"Agreeing to Disagree" Type Results under Ambiguity

Adam Dominiak * — Virginia Polytechnic Institute

Jean-Philippe Lefort † — Université Paris-Dauphine

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Abstract

In this paper we characterize conditions under which it is impossible that non-Bayesian agents "agree to disagree" on their individual decisions. The agents are Choquet expected utility maximizers in the spirit of Schmeidler (1989, *Econometrica* **57**, 571-587). Under the assumption of a common prior capacity distribution, it is shown that whenever each agent's information partition is made up of unambiguous events in the sense of Nehring (1999, *Mat. Soc. Sci.* **38**, 197-213), then it is impossible that the agents disagree on the common knowledge decisions, whether they are posterior capacities or posterior Choquet expectations. Conversely, an agreement on posterior Choquet expectations - but not on posterior capacities - implies that each agent's private information consists of Nehring-unambiguous events. These results indicate that under ambiguity - contrary to the standard Bayesian framework - asymmetric information matters and can explain differences in common knowledge decisions due to the ambiguous nature of agents' private information.

Keywords: Ambiguity, capacities, Choquet expected utility, unambiguous events, updating, asymmetric information, common knowledge, agreement theorem

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*Department of Economics, Virginia Polytechnic Institute and State University (Virginia Tech), Blacksburg, VA 24061-0316, USA, E-mail: dominiak@vt.edu

†Université Paris-Dauphine, LEDa Place du Maréchal de Lattre de Tassigny, 75 775 Paris cedex 16, France, E-mail: jplefort1@yahoo.fr

1 Introduction

In his celebrated article "Agreeing to Disagree", Aumann (1976) challenged the role that asymmetric information plays in the context of interpersonal decision problems under uncertainty. Presupposing that agents are Bayesian and share an identical prior probability distribution, Aumann showed that the agents cannot "agree to disagree" on their posterior beliefs. More precisely, whenever agents' posterior beliefs for some fixed event are common knowledge, then these posteriors must coincide, despite the fact that the posteriors may be conditioned on diverse information. This remarkable result implies that whenever a group of agents come to common knowledge of decisions then these decisions must be made as if there where no private information at all. In this paper, we scrutinize the role of asymmetric information among non-Bayesian agents. In essence, we demonstrate that differences in commonly known decisions are possible due to the ambiguous character of agents' private information.

Within the Bayesian framework, Aumann's impossibility result has been extended to more abstract decisions such as posterior expectations (Geanakoplos and Sebenius (1983)) and actions maximizing posterior expectations (Milgrom (1981), Milgrom and Stokey (1982) and Bacharach (1985)). These "agreeing to disagree" type results, also referred to as probabilistic agreement theorems, are often viewed as pointing out limitations of the explanatory power of asymmetric information. Differences in individual decisions cannot be explained *solely* by differences in agents' private information. Two approaches have been proposed in order to overcome these limitations. In the first one, Morris (1994, 1995) advocates to discard the "commonness" assumption of prior probabilities. The second approach, suggested by Monderer and Samet (1989), relies on weakening the notion of "common knowledge". Although, both of these approaches maintain the Bayesian paradigm. In this paper, we suggest an alternative approach. We maintain the assumption of common prior beliefs as well as the notion of common knowledge. Instead, we weaken the "additivity" property of subjective beliefs by allowing the agents to be non-Bayesian in the vein of the Choquet expected utility theory of Schmeidler (1989).

In Schmeidler's theory subjective beliefs are represented by a normalized and monotone (but-non-necessarily-additive) set function, called *capacity*. The notion of capacity allows to accommodate ambiguity and ambiguity attitudes into the decision making process. Ambiguity refers to situations in which probabilities for some uncertain events are known, whereas for other events they are unknown due to missing probabilistic information. The lack of probabilistic information and reaction to it, as manifested for instance in Ellsberg's (1961) type experiments, may affect agents' choices in the way that they are incompatible with subjective expected utility theory of Savage (1954). In the presence non-additive beliefs, individual decisions are made on the basis of maximizing expected utilities, which are computed by means of Choquet (1954) integrals.

Let's consider a finite group of agents facing a dynamic decision problem under ambiguity. The agents share a common prior capacity distribution over an algebra of events generated by a finite set of states. Moreover, each agent is endowed with a partition over the set of states which represents his private information. There are two stages of planning: an ex-ante and an interim stage. At the ex-ante stage all agents share identical information. At the ex-post stage, each agent receives his private information. Conditional on their private signals, the agents revise their prior preferences. Posterior preferences are derived by updating prior capacity and keeping the utility function unchanged. There are many reasonable updating rules for non-additive beliefs, with Bayes' rule being only one alternative (see Gilboa and Schmeidler, 1993). However, our results do not depend upon which updating rule is used. We only require that updating rules respect consequentialism, a property introduced by Hammond (1988). Consequentialism requires that posterior preferences are only affected by the conditioning events, i.e. agents' private information in our setup. Counterfactual events, as well as the past decision history, are immaterial for posterior decisions (see Hanany and Klibanoff (2007)). Once posterior preferences have been generated, the agents announce their individual decisions. An agreement on decisions designates situations in which it is impossible that the agents disagree on common knowledge posterior decisions. The decisions that we focus on are: posterior capacities for some fixed event, posterior Choquet expectations for a given action, and actions maximizing posterior Choquet expectations for a given set of feasible actions.

Our first objective is to characterize the properties of events in agents' information partitions which guarantee that disagreements on decisions are impossible. A natural choice for such events are events which are perceived by the agents as being unambiguous. In the Bayesian framework, in which probabilistic agreement theorems are established, all uncertain events are unambiguous. In non-Bayesian setups, however, some uncertain events may be subjectively seen as unambiguous while other events are perceived as ambiguous. Recently, several notions of revealed unambiguous events have been proposed, e.g., by Nehring (1999) and by Zhang (2002). We start our analysis by assuming that only the events which reflect agents' private information are unambiguous, while other events may be ambiguous. In turns out that Nehring's (1999) notion of unambiguous events suffices to rule out possibilities of disagreements on common knowledge decisions. More precisely, if each agent's information partition is made up of Nehring-unambiguous events then it is impossible that at some state agents' decisions are common knowledge and they are not the same. These decisions can be posterior capacities, posterior Choquet expectations, or actions maximizing posterior Choquet expectations. However, disagreements on commonly known decisions may occur as soon as one departs from the notion of Nehring-unambiguous events. Especially when adapting a slightly weaker notion of unambiguous events, as proposed by Zhang (2002), the agents may "agree to disagree" on their individual decisions. We exemplify a situation in which a disagreement on posterior beliefs among two agents whose information partitions are made up of Zhang-unambiguous events occurs. That is, the agents come to a common knowledge agreement of their posterior capacities for some fixed event. Nevertheless, these posteriors do not coincide.

Next we focus on a converse result. We consider situations in which it is impossible that the agents "agree to disagree" on their decisions. An immediate question that arises in this context is if knowing that disagreements are impossible can one infer something about the nature of the agents' private information? In principle, the answer is affirmative. However, what we may infer about the nature of agents' private information depends on the type of decisions that the agents "agree to agree" on. Assuming that disagreements on posterior capacities are impossible, we can show that nothing can be concluded about the properties of the events in agents' information partitions. This is because one can always find a capacity distribution and an updating rule for prior beliefs such that an agreement on posterior beliefs holds true. Nevertheless, the events in agents' information partitions will be neither Nehring- nor Zhang-unambiguous events. However, when an agreement is reached on posterior Choquet expectations, and on actions maximizing posterior Choquet expectations as well, then each agent's private information must be made up of Nehring-unambiguous events.

This paper is organized as follows. The next section introduces the capacity model of Schmeidler (1989). First, the Choquet expected utility preferences are defined, and then, the notion of unambiguous events in the sense of Nehring (1999) and Zhang (2002) are presented. In Section 3, the Choquet expected utility model is extended to dynamic choice situations. In Section 4, we introduce the standard epistemic framework used for modeling interpersonal decision problems with differential information. In Section 5, the sufficient condition for the impossibility of "agreeing to disagree" on individual decisions is established. This section ends with an example demonstrating a possibility of disagreement about common knowledge posterior capacities. In Section 6, the necessary condition for the impossibility of disagreement on posteriors Choquet expectations is established and proven. We close this section with a brief discussion on the meaningfulness of consequentialism in the context of interpersonal decision problems with differential information. In Section 7, a no-speculative trade corollary is established. Finally, we conclude in Section 8.

2 Static Choquet Preferences

In this section we recall the main tenets of the *Choquet expected utility* theory pioneered by Schmeidler (1989). We consider a finite set Ω of states. An event E is a subset of Ω. Let $\mathcal{A} = 2^{\Omega}$ be the set of all subsets of Ω. For any $E \subset \Omega$ we denote $\Omega \setminus E$, the complement of E, by E^c. Subjective beliefs over uncertain events are represented by *capacities.* A capacity $\nu : \mathcal{A} \to \mathbb{R}$ is a normalized and monotone set function, i.e., *i)* $\nu(\emptyset) = 0,\ \nu(\Omega) = 1$ and *ii)* $\nu(E) \leq \nu(F)$ whenever $E \subset F \subset \Omega$. Capacities are not required to be additive, although they must satisfy the monotonicity property. In terms of qualitative beliefs, monotonicity has a natural interpretation; "larger" events, with respect to the set inclusion, are regarded as "more likely".

