

7 Appendix

In the Appendix, we provide more extensive details on the methodology to detect structural changes developed by Zeileis et al. (2003) and Zeileis (2006).

7.1 MOSUM-based Processes

With reference to Section 5.1, another possibility to detect a structural change is to analyze moving sums of residuals (instead of using cumulative sums of the same residuals). The resulting empirical fluctuation process does not contain the sum of all residuals up to a certain time t , but the sum of a fixed number of residuals in a data window determined by the bandwidth parameter $h \in (0, 1)$, which is moved over the whole sample period. The OLS-based MOSUM process is defined by:

$$\begin{aligned} M_n^0(t | h) &= \frac{1}{\hat{\sigma}\sqrt{n}} \left(\sum_{i=\lfloor N_n t \rfloor + 1}^{\lfloor N_n t \rfloor + \lfloor nh \rfloor} \hat{u}_i \right) \quad (0 \leq t \leq 1 - h) \\ &= W_n^0 \left(\frac{\lfloor N_n t \rfloor + \lfloor nh \rfloor}{n} \right) - W_n^0 \left(\frac{\lfloor N_n t \rfloor}{n} \right) \end{aligned} \quad (13)$$

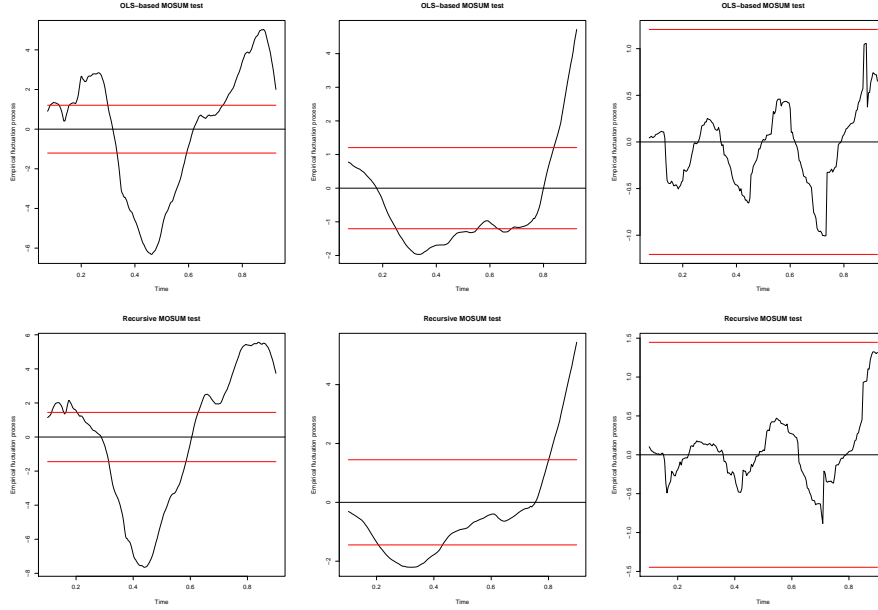
with $N_n = \frac{(n - \lfloor nh \rfloor)}{(1 - h)}$. Similarly, the Recursive MOSUM process is defined by:

$$\begin{aligned} M_n(t | h) &= \frac{1}{\tilde{\sigma}\sqrt{\eta}} \left(\sum_{i=k + \lfloor N_\eta t \rfloor + 1}^{k + \lfloor N_\eta t \rfloor + \lfloor \eta h \rfloor} \tilde{u}_i \right) \quad (0 \leq t \leq 1 - h) \\ &W_n \left(\frac{\lfloor N_\eta t \rfloor + \lfloor \eta h \rfloor}{\eta} \right) - W_n \left(\frac{\lfloor N_\eta t \rfloor}{\eta} \right) \end{aligned} \quad (14)$$

with $N_\eta = \frac{(\eta - \lfloor \eta h \rfloor)}{(1 - h)}$. The limiting process for the empirical MOSUM processes are a Brownian bridge or the increments of a Brownian motion, respectively (Chu et al. (1995a)). If again a single structural shift is assumed at t_0 , then both MOSUM paths should also have a strong shift around t_0 . We have

set the bandwidth parameter $h = 0.2$.

Figure 8: OLS- (top panel) and Recursive- (bottom panel) based MOSUM processes for EGARCH (left), IV (middle), and RV (right) Carbon Volatility Series



As shown in Figure 8, the MOSUM processes computed yield to the same conclusion as in Section 5.1: both EGARCH and IV carbon volatility series exhibit some instability (with peaks highlighting structural shifts), while the RV process remains within the boundaries.

