41	1337.66	0.00	2754.69
			k = 4	0.00	91.22	0.00	391.38	0.00	482.60
	II	Yes	k = 5	390.99	560.00	79.66	413.61	0.00	1444.26
			k = 4	0.00	560.00	71.97	147.25	0.00	779.22
B	I	No	k = 4	0.00	560.00	65.6767	391.021	0.00	1016.70
	II	Yes	k = 4	0.00	91.2246	0.00	391.375	0.00	482.60

4. Conclusion

In this article we propose a new dynamic blending method linked to minimum stock build up, tested in the case of a mining chain, to control risk related to demand volatility. These security stocks are defined only for critical source ores to enable a week's production of any output. This work has led to sizing minimal stocks to be built up in an uncertain universe and has enabled us to define an optimization criterion relevant to the mining sector. Finally, for different scenarios, it has enabled to define the minimum stock levels in relation to the objective variable.
To conclude, our study points to research avenues regarding: *i*) refining the objective function by taking into account a marginal cost of deviance per component (as defined by constraint (7)), and *ii*) introducing constraints on some input removals from the mine in order to free up access to lower mine layers that are planned to be extracted.

5. References

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