Let X be a set of consequences. A mapping $f : \Omega \to X$ assigning consequences to states is called an action. Let $\mathcal{F}$ be a set of all actions. We refer to a subset $\mathcal{B} \subset \mathcal{F}$ as a set of feasible actions. For a pair of actions $f, g \in \mathcal{F}$ and an event $E \in \mathcal{A}$, denote by $f_E g$ an action that assigns the consequence $f(\omega) \in X$ to each state ω in E and $g(\omega) \in X$ to each state $\omega \in E^c$. Let $\succcurlyeq$ be a preference relation defined on the set of actions $\mathcal{F}$. A preference relation $\succcurlyeq$ is said to admit *Choquet expected utility* representation if there exists a vN-M utility function $u : X \to \mathbb{R}$ and a capacity ν on $\mathcal{A}$ such that for any $f, g \in \mathcal{F}$:

$$f \succcurlyeq g \Leftrightarrow \int_\Omega u \circ f \; d\nu \geq \int_\Omega u \circ g \; d\nu. \tag{1}$$

Choquet expected utility preferences have been justified behaviorally by Schmeidler (1989), Gilboa (1987) and Sarin and Wakker (1992) for an infinite state space. Imposing some richness conditions on the set of outcomes and allowing for a finite state space, Choquet expected utility preferences has been axiomatized by **?**, Nakamura (1990) and Chew and Karni (1994).

In presence of non-additive beliefs, the expectations in (1) are computed by means of Choquet integrals. For a given action f, let $E_1, \ldots, E_n$ denote the partition ordered from the most to the least favorable events, i.e. such that $u\big(f(E_1)\big) \geq \cdots \geq u\big(f(E_n)\big)$. The ranking position of an event expresses its favorableness with respect to consequences associated with f. The *Choquet integral* of f with respect to ν and u is defined to be:

$$\int_\Omega u \circ f \; d\nu = \sum_{j=1}^{n-1} \Big[u\big(f(E_j)\big) - u\big(f(E_{j-n})\big) \Big] \; \nu\big(E_1, \ldots, E_j\big) + u\big(f(E_n)\big) \tag{2}$$

For a given capacity ν and an action f one can define a *rank-dependent probability* distribution p_f^ν on $E_1, \ldots, E_n$, where

$$p_f^\nu\big(E_j\big) = \nu\big(E_1, \ldots, E_j\big) - \nu\big(E_1, \ldots, E_{j-1}\big). \tag{3}$$

The probability $p_f^\nu(E_j)$ of E_j can be interpreted as a marginal capacity contribution of the event E_j to events $E_1, \ldots, E_{j-1}$ yielding better consequences. Accordingly, (2) can be equivalently written as an expected utility of f with respect to the rank-dependent probability distribution p_f^ν and u:

$$\int_\Omega u \circ f \; dp_f^\nu = \sum_{j=1}^{n-1} \Big[u\big(f(E_j)\big)\Big] \; p_f^\nu(E_j). \tag{4}$$

In general, actions generating distinct ranking position of states are evaluated with respect to different rank-dependent probability distributions. Only actions which induce the same ordering of events $E_1, \ldots, E_n$, also called *comonotonic* actions, are always evaluated with respect to the same rank-dependent probability distribution.[1]

In the face of ambiguity it is important to localize events that are somehow unambiguous. The intuition behind the notion of unambiguous events is, that they must support some kind of probabilistic beliefs. For Nehring (1999) ambiguity of an event is closely related to its rank dependence.[2] More precisely, Nehring calls an event U unambiguous, henceforth Nehring-unambiguous, if the probability $p_f^\nu(U)$ attached to the event does not depend on the ranking position of U; or equivalently, it does not depend upon the act f being evaluated. Accordingly, an event $U \in \mathcal{A}$ is called Nehring-unambiguous if $p_f^\nu(U) = p_g^\nu(U) = \nu(U)$ for all $f, g \in \mathcal{F}$. Let $\mathcal{A}_N^U$ be the collection of Nehring-unambiguous events.[3] Any event in $\mathcal{A}_N^U$ can be also characterized in terms of a given capacity ν. Nehring (1999) showed namely that ν is additively separable across its unambiguous events. That is, $U \in \mathcal{A}_N^U$ if and only if for all $E \in \mathcal{A}$:

$$\nu(E) = \nu(E \cap U) + \nu(E \cap U^c). \tag{5}$$

From behavioral point of view, the notion of Nehring-unambiguous events can be also characterized by applying Savage's (1954) Sure-Thing-Principle (see Sarin and Wakker (1992) and Dominiak and Lefort (2011)). That is to say, $U \in \mathcal{A}_N^U$ if and only if for any $f, g, h, h' \in \mathcal{F}$:

$$f_U h \succcurlyeq g_U h \;\Rightarrow\; f_U h' \succcurlyeq g_U h', \tag{6}$$

[1]Formally, two actions f and g are called *comonotonic* if there are now two states, ω and ω', such that $f(\omega) > g(\omega)$ and $f(\omega') < g(\omega')$.

[2]Recently, other notions of unambiguous events have been suggested in the literature, see for instance Epstein and Zhang (2001), Zhang (2002) and Ghirardato, Maccheroni, and Marinacci (2004).

[3]Nehring (1999) proved that for any capacity ν the set $\mathcal{A}_N^U$ is always an algebra.

and (6) is also satisfied when U is everywhere replaced by U^c. Otherwise, U is called ambiguous. The Sure-Thing-Principle constrained to the events U and U^c guarantees that the ranking acts $f_U h$ and $g_U h$ remains unchanged whatever are the common consequences assigned to states outside of U.

Zhang (2002) suggested an alternative definition of unambiguous events by weakening the Sure-Thing-Principle. He refers an event U to be unambiguous, henceforth Zhang-unambiguous, if replacing a constant outcome x outside of U by any other constant outcome x' does not change the ranking of acts being compared. Accordingly, an event U is Zhang-unambiguous if and only if for any action $f, g \in \mathcal{A}$ and for any outcome $x, x' \in X$:

$$f_U x \succcurlyeq g_U x \;\Rightarrow\; f_U x' \succcurlyeq g_U x'. \tag{7}$$

and (7) is also true when U is everywhere replaced by U^c. Otherwise, U is called ambiguous. Let $\mathcal{A}_Z^U$ be the collection of all Z-unambiguous events. Again, in terms of capacities Zhang (2002) showed that $U \in \mathcal{A}_Z^U$ if and only if for all $E \in \mathcal{A}$ such that $E \subset U^c$:

$$\nu(E \cup U) = \nu(E) + \nu(U). \tag{8}$$

Thus, the additive separability property of ν is satisfied only on subevents of their unambiguous complements. It is worth to mention that $\mathcal{A}_Z^U \subset \mathcal{A}_N^U$, since $\mathcal{A}_Z^U$ does not need to be an algebra.[4] It is a λ-system, a collection of events that contains Ω and that is closed under complements and disjoint unions, but not under intersections.

3 Dynamic Choquet Preferences

In the sequel, we extend the previous setup to dynamic choice situations. In dynamic choice problems there are two stages of planning: ex ante stage and interim stage. At interim stage, agents are informed that some event has occurred and incorporate this information by updating their preferences. When beliefs are probabilistic and preferences are of expected utility type, then prior preferences are updated in the Bayesian way. That is, the conditional preferences are derived by updating prior beliefs

[4]Nehring (1999) proved that for any capacity ν the set $\mathcal{A}_N^U$ is an algebra.

with accordance to Bayes' rule and by leaving the utility function unchanged. However, when beliefs are non-additive there are many possible updating procedures. In this paper, we constrain our analysis to updating rules generating conditional preferences which respect *consequentialism* and are representable by Choquet expected utilities with respect to an updated capacity and the same unconditional vNM-M utility function.

At ex-ante stage, when no information is available, we denote by $\succcurlyeq$ the unconditional Choquet expected utility preferences. At interim stage, an event E has been observed and conditional preferences are generated. Throughout the paper, we assumed that all conditioning events are non-null, i.e., $\nu(E) > 0$, and denote by $\succcurlyeq_E$ the conditional preferences over the set of actions $\mathcal{F}$. Such a conditional preference relation is viewed as governing decisions upon the realization of E. As Machina (1989) observes, updating of non-expected utility preferences may lead to conditional choices which are affected not just by the conditioning event E. Conditional preferences may be also affected by states in the counterfactual event, E^c, as well as by the the whole choice history, i.e., by prior choices and the feasible set of actions. Such a updating rule is referred to as non-consequantialist, a property introduced by Hammond (1988).

In this paper, we require that updating rules maintain *consequentialism*. That is, we consider updating rules generating conditional Choquet preferences which depend *only* on the conditioning events E by leaving the forgone uncertainty as well as the decision history immaterial for future choices.[5] Let $\mathcal{A}_E$ be the algebra of events generated by all sub-events of the the conditional event E. A consequentialist updating rule delivers conditional preferences $\succcurlyeq_E$ representable by Choquet expected utilities with respect to the unconditional vN-M utility function u and a well-defined conditional capacity $\nu(\cdot \mid E)$ on $\mathcal{A}_E$, that is, such that for all $f, g \in \mathcal{F}$:

$$f \succcurlyeq_E f \Leftrightarrow \int_\Omega u \circ f \; d\nu(\cdot \mid E) \geq \int_\Omega u \circ g \; d\nu(\cdot \mid E). \tag{9}$$

and

$$\nu(\cdot \mid E) : \mathcal{A}_E \to [0; 1]. \tag{10}$$

[5]From behavioral point of view, consequentialism states that for any action $f, g \in \mathcal{F}$ and event $E \in \mathcal{A}$, whenever $f(\omega) = g(\omega)$ for all $\omega \in E$ then $f \sim_E g$. The meaningfulness of maintaining consequentialism in the context of interpersonal decision problems will be further discussed and justified in Section 6.