7.2 Estimates-based Processes

Instead of defining fluctuation processes on the basis of residuals, they can be equally based on estimates of the unknown regression coefficients. With the same ideas as for the residual-based CUSUM- and MOSUM-type processes, the $k \times 1$ vector β is estimated either recursively with a growing number of observations (Ploberger et al. (1989)):

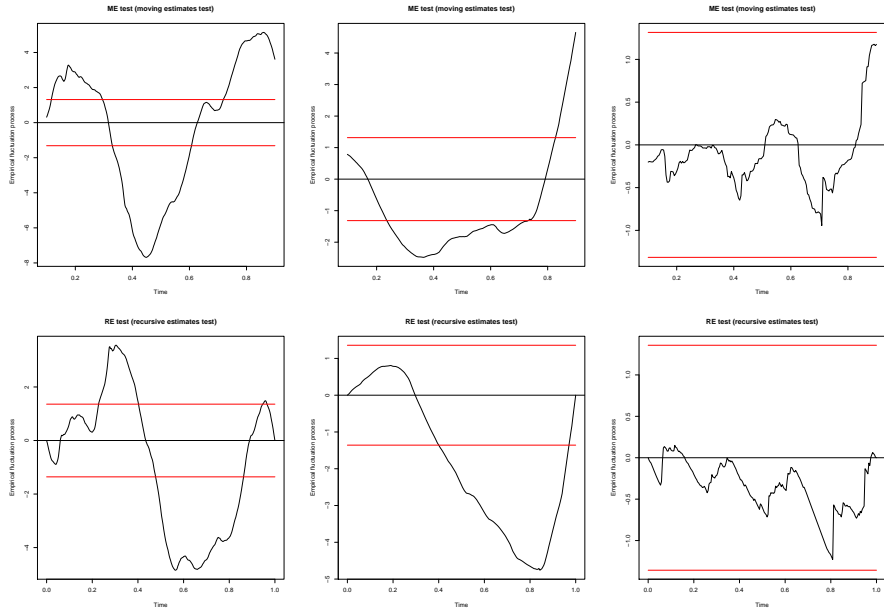
$$Y_n(t) = \frac{\sqrt{i}}{\hat{\sigma}\sqrt{n}} (X^{(i)\top} X^{(i)})^{\frac{1}{2}} (\hat{\beta}^{(i)} - \hat{\beta}^{(n)}) \quad (0 \leq t \leq 1 - h) \quad (15)$$

(with $i = \lfloor k+t(n-k) \rfloor$, $t \in [0, 1]$), or with a moving data window of constant bandwidth h and then compared to the estimates based on the whole sample (Chu et al. (1995b)):

$$Z_n(t | h) = \frac{\sqrt{\lfloor nh \rfloor}}{\hat{\sigma} \sqrt{n}} (X^{\lfloor nt \rfloor, \lfloor nh \rfloor})^\top X^{\lfloor nt \rfloor, \lfloor nh \rfloor})^{\frac{1}{2}} (\hat{\beta}^{\lfloor nt \rfloor, \lfloor nh \rfloor} - \hat{\beta}^{(n)}) \quad (0 \leq t \leq 1-h) \quad (16)$$

The limiting processes are a k -dimensional Brownian Bridge or the increments thereof, respectively. Under a single shift alternative, the recursive estimates processes should have a peak, and the moving estimates process should again have a shift close to the shift point t_0 .

Figure 9: Moving Estimates (top panel) and Recursive Estimates (bottom panel) processes for EGARCH (left), IV (middle), and RV (right) Carbon Volatility Series



It can be seen in Figure 9 that the EGARCH and IV processes exceed their boundaries (both for the Moving and Recursive Estimates). Hence, there is evidence for structural change, as commented in Section 5.1.

7.3 F -Statistics

Formally, F -statistics allow to test against a single-shift alternative of unknown timing, based on a sequence of F -statistics for a change at time i . OLS residuals $\hat{u}(i)$ from a segmented regression (i.e. one regression for each subsample with breakpoint i) are compared to the residuals \hat{u} from the unsegmented model:

$$F_i = \frac{\hat{u}^\top \hat{u} - \hat{u}(i)^\top \hat{u}(i)}{\frac{\hat{u}(i)^{\text{top}} \hat{u}(i)}{(n-2k)}} \quad i = n_h, \dots, n - n_h (n_h \geq k) \quad (17)$$

with $n_h = \lfloor nh \rfloor$ a trimming parameter set to $h = 0.2$. As detailed in Section 5.2.1, H_0 is rejected if the supremum ($\text{sup}F$) or the average ($\text{ave}F$) of the F -statistics is too large (Andrews and Ploberger (1994)). Asymptotic p -values of these tests are given by Hansen (1997).

7.4 Breakpoints Dectection

According to Bai and Perron (2003), the optimal segmentation satisfies the recursion:

$$RSS(I_{m,n}) = \min_{mn_h \leq i \leq n - n_h} [RSS(I_{m-1,i} + r_{ss}(i+1, n))] \quad (18)$$

For each point i , we need to find the ‘‘optimal previous partner’’ if i was the last breakpoint in an m -partition. This can be derived from a triangular matrix of $r_{ss}(i, j)$ with $ji \geq n_h$, the computation of which is made easier by the recursive relation $r_{ss}(i, j) = r_{ss}(i, j1) + r(i, j)^2$, where $r(i, j)$ is the recursive residual at time j of a sample starting at i .