There are many reasonable revision rules which guaranty that conditional capacity satisfies the required property (10). Beside the Bayes rule, proposed by Gilboa and Schmeidler (1993), there are two other prominent revision rules for capacities; the Maximum-Likelihood updating rule, introduced by Dempster (1968) and Shafer (1976), the Full-Bayesian updating rule suggested by Jaffray (1992) and Walley (1991), and the h-Bayesian updating rule of Gilboa and Schmeidler (1993). It is worth to mention, however, that the results we obtain are independent of which among the consequentialsit updating rule is used.

4 Interpersonal Decision Model

In this section we describe an epistemic framework in which agreement theorems are established. There is a finite group of agents I indexed by $i = 1, \ldots, N$. Each agent i is endowed with a partition $\mathcal{P}_i$ of Ω. The partition $\mathcal{P}_i$ represents i's private information. That is, if the true state is ω, then i is informed of the atom $\mathcal{P}_i(\omega)$ of $\mathcal{P}_i$ to which ω belongs. Intuitively, $\mathcal{P}_i(\omega)$ is the set of all states that agent i considers possible at ω, other states are deemed impossible at ω. Given this information structure it is said that the agent i knows an event E at ω if $\mathcal{P}_i(\omega) \subset E$. The event that i knows E, denoted by K_iE, is a set of all states in which i knows E, i.e.

$$K_iE = \{\omega \in \Omega : \mathcal{P}_i(\omega) \subset E\}. \tag{11}$$

An event E is common knowledge at ω if everyone knows E at ω, everyone knows that everyone knows E at ω, and so on, ad infinitum. The event that everyone knows an event E is captured by an operator $K^1 : \mathcal{A} \to \mathcal{A}$ defined as:

$$K^1 = K_1E \cap \cdots \cap K_nE = \bigcap_{i=1}^{N} K_iE. \tag{12}$$

A common knowledge operator $CK : \mathcal{A} \to \mathcal{A}$ is defined as an infinite application of the operator K^1, i.e.

$$CKE = K_1E \cap K_1K_1E \cap K_1K_1K_1K_1E \cdots = \bigcap_{m=1}^{\infty} K^m(E). \tag{13}$$

An event E is *commonly known* at ω if $\omega \in CKE$. Following Aumann (1976) and Milgrom (1981), the concept of common knowledge can be expressed equivalently as follows. Let $\mathcal{M} = \wedge_{i=1}^{N}\mathcal{P}_i$ be the meet (i.e. finest common coarsening) and $\mathcal{J} = \vee_{i=1}^{N}\mathcal{P}_i$ the joint (i.e. coarsest common refinement) of all agents' partitions. Denote by $\mathcal{M}(\omega)$ the member of $\mathcal{M}$ that contains ω. Then, E is commonly known at ω if and only if $\mathcal{M}(\omega) \subset E$.

At each state $\omega \in \Omega$, each agent $i \in I$ makes an individual decision. Let $\mathcal{D}$ be a non-empty set of possible decisions. An individual decision is determined by i's *decision rule* $d_i : \Omega \to \mathcal{D}$ which is a function of i's private information, i.e., $d_i(\omega) = d_i(\mathcal{P}_i(\omega))$. Furthermore, let $D_i(\xi_i) = \{\omega : d(\mathcal{P}_i(\omega)) = \xi_i\}$ be the event that the agent i makes a decision $\xi_i \in \mathcal{D}$. The partitional information structure can also be applied for reasoning about what the agents know about the other agents' decisions. In particular, agents' individual decisions are common knowledge at ω if and only if $\mathcal{M}(\omega) \subseteq D_1(\xi_1) \cap \cdots \cap D_N(\xi_N)$.

A collection $\mathcal{I} = (I, \Omega, \mathcal{D}, (\mathcal{P}_i, d_i)_{i\in I})$ where I is the set of agents, Ω the set of states, $\mathcal{D}$ the set of decisions, $(\mathcal{P}_i)_{i\in I}$ the agents' information partitions, and $(d_i)_{i\in I}$ the agents' decision rule is termed an *interpersonal decision model.* For a given interpersonal decision problem, the impossibility of "agreeing to disagree" in decisons designates a situation in which there is no state at which agents' individual decisions are common knowledge and not the same, regardless which private signals the agent received. Formally, agreement theorem can be stated as follows.

Agreement Theorem. *Let $\mathcal{I}$ be a interpersonal decision model. Then, if at some state ω^* the event $\bigcap_{i\in I} D_i(\xi_i)$ is common knowledge then $\xi_1 = \xi_2 = \cdots = \xi_N$.*

In general, individual decisions may described be arbitrary functions. However, in this study, as in the case of probabilistic agreement theorems, we consider individual decisions that are based on agent's subjective beliefs over uncertain events. The probabilistic theorems rely on the assumption that the agents are Bayesian and that they share a common prior probability distribution π over Ω. If the true state is ω, then the agent i is informed of the atom $\mathcal{P}_i(\omega)$ of her partition $\mathcal{P}_i$ to which ω belongs and revises the prior π given $\mathcal{P}_i(\omega)$ according to Bayes' rule. The posterior probability distribution $\pi(\cdot \mid \mathcal{P}_i(\omega))$ serves as a basis for agents' individual decions. These decisions may be just posterior probabilities for a given event E, posterior expectations for a given action f, or actions maximizing posterior expectations from a given set of feasible actions $\mathcal{B}$. Adopting the Bayesian paradigm, agreement theorem for posterior probabilities was proven by Aumann (1976) and for posterior expectations by Milgrom (1981), Geanakoplos and Sebenius (1983), and Rubinstein and Wolinsky (1990). Bacharach (1985) and Cave (1983) extended the previous results to actions maximizing posterior expectations.

In the sequel, we extend the probabilistic agreement theorems to the non-Bayesian setup in which individuals decisions are based on subjective beliefs represented by a common-but-non-necessarily-additive prior distribution.

5 Agreement Theorems under Ambiguity

Throughout the paper we consider interpersonal decision models $\mathcal{I}$ with agents being endowed with Choquet expected utility preferences. Furthermore, it is assumed that the agents share a common capacity distribution ν on the state space Ω where $\nu(\mathcal{P}_i(\omega)) > 0$ for all states $\omega \in \Omega$ and for all $i \in I$. If the true state is ω, each agent i revises the prior capacity ν given her private information $\mathcal{P}_i(\omega)$ by applying a consequantialist updating rule (see Section 3). The updated $\nu(\cdot \mid \mathcal{P}_i(\omega))$ serves as a basis for agent i's decisions. In the perfect analogy to the Bayesian framework we focus on three types of individual decisions:

i) conditional capacities for some event $E \in \mathcal{A}$,

$$d_i(\omega) = \nu(E \mid \mathcal{P}_i(\omega)), \tag{14}$$

ii) conditional Choquet expectations for some action $f \in \mathcal{F}$,

$$d_i(\omega) = \int_\Omega u \circ f \; d\nu(\cdot \mid \mathcal{P}_i(\omega)). \tag{15}$$

iii) optimal actions from a given set of feasible actions $\mathcal{B} \subset \mathcal{F}$:

$$d_i(\omega) = \max_{f \in \mathcal{B}} \int_\Omega u(f) \; d\nu(\cdot \mid \mathcal{P}_i(\omega)). \tag{16}$$

We refer to situations in which agreement theorem holds true and agents' individual decisions are conditional capacities (14) as *Agreement in Choquet Beliefs*, when decisions are conditional Choquet expectations (15) as *Agreement in Choquet Expectations*, and when decisions are actions maximizing conditional Choquet expectations (16) as *Agreement in Choquet Actions.*

It is well-known that in the presence of arbitrary decision rules agreement theorem holds true whenever agents are "like-minded", i.e. they follow the same decision rule, and their decisions satisfy the *union-consistency* condition (see e.g. Cave (1983), Bacharach (1985) and Samet (2010)). Let $E_1, \ldots, E_n$ be a partition of Ω. The union-consistency condition requires that, if an agent i makes the same decision ξ_i knowing which of the mutually exclusive events E_j has occurred, then she also should make the same decision ξ_i without knowing which one occurred, i.e. $E_1 \cup \ldots \cup E_n$.

Union Consistency. *Let $E_1, \ldots, E_n$ of Ω be a partition of Ω. The decision function d_i satisfies union-consistency if and only if:*

$$d_i(E_1) = \cdots = d_i(E_n) = \xi_i \quad \Rightarrow \quad d(\bigcup_{j=1}^{n} E_j) = \xi_i. \tag{17}$$

Bacharach (1985) refers to condition (17) as a"[...] fundamental principle of rational decision-making". Note, in the class of probabilistic models, decision functions such as conditional probabilities, conditional expectations, as well as actions maximizing conditional expectations, satisfy the union-consistency condition on *any* partition. In non-probabilistic models, however, the decision function may satisfy union-consistency on some partitions, but not on other ones.

For this reason we focus on a fix partition and look at properties of events of that partition which are sufficient for a decision function $d(\cdot)$ to satisfy the union-consistency condition. It turns out that the decision function $d_i(\cdot)$, defined either as conditional capacities, or conditional Choquet expectations, or actions maximizing conditional expectations, satisfies the union-consistency on partitions made up of N-unambiguous events. This condition on its own is sufficient for agreement theorem to hold true under ambiguity. That is, if each agent i's private information is represented by a partition $\mathcal{P}_i$ made up of N-unambiguous events, then the agents cannot agree to disagree on their individual decisions, whatever these decisions are; whether conditional capacities, conditional Choquet expectations or actions maximizing conditional Choquet expectations. In other words, the unambiguous character of agents' private information precludes the possibility of agreeing to disagree on individual decisions.

Again, this result occurs despite the fact that agents' decisions may be conditioned on diverse information. It is formally stated in Theorem 1.

Theorem 1. *Let ν be a common capacity distribution on Ω and let $\mathcal{A}_N^U \subset \mathcal{A}$ be a collection of N-unambiguous events. Let $P_1^i, \ldots, P_k^i, \ldots, P_K^i$ be the events in i's partition $\mathcal{P}_i$. If $P_k^i \in \mathcal{A}_N^U$ for all $k = 1, \ldots, K$ and all agents $i \in I$, then the following statements are true:*

(i) Agreement in Choquet Beliefs holds true,

(ii) Agreement in Choquet Expectations holds true.

(iii) Agreement in Choquet Actions holds true.

How strong is the sufficiency condition in Theorem 1? In particular, suppose that we adapt a weaker notion of unambiguous events, for instance, the one proposed by Zhang (2002). Is the claim still true that disagreements in commonly known decisions are impossible? As Example 1 demonstrates, the answer is negative. Even a small departure from Nehring's notion of unambiguous events may create disagreement opportunities. That is, if agent i's information partition $\mathcal{P}_i$ is made up of Z-unambiguous events, which are not N-unambiguous, then her decision function may violate the union-consistency on $\mathcal{P}_i$. Consequently, one may construct information partitions for the other agents (made up of Z or N-unambiguous events) such as there will be a state at which agents' decisions are common knowledge and do not coincide after all. In a two agent setup, Example 1 shows that even in the case when one agent's information partition consists of Z-unambiguous events, then the two agents may disagree on their common known posterior capacities.

Example 1 (*Disagreement in Choquet Beliefs*)**.** *Consider an interpersonal decision model $\mathcal{I}$ with two agents $I = \{A, B\}$, called Anna and Bob, the set of states $\Omega = \{\omega_1, \omega_2, \omega_3, \omega_4\}$, the set of decisions $\mathcal{D} = [0, 1]$ and the decision function defined as in (14). Let $\mathcal{P}_A = \{\{\omega_1, \omega_2\}, \{\omega_3, \omega_4\}\}$ and $\mathcal{P}_B = \{\omega_1, \omega_2, \omega_3, \omega_4\}$ be the agents' information partitions. Anna and Bob face the following capacity distribution on $\mathcal{A}$:*

$$
\begin{aligned}
&\nu(\omega_j) = \tfrac{1}{10}, && \textit{for any} \quad j = 1, \ldots, 4,\\
&\nu(\omega_j, \omega_k) = \tfrac{1}{2}, && \textit{for any} \quad j + k \neq 5,\\
&\nu(\omega_j, \omega_k) = \alpha, && \textit{for any} \quad j + k = 5 \ \textit{where}\ \alpha \in [\tfrac{1}{10}; \tfrac{1}{2}),\\
&\nu(\omega_j, \omega_k, \omega_l) = \tfrac{6}{10}, && \textit{for any} \quad j, k, l = 1, \ldots, 4.
\end{aligned}
$$

Note, all events $\{\omega_j, \omega_k\}$ with $j + k \neq 5$ are Z-unambiguous, but not N-unambiguous. To see this, consider the event $\{\omega_1, \omega_2\}$ and its complement $\{\omega_3, \omega_4\}$. On this partition the capacity sums up to one. Now, if these events were N-unambiguous, then according to the additive separability property (5) the capacity for the event $\{\omega_1, \omega_3\}$ were $\nu(\omega_1, \omega_3) = \nu(\omega_1) + \nu(\omega_3) = \frac{1}{5}$, but not $\frac{1}{2}$. One can verify that the capacity ν satisfies the additive separability property (8) only on subevents of its unambiguous complements. For instance, $\nu(\omega_1, \omega_2, \omega_3) = \nu(\omega_1, \omega_2) + \nu(\omega_3) = \frac{6}{10}$. Accordingly, $\mathcal{A}_Z^U = \{\emptyset, \{\omega_1, \omega_2\}, \{\omega_3, \omega_4\}, \{\omega_1, \omega_3\}, \{\omega_2, \omega_4\}, \Omega\}$ is the collection of Z-unambiguous events.

Thus, Anna's partition is made up of Z-unambiguous events, and which are not N-unambiguous. Consider the event $E = \{\omega_1, \omega_3\}$. At each state $\omega \in \Omega$, Anna and Bob announce their posterior beliefs for the occurrence of E given their private information at ω. Due to Bob's private information his decision is $d_B(\omega) = \nu(E \mid \mathcal{P}_B(\omega)) = \frac{1}{2}$ at each state. Anna has finer information than Bob, and therefore her conditional capacity is $d_A(\omega) = \nu(E \mid \mathcal{P}_A(\omega)) = \frac{1}{5}$ on all states. Note, Anna's decision function $d_A(\cdot)$ violates the union-consistency condition on $\mathcal{P}_A$. Furthermore, since $\mathcal{M} = \Omega$, the event that Anna's decision is $\frac{1}{5}$ and that Bob's decision is $\frac{1}{2}$ is commonly known at any state. That is, $\mathcal{M}(\omega) \subseteq D_A(\frac{1}{5}) \cap D_B(\frac{1}{2}) = \Omega$ for all $\omega \in \Omega$. But, these decisions are in fact not the same! This shows that, if for one agent her private information is made up of Z-unambiguous events, which are not N-unambiguous, than the union-consistency condition may be violated and it is possible that the agents end up agreeing to disagree after all!

Suppose now Anna's partition $\mathcal{P}_A = \{\{\omega_1, \omega_2\}, \{\omega_3, \omega_4\}\}$ were made up of N-unambiguous events. In this case, the capacity for the event E must be equal to $\frac{1}{5}$ due to the additive separability property (5). Now, $\frac{1}{5}$ is Bob's decision which he announces in all states. Therefore, agents decisions are commonly known at any state and in fact they are the same. Thus, when agents' private information is made up of N-unambiguous events *it is impossible for them to agree to disagree on their posterior capacities.*

6 Agreement Theorems - The Converse Result

In this section we address the following issue. Suppose that individual decisions satisfy the union consistency condition on agents' information partitions and consequently the agents cannot "agree to disagree". A natural question that arises in this context is, whether one can infer something about the nature of agents' private information, presupposing that disagreements are impossible? In principle, the answer is affirmative. However, what may be inferred depends upon the type of decisions on which agents "agree to agree". There are situations in which Agreement in Choquet Beliefs is on hand and nothing can be said about the nature of agents' private information. The reason is the following observation. One can find a common capacity distribution on $\mathcal{A}$ and a consequentialist updating rule for the prior capacity for which the conditional capacities satisfy the union consistency condition on agents' partition. Hence, as soon as the agents' conditional capacities for some fixed event $E \in \mathcal{A}$ are common knowledge then they must coincide and Agreement in Choquet Beliefs holds true. Nevertheless, agents' information partitions are ambiguous, i.e., they will be neither made up of N-unambiguous, nor of Z-unambiguous events. We elaborate this situation in Example 2.

Example 2. *Consider the interpersonal decision model $\mathcal{I}$ as it was described in Example 1. Suppose now, Anna and Bob face the following capacity distribution on $\mathcal{A}$:*

$$\nu(\omega_j) = \tfrac{1}{9}, \quad \nu(\omega_j, \omega_k) = \tfrac{1}{3}, \quad \nu(\omega_j, \omega_k, \omega_l) = \tfrac{4}{9},$$

where $j, k, l \in \{1, \ldots, 4\}$ are distinct indexes. Consider the event $E = \{\omega_j, \omega_k\}$ with $j + k = 5$. At $\omega \in \Omega$, the agents observe their private information, revise their prior beliefs in accordance with Bayes' rule, and then announce their posterior capacities for E. Note that the Bayesian update coincides here with the Maximum-Likelihood and the Full-Bayesian update. Since $d_A(\omega) = \nu(E \mid \mathcal{P}_A(\omega)) = \frac{1}{3}$ for all ω, Anna announces $\frac{1}{3}$ at any state. Since $d_B(\omega) = \nu(E \mid \mathcal{P}_B(\omega)) = \frac{1}{3}$ for all ω, Bob's announcement is also $\frac{1}{3}$ at any state. Anna's as well as Bobs's conditional capacities satisfy the union consistency condition on their partitions. At each state agents' posteriors for the event E

are common knowledge and the same. Thus, Agreement in Choquet Beliefs is on hand. However, the impossibility of "agreeing to disagree" on posterior beliefs does not indicate that Anna's or Bob's private information is in some sense unambiguous. Events in Anna's partition are made up neither of N-unambiguous, nor of Z-unambiguous events. The capacity ν does not even add up to one on Anna's partition.

Similar examples can by easily constructed within the class of ϵ-contaminated capacities. An ϵ-contaminated capacity is defined as a distortion of a probability measure π on Ω. That is, $\nu(E) = \epsilon\pi(E)$ with $\epsilon \in [0, 1]$ and $E \subsetneq \Omega$. The parameter ϵ may be interpreted as the agent's degree of perceived ambiguity. Now, conditionally on an event E define an ϵ-contaminated update of ν as Bayesian updated distorted by the parameter ϵ, i.e., $\nu(\cdot \mid E) = \epsilon\pi(\cdot \mid E)$. Obviously, this updating rule satisfies consequentialism and ensures that conditional capacities maintains the union consistency condition. Consequently, the agents sharing a common ϵ-contaminated capacity and following the ϵ-contaminated updating rule cannot agree to disagree on the values of their posteriors beliefs. However, the ϵ-contaminated capacity, by construction, is not suitable to model ambiguous and unambiguous events at same time. Then, whenever the parameter $\epsilon < 1$ then all uncertain events are perceived as being ambiguous. That is, these events are neither unambiguous in the sense of Nehring (1999) nor in the sense of Zhang (2002). Consequently, nothing can be said about the nature of events representing agents' private information knowing that the agents cannot agree to disagree on the values of ϵ-contaminated conditional capacities. In the light of these observations we conclude that Agreement in Beliefs is simply too "weak" for making any inference about the nature of agents' private information.

Now, can we say something more about the nature agents' information partitions knowing that the agents reach an Agreement in Expectations for some bets? A bet $b = x_E y$ is a function which assigns the constant outcome $f(\omega) = x \in X$ to each state ω in E and the constant outcome $f(\omega) = y \in X$ to each ω in E^c. It turns out that Agreement in Bets does not provide any information about the quality of agents' private information. This negative result is due to Proposition 1. It states that, if at some state ω it impossible that the agents agree to disagree on conditional capacities for some fixed event E, then it is also impossible at ω that they agree to disagree on conditional Choquet expectations of binary actions defined on the event E.

Proposition 1. *Let ν be a common capacity distribution ν on Ω. Let $\mathcal{P}_i$ be i's information partition and let $d_i(\cdot)$ be i's conditional capacity for some event $E \in \mathcal{A}$. Suppose that at some state ω^* Agreement in Choquet Beliefs holds true for the event E. Consider a bet $b = x_E y$ defined on the event E with $x, y \in X$. Let $\tilde{d}_i$ be i's conditional Choquet expectation of b. Then, Agreement in Expectations holds true at ω^* for the bet b.*

Hence, knowing that agents cannot agree to disagree on expectations for some bet, nothing can be said about the nature of events representing the agents' private information. Then, in the view of Example 2 we can always find a common capacity distribution and an updating rule such that for a some fixed event E Agreement in Beliefs is on hand. According to Proposition 1, the agents will also reach Agreement in Expectations for bets on the event E. Nevertheless, the agents' information partitions will neither be made up of N-unambiguous, nor of Z-unambiguous events.

Motivated by these observations we move to situations in which agents cannot agree to disagree on the expected values of many-valued actions. Again, we ask whether one can infer something about the nature of events in agents' information partitions knowing that Agreement in Expectations for some fixed action $f \in \mathcal{F}$ is present. It can be shown that Agreement in Expectations implies that agents' information partitions must be made up of N-unambiguous events. Theorem 2 state this result formally.

Theorem 2. *Let ν be a common capacity distribution on $\mathcal{A}$. Let $\mathcal{A}'$ be a sub-algebra of $\mathcal{A}$. Let $d_i(\cdot)$ be the Choquet conditional expectation for some action f in $\mathcal{F}$. If for any information partition $\mathcal{P}_i = P_1^i, \ldots, P_k^i, \ldots, P_K^i$ such that $P_k^i \in \mathcal{A}'$ for all $k = 1, \ldots, K$ and all agents $i \in I$, $d_i(\cdot)$ satisfies the union-consistency on $\mathcal{P}_i$, then $\mathcal{A}'$ is the algebra made up of N-unambiguous events.*

Theorem 1 and 2 highlight the relevance of asymmetric information in the context of interpersonal decision problems under ambiguity. Differences in commonly known expectations are possible and can be attributed to ambiguous character of agents' private information.

Note, both theorems have been established under the assumption that agents share a common capacity distribution. However, in the light of probabilistic agreement theorems, it has been argued that the assumption of common prior beliefs may be too strong and not realistic (see e.g. Aumann (1976), Morris (1995). Our results suggest it is not necessary the "commonness" assumption of prior beliefs, that make the probabilistic agreement theorems working. Instead, it is the "additivity" assumption of the prior beliefs that make asymmetric information less powerful when explaining differences of individual decisions. Put differently, the fact that common knowledge of decisions neglect asymmetric information about uncertain events can be ascribed to the unambiguous nature of these events.

Furthermore, we assumed that agents' updated preferences maintain consequentialism. However, when updating non-expected utility preferences there is another attractive property of preferences, called dynamic consistency. Dynamic consistency requires that choices made at ex-ante stage are respected by updated preferences and vice versa.[6] When both properties, consequentialism and dynamic consistency are satisfied then preferences admit subjective expected utility representation and conditional preferences are obtained by applying Bayes' rule to probabilistic beliefs (see Ghirardato (2002), Siniscalchi (2011)). Thus, when modeling dynamic choice problems in the class of non-expected utility preferences, then either consequentialism or dynamic consistency (or both) must be weakened in some respect. We argue that consequentialism cannot be dispensed with in the context of interpersonal decision problems with asymmetric information. Recall, consequentialism requires that conditional preferences are independent of states in forgone evens. If, to the contrary, one would allow updated preferences to depend upon contractual events then it would be impossible to infer what is the true impact of private information on individual decisions. Put differently, it would be possible that individual decisions diverge even though their are based on the same information. Thus violating, what Aumann (1976) calls, *"Harsanyi consistency"*.

Consider an updating rule generating the conditional capacity which depends on

[6] Formally, for any $E \in \mathcal{A}$ and $f, g \in \mathcal{F}$ such that $f(\omega) = g(\omega)$ for all $\omega \in E^c$, it is true that $f \succcurlyeq g$ if and only if $f \succcurlyeq_E g$.

the actual conditional event and on agent's information about events which are conditionally impossible. That is, if the true state is ω, then posterior beliefs, $\nu(\cdot \mid \mathcal{P}_i(\omega), \mathcal{P}_i)$, are affected not only by the actual signal $\mathcal{P}_i(\omega)$, but also by $\mathcal{P}_i \setminus \mathcal{P}_i(\omega)$. Obviously this updating rule violates consequentialism. Consequently, two agents, i and j, with the same information at some ω, $\mathcal{P}_i(\omega) = \mathcal{P}_j(\omega)$, but diiferent one outside of ω, $\mathcal{P}_i(\omega') \neq \mathcal{P}_j(\omega')$ for some $\omega \in \Omega \setminus \omega'$, will in general have diferent posterior beliefs at ω, i.e., $\nu(\cdot \mid \mathcal{P}_i(\omega), \mathcal{P}_i) \neq \nu(\cdot \mid \mathcal{P}_j(\omega), \mathcal{P}_j)$.

7 No Speculative Trade - A Corollary

In this section the no-trade theorem of Milgrom and Stokey (1982) is generalized within the class of Choquet expected utility preferences. In the view of the aforementioned results, we are able to characterize the properties of agents' private information which are sufficient to guarantee that asymmetric information alone cannot generate any profitable trade opportunities under ambiguity.

We interpret an interpersonal decision model $\mathcal{I}$ as a pure exchange economy with as a single commodity. That is, let $X = \mathbb{R}_+$ be the commodity space and call elements of $\mathcal{F}$ contingent consumption bundles. An allocation a is a family $a = [a_1, \dots, a_N]$ where each $a_i \in \mathcal{F}$ represents i's contingent consumption bundle. An initial allocation is denoted by $e = [e_1, \dots, e_N]$, where each $e_i \in \mathcal{F}$ is referred to as i's endowment. As in the previous sections, it is assumed that the agents share an identical capacity distribution ν on $\mathcal{A}$. Moreover, each agent i is characterized by her preferences over $\mathcal{F}$ which are supposed to admit Choquet expected utility representation, an initial endowment $e_i \in \mathcal{A}$, and her private information $\mathcal{P}_i$. A trade $t = [t_1, \dots, t_N]$ is an N-tuple of functions $t_i : \Omega \to \mathbb{R}$. If the true state is ω, $t_i(\omega)$ corresponds to i's net trade of the single commodity. We say that the trade t is *feasible*, if:

$$\begin{aligned} \sum_{i=1}^{N} t_i(\omega) &\leq 0 \quad \forall\, \omega \in \Omega, \\ e_i(\omega) + t_i(\omega) &\geq 0 \quad \forall\, \omega \in \Omega,\ \forall i \in I. \end{aligned} \tag{18}$$

An initial allocation e is called *ex-ante efficient* if there does not exist a feasible trade t such that at ex-ante stage each agent i prefers the contingent consumption bundle

$e_i + t_i$ to her endowment e_i, i.e.:

$$\int_\Omega u \circ (e_i + t_i)\ d\nu \geq \int_\Omega u \circ e_i\ d\nu \quad \forall i \in I. \tag{19}$$

Suppose that the agents trade to an ex-ante efficient allocation e before any information is revealed. After the receipt of private information the market is reopened and the agents have the chance to reallocate the initial allocation e through a feasible trade t. That is, when the true state is ω, each agent i observes $\mathcal{P}_i(\omega)$ and then the feasible trade t is proposed. We call the feasible trade t *acceptable* (or weakly preferable to a zero trade) if each agent i prefers the contingent consumption bundle $e_i + t_i$ to her endowment e_i given $\mathcal{P}_i(\omega)$ for all $\omega \in \Omega$, i.e.:

$$\int_\Omega u \circ (e_i + t_i)\ d\nu(\cdot \mid \mathcal{P}_i(\omega)) \geq \int_\Omega u \circ e_i\ d\nu(\cdot \mid \mathcal{P}_i(\omega)) \quad \forall \omega \in \Omega, \tag{20}$$

with strictly equality for at least one ω. In Bayesian frameworks, where all uncertainty is quantifiable by a common additive probability distribution, the receipt of private information can not create any incentives to re-trade an ex-ante efficient allocation, even though the information the agents receive may be distinct. What are the conditions on agents private information which are sufficient to ensure that the no-trade theorem still holds in the presence of common, but non-additive priors? It turns out that as long as agents' information partitions are made up of N-unambiguous events, at interim stage the agents will not find it advantageous to re-trade an initially efficient allocation. In other words, when each agent's private information is free from ambiguity it is impossible that purely speculative trade occurs only due to differences in their private information. This result is stated in the following theorem.

Corollary 7.1 (No-Trade Theorem)**.** *Let ν be a common capacity distribution on Ω and let $\mathcal{A}_N^U \in \mathcal{A}$ be a collection of N-unambiguous events. Let $P_1^i, \ldots, P_k^i, \ldots, P_K^i$ be the events in i's partition $\mathcal{P}_i$. Suppose that $P_k^i \in \mathcal{A}_N^U$ for all $k = 1, \ldots, K$ and for all agents $i \in I$. Suppose the initial allocation $e = [e_1, \ldots, e_N]$ is ex-ante efficient. Let $t = [t_1, \ldots, t_N]$ be a trade proposed at interim stage. If it is common knowledge at ω^* that t is feasible and acceptable, then $t_1(\omega^*) = \ldots = t_N(\omega^*) = 0$.*

Corollary 7.1 provides an intuitive explanation for the existence of speculative trade. As it was already stipulated by Knight (1921), it is the presence of ambiguity, or what he called "unmeasurable uncertainty", that generates profitable trade opportunities. When agents' private information is ambiguous, then, conditional on different information agents may expect gains from re-trading an initially efficient allocation. Example 3 illustrates how gains from trade may occur even when one agent's private information partition is made up of Z-unambiguous events, which are not N-unambiguous.

Example 3. *Let $X = \mathbb{R}_+$ be the set of outcomes. Consider an interpersonal decision model $\mathcal{I}$ with the set of contingent consumption bundles $\mathcal{F} = \{a \mid a : \Omega \to \mathbb{R}_+\}$ and the same information structure and the same capacity distribution as in Example 1. Let $e = [e_A = (2,0,2,0), e_B = (1,2,1,0)]$ be the initial allocation. Suppose Anna and Bob are risk neutral. By computing the Choquet expectations of e_A and e_B with respect to u and ν for both agents, we get:*

$$\int u \circ e_A \; d\nu = 2\frac{1}{10} + 1[\frac{6}{10} - \frac{1}{10}] + 0[1 - \frac{6}{10}] = \frac{7}{10}, \tag{21}$$

$$\int u \circ e_B \; d\nu = 2\frac{1}{2} + 0[1 - \frac{1}{2}] = 1. \tag{22}$$

At ex-ante stage there is no feasible trade t that would make both agents better off. In fact, the contingent consumption bundle e_B makes Anna better off, but any feasible trade would make Bob worse off. Hence, e is ex-ante efficient. Now, let ω_1 be the true state. Because of Bob's information at ω_1, i.e. $\mathcal{P}_B(\omega_1) = \Omega$, his evaluation of e_A and e_B does not change. Given Anna's information at ω_1, i.e. $\mathcal{P}_A(\omega_1) = \{\omega_1, \omega_2\}$, she updates her preferences by taking into account the conditional capacities $\nu(\omega_1 \mid \mathcal{P}_A(\omega_1)) = \nu(\omega_2 \mid \mathcal{P}_A(\omega_1)) = \frac{2}{10}$ and calculates the conditional Choquet expectations of e_A and e_B:

$$\int u \circ e_A \; d\nu(\cdot \mid \mathcal{P}_A(\omega_1)) = 2\frac{2}{10} + 1[1 - \frac{2}{10}] = \frac{12}{10}, \tag{23}$$

$$\int u \circ e_B \; d\nu(\cdot \mid \mathcal{P}_A(\omega_1)) = 2\frac{2}{10} + 0[1 - \frac{2}{10}] = \frac{4}{10}. \tag{24}$$

Now, consider the trade $t := [t_A = (1,-2,1,0), t_B = (-1,2,-1,0)]$ proposed at the interim stage. Note, since $e_A + t_A = e_C$ and $e_C + t_C = e_A$ the trade t is feasible. By (23) and (24) Anna prefers e_B to e_A and by (21) and (22) Bob prefers e_A to e_B making the trade t acceptable at ω_1. At ω_1 it is commonly known between Anna and Bob that the trade t is feasible and acceptable and t is not the null-trade. The events in Anna's *partition are Z-unambiguous, but not N-unambiguous; due to this fact differences in agents' private information matter and make a profitable trade possible.*

A few remarks with regard to the related literature are in order. Close to our approach are the contributions of Rubinstein and Wolinsky (1990) and Dow, Madrigal, and Werlang (1990). Their results are obtained without constraining the analysis to a particular class of ambiguity-sensitive preferences. Rubinstein and Wolinsky (1990) argued that Milgrom-Stokey's result is valid for any theory of decision making under uncertainty as long as preferences satisfy dynamic consistency. Dow, Madrigal, and Werlang (1990) showed that the no-trade theorem is true if and only if preferences are representable by a state-additive utility function. Corollary 7.1 can be viewed as characterizing those properties of events in information partitions on which dynamic consistency as well as state-additivity of Choquet preferences are satisfied. Then, if a fixed partition is made up of N-unambiguous events, then Choquet expected utility preferences respect dynamic consistency on that partition (see Section **??**). Furthermore, Choquet preferences respect dynamic consistency on a fixed partition if and only if the Choquet integral satisfies the additivity property constrained to that partition (see Sarin and Wakker, 1998). In two other related works, Ma (2002) and Halevy (2004) attempt to establish sufficient condition for the no-trade theorem to be true for the class of preferences violating consequentialism in some respect.

8 Conclusion

A Appendix

Proof of Theorem 1. First we show that (ii) is true.

Step 1 Consider an agent $i \in I$. Let $P_1, \ldots, P_k, \ldots, P_K$ be the events in the agent i's partition $\mathcal{P}_i$. That is $\mathcal{P}_i(\omega) = \mathcal{P}_i(\omega^{'})$ for all states $\omega, \omega' \in P_k$. Suppose that the i's information partition $\mathcal{P}_i$ is made up off N-unambiguous events, i.e. $P_k \in \mathcal{A}_N^U$ for any $k = 1, \ldots, K$. Fix an action $f \in \mathcal{F}$. Let d_i be the Choquet decision rule defined as in (15). Furthermore, we assume that the agent i computes the posterior capacity $\nu(\cdot \mid P_k)$ conditional on P_k by applying Bayes' rule. This assumption is reasonable, since all other updating rules, among others those defined in Section **??**, coincide with Bayes' rule when conditioning on partitions made up off N-unambiguous events (see Proposition **??**). Suppose that for any index $k = 1, \ldots, K$ the conditional Choquet expectation of f given P_k is equal to ξ:

$$d_i(P_1) = \ldots d_i(P_k) = \ldots = d_i(P_K) = \xi, \tag{25}$$

where:

$$\begin{aligned} d_i(P_k) &= \int_\Omega u \circ f \; d\nu(\cdot \mid P_k) \\ &= \sum_{j=1}^{n-1} [u(x_j) - u(x_{j+1})] \; \frac{\nu(E_1, \ldots, E_j \; \cap P_k)}{\nu(P_k)} \\ &= \xi. \end{aligned}$$

By the additive separability condition (5) of N-unambiguous events the Choquet expected value of f with respect to the prior capacity ν can be written as:

$$\int_\Omega u \circ f \; d\nu \;\; = \;\; \sum_{k=1}^{n} \int_{P_k} u \circ f \; d\nu. \tag{26}$$

Thus, we obtain:

$$\int_\Omega u \circ f \; d\nu(\cdot) \;\; = \;\; \sum_{k=1}^{n} \nu(P_k) \int_{P_k} u \circ f \; d\nu(\cdot \mid P_k) = \sum_{k=1}^{n} \nu(P_k) \; \xi = \xi.$$

Therefore, $d_i(\bigcup_{j=1}^{K} P_j) = \xi$ shows that the Choquet conditional expectations of f satisfy the union-consistency on partitions made up off N-unambiguous events.

Step 2 Fix an agent i. Let $D_i(\xi_i) = \{\omega : d(\mathcal{P}_i(\omega)) = \xi_i\}$ be the event that the i's decision is ξ_i. Suppose at some state ω^* the event $\bigcap_{i \in I} D_i(x_i)$ is common knowledge, i.e. $\mathcal{M}(\omega^*) \subseteq \bigcap_{i \in I} D_i(x_i)$. Denote by $Q = \mathcal{M}(\omega^*)$ the member of $\mathcal{M}$ that contains ω^*. Let $Q_1, \ldots, Q_l, \ldots, Q_L$ be events in i's partition $\mathcal{P}_i$ such that $Q = \bigcup_{l=1}^{L} Q_l$. By assumption, $\mathcal{M}(\omega^*) \subseteq D_i(\xi_i)$ and $d_i(\mathcal{P}_i(\omega)) = \xi_i$ for any $\omega \in Q_l$ with $l = 1, \ldots, L$. Furthermore, since each event Q_l is N-unambiguous the decision function $d_i(\cdot)$ satisfies the union-consistency by Step 1. Thus, $d_i(Q) = \xi_i$. The same argument is true for any agent $j \in I \setminus \{i\}$. That is, $d_j(Q) = \xi_j$. Thus, $\xi_1 = \ldots = \xi_N$. The fact that the Sure-Thing-Principle is sufficient for agreement theorem to be true has been proved, among others, by Bacharach (1985, Theorem 3, p.182).

□

Proof of Proposition 1. Fix an event E. Let $D_i(\alpha_i) = \{\omega : \nu(E \mid \mathcal{P}_i(\omega)) = \alpha_i\}$ be the event that i's conditional capacity of E is α_i. Suppose that at some state ω^* the agents reached Agreement in Beliefs. That is, the event $\bigcap_{i \in I} D_i(\alpha_i)$ is common knowledge at ω^* and agents' conditional capacities for E are the same, $\alpha_1 = \ldots = \alpha_N$.

For any $x, y \in X$ such that $x \succ y$ let $b = x_E y$ be a bet. Fix an agent i. Let $P_1, \ldots, P_k, \ldots, P_n$ be events in i's information partition $\mathcal{P}_i$. Let $d_i(P_k)$ be the i's conditional Choquet expectation of b given P_k. Suppose that $d_i(P_k) = \beta_i$ for any $k = 1, \ldots, K$, i.e.:

$$\begin{aligned} d_i(P_k^i) &= \int_\Omega u \circ b \; d\nu(\cdot \mid P_k) \\ &= [u(x) - u(y)]\nu(E \mid P_k^i) + u(y) \\ &= \beta_i, \end{aligned}$$

Rearranging the above equation we get for any $k = 1, \ldots, n$:

$$\begin{aligned} \nu(E \mid P_k^i) &= \frac{\beta - u(y)}{u(x) - u(y)} \\ &= \alpha_i. \end{aligned}$$

Thus, since Agreement in Beliefs holds it follows that:

$$\begin{aligned} \nu(E \mid \bigcup_{k=1}^{K} P_k) &= \frac{\beta - u(y)}{u(x) - u(y)} \\ &= \alpha_i. \end{aligned}$$

Therefore:

$$\begin{aligned} d_i(\bigcup_{k=1}^{K} P_k^i) &= \int_\Omega u \circ b \; d\nu(\cdot) \\ &= [u(x) - u(y)]\nu(E) + u(y) \\ &= \beta_i. \end{aligned}$$

Let $D_i(\beta_i) = \{\omega : d(\mathcal{P}_i(\omega)) = \beta_i\}$ be the event that i's conditional Choquet expectation of b is β_i. Since the Sure-Thing Condition holds, the event $\bigcap_{i \in I} D_i(\beta_i)$ is common knowledge at ω^* and in fact $\beta_1 = \ldots = \beta_N$. Therefore, we conclude that an Agreement in Beliefs implies an Agreement in Expectations for binary actions. □

Proof of Theorem 2. Let $\mathcal{A}'$ be a sub-algebra of $\mathcal{A}$. In Step 1 we show that for any event $E \in \mathcal{A}'$ and all events $F, G \in \mathcal{A}$ such that $\emptyset \not\subseteq F, G \not\subseteq E^c$, the capacity ν has the following property:

$$\nu(E \cup G) - \nu(G) = \nu(E \cup F) - \nu(F). \tag{27}$$

In Step 2 it is shown that for any event $E \in \mathcal{A}'$ the capacity ν is separable among all subevents of E^c, i.e. for any $F \subset E^c$:

$$\nu(E) = \nu(E \cup F) - \nu(F) = 1 - \nu(E^c). \tag{28}$$

Step 1. Let $A_1, A_2, A_3 \in \mathcal{A}$ be a collection of disjoint events partitioning the event E^c. Consider an action $f = (x_1 A_1, x_2 A_2, x_3 A_3)$ with outcomes $x_1, x_2, x_3 \in X$ such that $x_1 < x_2 < x_3$. Suppose that the Choquet expected utility of f conditional on E^c equals x, i.e.:

$$f \sim_{E^c} x. \tag{29}$$

By computing the conditional Choquet expectation of f we get:

$$\begin{aligned} \int u \circ f \; d\nu(\cdot \mid E^c) = u(x_1) \; \Big[1 - \nu(A_2, A_3 \mid E^c)\Big] \\ + u(x_2) \; \Big[\nu(A_2, A_3 \mid E^c) - \nu(A_3 \mid E^c)\Big] + u(x_3) \; \nu(A_3 \mid E^c) = \; x. \end{aligned} \tag{30}$$

Now, consider an action $g = f_{E^c} x$. By the assumption (29) the conditional Choquet expectation of g satisfies the union-consistency on the partition E, E^c, i.e.:

$$\int u \circ g \; d\nu(\cdot \mid E^c) = x \quad \text{, and} \quad \int u \circ g \; d\nu(\cdot \mid E) = x$$

implies

$$\int u \circ g \; d\nu(\cdot \mid \Omega) = x. \tag{31}$$

When computing the unconditional Choquet integral (31) of g with respect to ν we consider two cases. In Case 1 we consider any x such that $x_2 < x < x_3$. In Case 2, we consider any x such that $x_1 < x < x_2$.

Case 1. For any x such that $x_2 < x < x_3$ the unconditional Choquet integral of g yields:

$$\begin{aligned} \int u \circ g \; d\nu &= u(x_1) \Big[1 - \nu(A_2, E, A_3)\Big] + u(x_2) \Big[\nu(A_2, E, A_3) - \nu(E, A_3)\Big] \\ &\quad + u(x) \Big[\nu(E, A_3) - \nu(A_3)\Big] + u(x_3) \; \nu(A_3) \\ &= x. \end{aligned} \tag{32}$$

Solving Equation (32) for x we get:

$$\begin{aligned} \int u \circ g \; d\nu(\cdot \mid E) &= \frac{1}{1 - \nu(E, A_3) + \nu(A_3)} \Big\{ u(x_1) \Big[1 - \nu(A_2, E, A_3)\Big] \\ &\quad + u(x_2) \Big[\nu(A_2, E, A_3) - \nu(E, A_3)\Big] + \; u(x_3) \; \nu(A_3) \Big\} \\ &= x. \end{aligned} \tag{33}$$

Equation (33) is true for any x_1, x_2, x_3 such that $x_1 < x_2 < x_3$ and any $g = f_{E^c}x$ with x such that $x_2 < x < x_3$. Thus, when fixing the values x_1, x_2 and varying the value of x_3 we get from Equation (30) and (33):

$$\nu(A_3 \mid E) = \frac{\nu(A_3)}{1 - \nu(E, A_3) + \nu(A_3)} \tag{34}$$

Case 2. For x such that $x_1 < x < x_2$ computing the unconditional Choquet integral of g yields:

$$\begin{aligned} \int u \circ g \; d\nu &= u(x_1) \Big[1 - \nu(E, A_2, A_3)\Big] + u(x) \Big[\nu(E, A_2, A_3) - \nu(A_2, A_3)\Big] \\ &\quad + u(x_2) \Big[\nu(A_2, A_3) - \nu(A_3)\Big] + u(x_3) \; \nu(A_3) \\ &= x. \end{aligned} \tag{35}$$

Solving the above Equation (35) for x'' we get:

$$\begin{aligned}\int u \circ g \, d\nu(\cdot \mid E) &= \frac{1}{1-\nu(E,A_2,A_3)+\nu(A_2,A_3)}\Big\{u(x_1)\,\Big[1-\nu(E,A_2,A_3)\Big] \\ &\quad + u(x_2)\,\Big[\nu(E,A_2,A_3)-\nu(A_2,A_3)\Big] + \; u(x_3)\;\nu(A_3)\Big\} \\ &= x. \end{aligned} \tag{36}$$

Again, Equation (36) is true for any x_1, x_2, x_3 such that $x_1 < x_2 < x_3$ and any $g = f_{E^c}x$ with x such that $x_1 < x < x_2$. Thus, when fixing the values x_1, x_2 and varying the value of x_3 we get from Equations (30) and (36):

$$\nu(A_3 \mid E) = \frac{\nu(A_3)}{1-\nu(E,A_2,A_3)+\nu(A_2,A_3)}. \tag{37}$$

From Equations (34) and (37) we conclude that:

$$\nu(E,A_3)-\nu(A_3) = \nu(E,A_2,A_3)-\nu(A_2,A_3). \tag{38}$$

Now, we repeat the same argument for an action $h = (y_1A_1, y_2A_2, y_3A_3)$ with outcomes $y_1, y_2, y_3 \in X$ such that $y_1 < y_3 < y_2$. Suppose that $h \sim_{E^c} y$ and construct an action $k = h_{E^c}y$. By construction, the conditional Choquet expectation of k satisfies the union-consistency on the partition E, E^c. After having considered two cases, Case 1 in which y is such that $y_3 < y < y_2$ and in Case 2 in which y is such that $y_1 < y < y_3$, we conclude:

$$\nu(E,A_2)-\nu(A_2) = \nu(E,A_2,A_3)-\nu(A_2,A_3). \tag{39}$$

From Equation (38) and Equation 39 it follows then that:

$$\nu(E,A_2)-\nu(A_2) = \nu(E,A_3)-\nu(A_3). \tag{40}$$

Therefore, it is true that for any event $E \in \mathcal{A}'$ for all events $F, G \in \mathcal{A}$ such that $\emptyset \not\subseteq F, G \not\subseteq E^c$:

$$\nu(E \cup G)-\nu(G) = \nu(E \cup F)-\nu(F). \tag{41}$$

Step 2. Let $A_1, A_2 \in \mathcal{A}$ be two disjoint events partitioning the event E and $A_3, A_4 \in \mathcal{A}$ two events partitioning E^c. Consider an action $f = (x_1A_1, x_2A_2, x_3A_3, x_4A_4)$

with outcomes $x_1, x_2, x_3, x_4 \in X$ such that $x_1 < x_3 < x_4 < x_2$. Suppose the conditional Choquet expectation of f satisfies the union-consistency on the partition E, E^c, i.e.:

$$\int u \circ f \; d\nu(\cdot \mid E^c) = x \quad \text{and} \quad \int u \circ f \; d\nu(\cdot \mid E) = x$$

implies

$$\int u \circ f \; d\nu(\cdot \mid \Omega) = x. \tag{42}$$

By computing the respective conditional Choquet integrals of f we have:

$$\int u \circ f \; d\nu(\cdot \mid E) \;=\; x_1 \left[1 - \nu(A_2 \mid E)\right] + x_2 \; \nu(A_2 \mid E) = x, \tag{43}$$

$$\int u \circ f \; d\nu(\cdot \mid E^c) \;=\; x_3 \left[1 - \nu(A_4 \mid E^c)\right] + x_4 \; \nu(A_4 \mid E^c) = x. \tag{44}$$

The unconditional Choquet integrals of f is:

$$\begin{aligned}\int u \circ f \; d\nu \;=\; & x_1 \left[1 - \nu(A_3, A_4, A_2)\right] + x_3 \left[\nu(A_3, A_4, A_2) - \nu(A_4, A_2)\right] \\ & + x_4 \left[\nu(A_4, A_2) - \nu(A_2)\right] + x_2 \; \nu(A_2).\end{aligned} \tag{45}$$

From Step 1 and Equation (43) we obtain the following equation:

$$x_1 \left[1 - \nu(A_3, A_4, A_2)\right] + x_4 \; \nu(A_4) = x \left[1 - \nu(A_4, A_3, A_2) + \nu(A_4)\right], \tag{46}$$

and thus:

$$x_1 \left[1 - \nu(A_3, A_4, A_2)\right] + x \left[\nu(A_4, A_3, A_2) - \nu(A_4)\right] + x_4 \; \nu(A_4) = x. \tag{47}$$

From Equation (45) and (47) we get:

$$x_2 \left[\nu(A_3, A_4, A_2) - \nu(A_3, A_4)\right] + x_3 \left[\nu(A_3, A_4) - \nu(A_4)\right] = x \left[\nu(A_3, A_4, A_2) - \nu(A_4)\right]. \tag{48}$$

and thus:

$$x_2 \frac{\left[\nu(A_3, A_4, A_2) - \nu(A_3, A_4)\right]}{\left[\nu(A_3, A_4, A_2) - \nu(A_4)\right]} + x_3 \frac{\left[\nu(A_4, A_2) - \nu(A_4)\right]}{\left[\nu(A_3, A_4, A_2) - \nu(A_4)\right]} = x. \tag{49}$$

Recall, in Equation (44) we had:

$$x_3 \left[1 - \nu(A_4 \mid E^c)\right] + x_4 \; \nu(A_4 \mid E^c) = x, \tag{50}$$

Therefore, for any $x_3, x_4 \in X$ such that $x_3 < x_4$ we have:

$$\nu(A_4 \mid E^c) = \frac{\nu(A_4, A_2) - \nu(A_4)}{\nu(A_3, A_4, A_2) - \nu(A_4)}, \tag{51}$$

and

$$1 - \nu(A_4 \mid E^c) = \frac{\nu(A_3, A_4, A_2) - \nu(A_3, A_4)}{\nu(A_3, A_4, A_2) - \nu(A_4)}. \tag{52}$$

Now, let us consider an action $g = (x_1 A_1, x_2 A_2, x_3 A_3, x_4 A_4)$ with outcomes x_1, x_2, x_3, $x_4 \in X$ such that $x_1 < x_4 < x_3 < x_2$. The same argument as above leads to the conclusion that:

$$1 - \nu(A_3 \mid E^c) = \frac{\nu(A_3, A_4, A_2) - \nu(A_3, A_4)}{\nu(A_3, A_4, A_2) - \nu(A_4)}. \tag{53}$$

After applying Step 1 to the partition A_4, A_4^c we get:

$$\nu(A_3, A_4, A_2) - \nu(A_3, A_2) = \nu(A_4, A_2) - \nu(A_2). \tag{54}$$

Thus, by Equation (51), (53) and (54) we have:

$$\nu(A_4 \mid E^c) = 1 - \nu(A_3 \mid E^c). \tag{55}$$

After applying Step 1 to the partition E, E^c we get:

$$\nu(A_4 \mid E^c) = \frac{\nu(A_4)}{1 + \nu(A_4) - \nu(A_4, E)}, \tag{56}$$

and

$$\nu(A_3 \mid E^c) = \frac{\nu(A_3)}{1 + \nu(A_3) - \nu(A_3, E)}. \tag{57}$$

Thus, by Equation (55), (56) and (57) we obtain:

$$\frac{\nu(A_4)}{1 + \nu(A_4) - \nu(A_4, E)} = \frac{1 - \nu(A_3, E)}{1 + \nu(A_3) - \nu(A_3, E)}, \tag{58}$$

From Step 1 we know that:

$$\nu(A_4, E) - \nu(A_4) = \nu(A_3, E) - \nu(A_3). \tag{59}$$

and therefore:

$$\begin{aligned} \nu(A_4) + \nu(A_4, E) &= 1, & (60) \\ \nu(A_4) + \nu(A_4^c) &= 1. & (61) \end{aligned}$$

Step 3. Let $A_1, A_2, A_3 \in \mathcal{A}$ be events partitioning the event E and Let $A_4, A_5 \in \mathcal{A}$ be events partitioning the complementary event E^c. By applying the argument from Step 1 when deriving the updating rule we obtain:

$$\nu(A_2, A_3 \mid E) - \nu(A_3 \mid E) = \frac{\nu(A_2, A_3)}{1 + \nu(A_2, A_3) - \nu(E^c, A_2, A_3)} - \frac{\nu(A_2)}{1 + \nu(A_2) - \nu(E^c, A_2)}.$$

From the property of the capacity ν derived in Step 1 we get:

$$\nu(A_2, A_3 \mid E) - \nu(A_3 \mid E) = \frac{\nu(A_2, A_3) - \nu(A_2)}{1 + \nu(A_1) - \nu(E^c, A_1)}. \tag{62}$$

Furthermore, from Step 2 we get:

$$\nu(A_2, A_3 \mid E) = \frac{\nu(A_2, A_3, A_4) - \nu(A_4)}{\nu(A_1, A_2, A_3, A_4) - \nu(A_4)}, \tag{63}$$

$$\nu(A_3 \mid E) = \frac{\nu(A_2, A_4) - \nu(A_4)}{\nu(A_1, A_2, A_3, A_4) - \nu(A_4)}. \tag{64}$$

Some computations yield:

$$\nu(A_2, A_3 \mid E) - \nu(A_3 \mid E) = \frac{\nu(A_2, A_3, A_4) - \nu(A_4)}{\nu(A_1, A_2, A_3, A_4) - \nu(A_2, A_4)}, \tag{65}$$

$$= \frac{\nu(A_2, A_3) - \nu(A_3)}{\nu(A_1, A_2, A_3, A_4) - \nu(A_4)}, \tag{66}$$

$$= \frac{\nu(A_2, A_3) - \nu(A_3)}{1 + \nu(A_1) - \nu(E^c, A_4)}. \tag{67}$$

and thus:

$$\nu(A_1, A_2, A_3, A_4) - \nu(A_4) = 1 + \nu(A_1) - \nu(E^c, A_4). \tag{68}$$

Again, from Step 1 and 2 we get the following equality:

$$\nu(A_1 \mid E) = \frac{\nu(A_1, A_4) - \nu(A_4)}{\nu(A_1, A_2, A_3, A_4) - \nu(A_4)} = \frac{\nu(A_1)}{1 + \nu(A_1) - \nu(E^c, A_4)}. \tag{69}$$

By Equation (68) the denominators are the same and thus:

$$\nu(A_1) = \nu(A_1, A_4) - \nu(A_4), \tag{70}$$

and the capacity ν is updated according to Bayes' rule, i.e.:

$$\nu(A_1 \mid E) = \frac{\nu(A_1)}{\nu(E)}. \tag{71}$$

Step 4. Fix an event $E \in \mathcal{A}'$ and let $A \in \mathcal{A}$ be an event such that $E \cap A \neq \emptyset$ and $E^c \cap A \neq \emptyset$. Suppose that:

$$\nu(A \mid E) = \alpha, \tag{72}$$

$$\nu(A \mid E^c) = \beta < \alpha. \tag{73}$$

Let x be an outcome for which Choquet conditional expectation of the action $f = x_A 0$ is equal to α, i.e.:

$$\begin{aligned} \int u \circ f \; d\nu(\cdot \mid E^c) &= x \; \nu(A \mid E^c), \\ &= \alpha. \end{aligned} \tag{74}$$

Now, consider an action $g = (x, E^c \cap A; 1, E \cap A; 0)$. Suppose the conditional Choquet expectation of g satisfies the union-consistency on the partition E, E^c, i.e.:

$$\int u \circ g \; d\nu(\cdot \mid E^c) = \alpha \quad \text{and} \quad \int u \circ g \; d\nu(\cdot \mid E) = \alpha$$

implies

$$\int u \circ g \; d\nu(\cdot \mid \Omega) = \alpha. \tag{75}$$

The unconditional Choquet expectation of g is:

$$\begin{aligned} \int u \circ g \; d\nu &= 1\Big[\nu(A) - \nu(A \cap E^c)\Big] + x\nu(A \cap E^c), \\ &= \alpha. \end{aligned} \tag{76}$$

From Step 3 we know that the updating rule is Bayes' rule:

$$\nu(A \mid E^c) = \frac{\nu(E^c \cap A)}{\nu(E^c)},$$

and thus:

$$x \; \nu(A \mid E^c) = \alpha \; \nu(E^c). \tag{77}$$

From Equation (76) and (77) we have:

$$\begin{aligned} \int u \circ g \; d\nu &= 1\Big[\nu(A) - \nu(A \cap E^c)\Big] + \alpha \; \nu(E^c), \\ &= \alpha. \end{aligned} \tag{78}$$

From Equation (77) and Step 2 we obtain:

$$\int u \circ g \; d\nu = 1\Big[\nu(A) - \nu(A \cap E^c)\Big] \;=\; \alpha(1 - \nu(E^c)),$$
$$= \; \alpha \; \nu(E). \tag{79}$$

Thus we have:

$$\frac{\nu(A) - \nu(A \cap E^c)}{\nu(E)} \;=\; \frac{\nu(A \cap E)}{\nu(E)} = x, \tag{80}$$

showing that E is N-unambiguous event, that is for any $A \in \mathcal{A}$ the capacity ν is additive separable:

$$\nu(A) \;=\; = \nu(A \cap E^c) + \nu(A \cap E). \tag{81}$$

□

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